

Split societies, elections, polls

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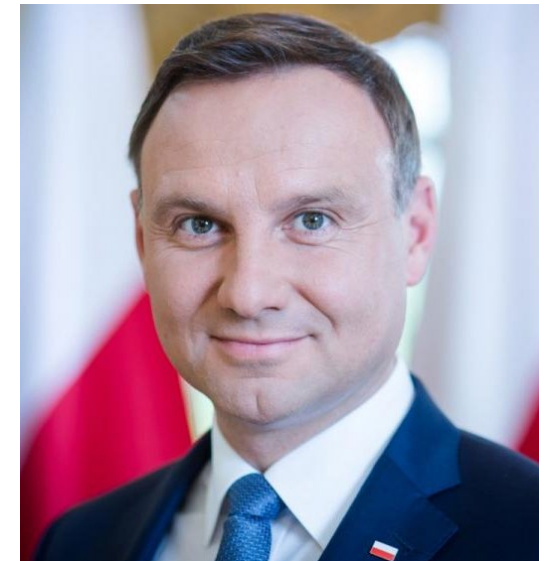
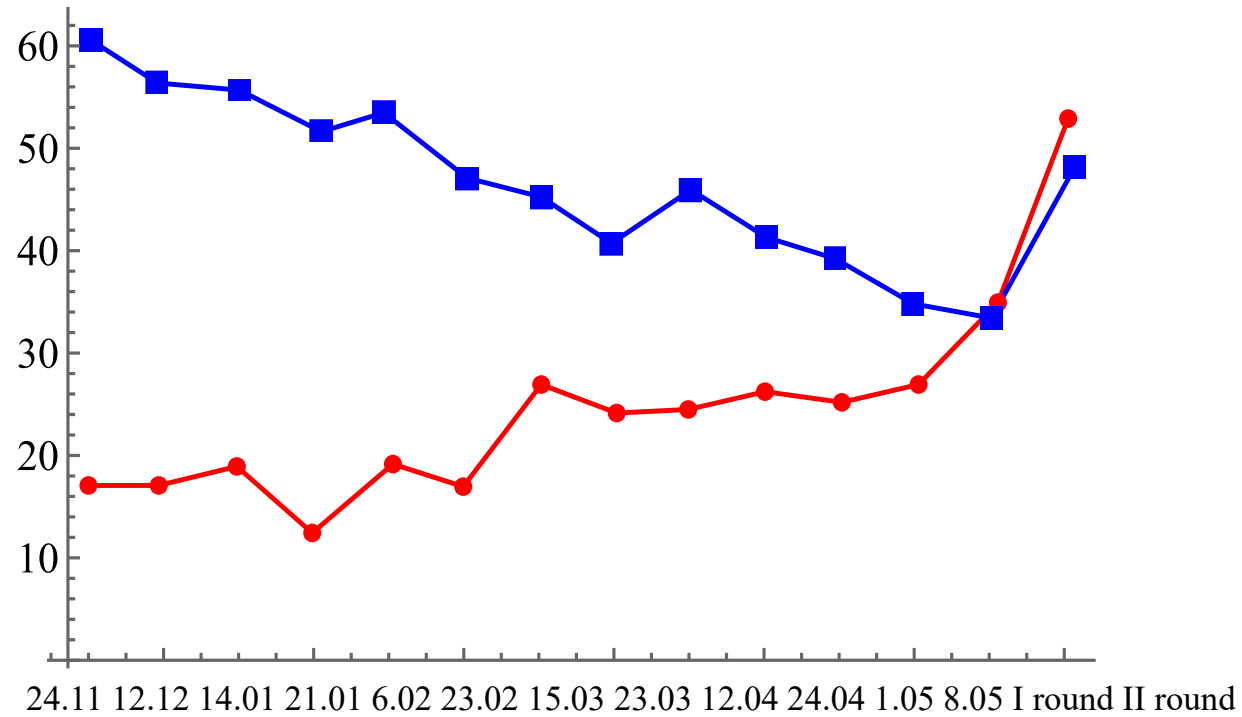


In many recent elections, votes tend to split 50/50

Komorowski-Duda 2015



48.5%



51.5%

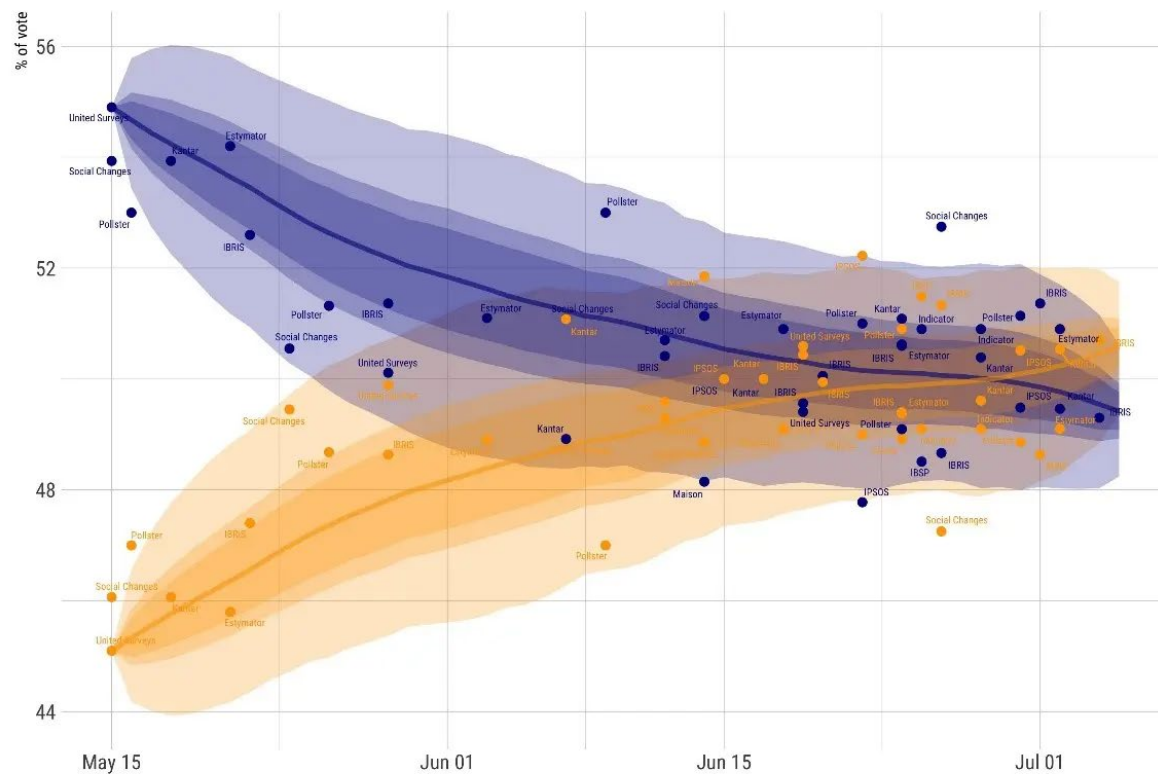
Duda – Trzaskowski 2020



51.03%

Polish presidential elections, round 2: trends

Data from Estymator, IBRIS, IBSP, Indicator, IPSOS, Kantar, Maison & Partners, Pollster, Social Changes, United Surveys



@BDStanley; benstanley.org

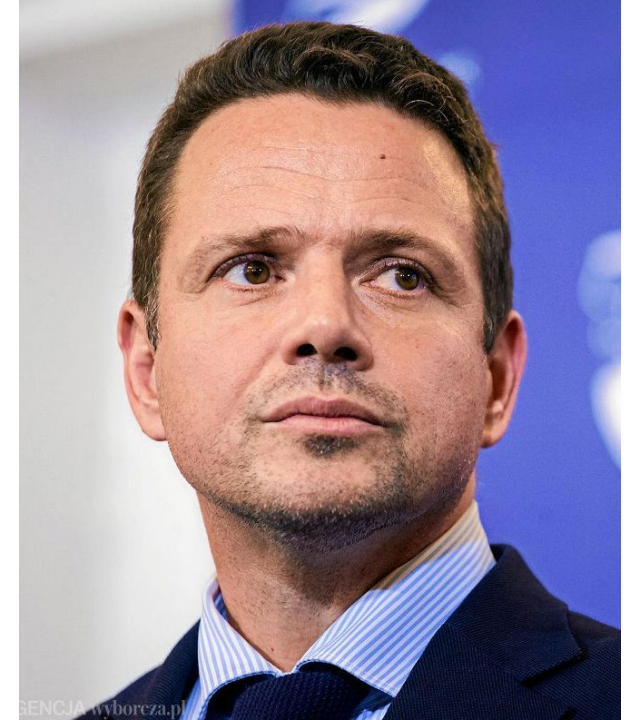
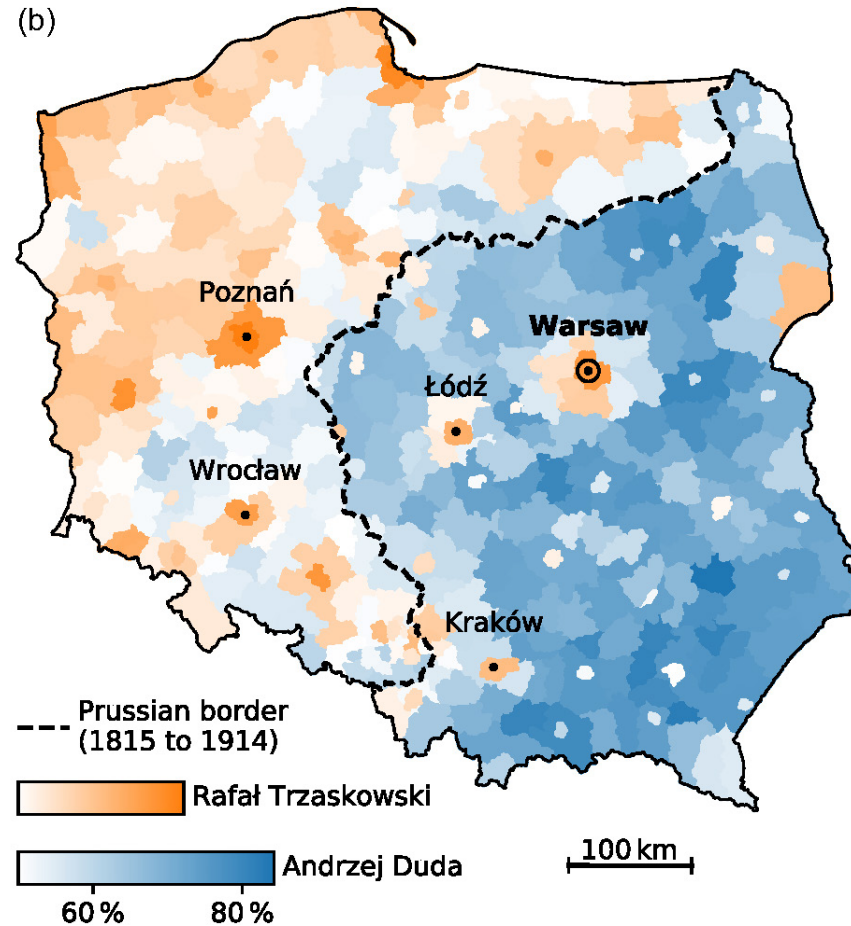


48.97%

Duda – Trzaskowski 2020



51.03%

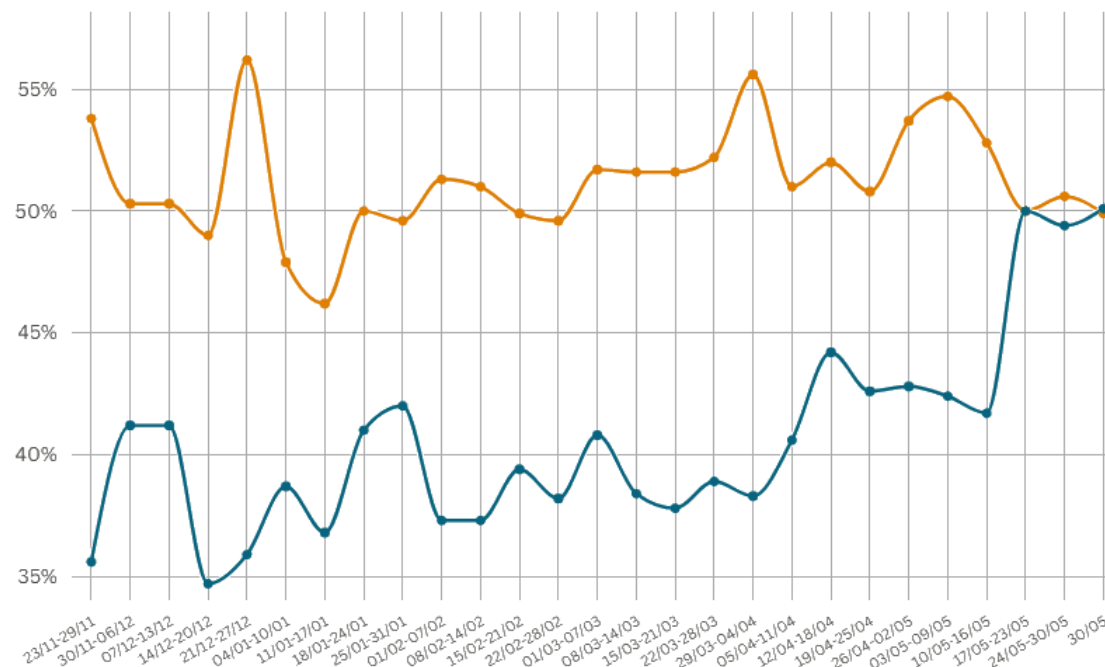


48.97%

Trzaskowski-Nawrocki 2025

Wybory 2025 – średnie poparcie w ujęciu tygodniowym (II tura)

Trzaskowski Nawrocki



Infografika: Tomasz Lebioda, Wyborcza.pl / BIQdata • Źródło: Ewybory.eu • Metodologia

wyborcza.pl BIQDATA.pl

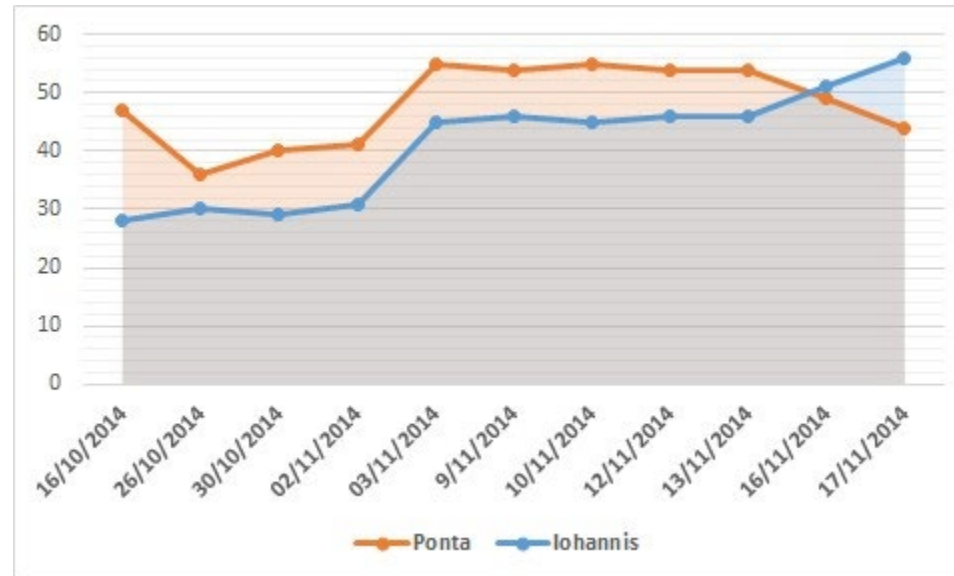


49.11%



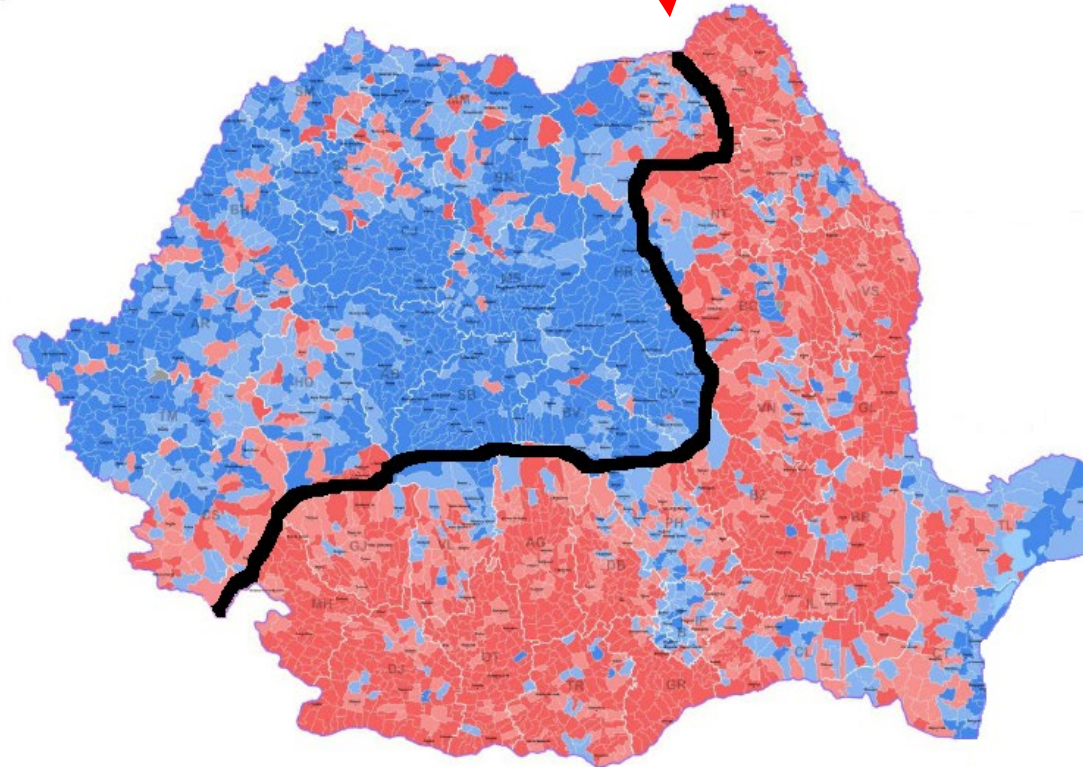
50.89%

Iohannis-Ponta 2014

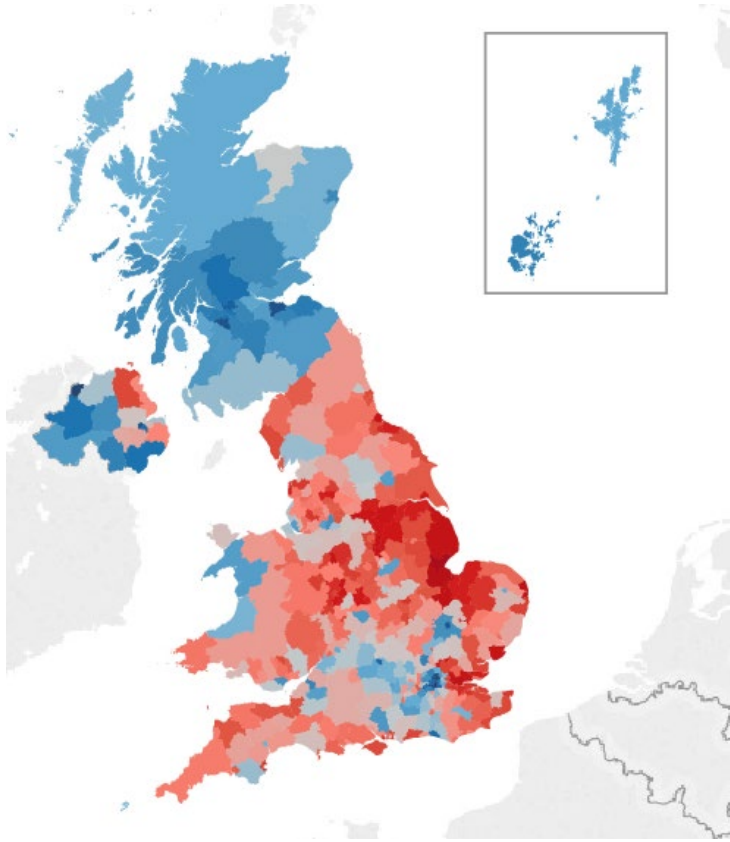


Iohannis-Ponta 2014

Hungarian border (before Treaty of Trianon)

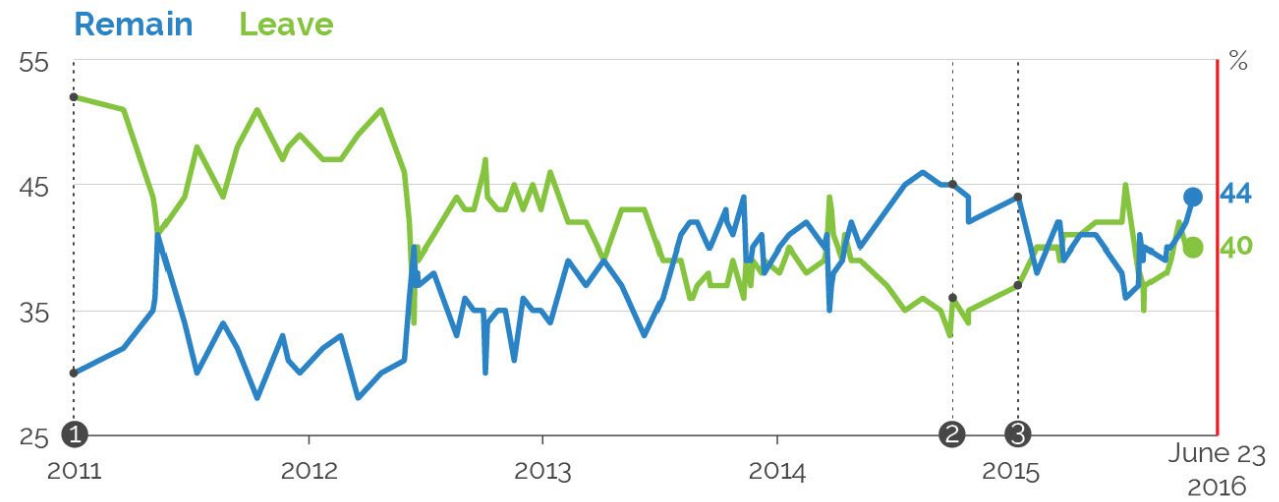


Brexit



52%

48%



USA

THE WALL STREET JOURNAL

WEDNESDAY, NOVEMBER 4, 2020 • VOL. CCLXXVI NO. 107

Swing States Are Tight

Pennsylvania, Michigan, Wisconsin not yet called amid high turnout

Economy, Virus and Race Split Electorate

By James McCreane
Ann-Cath Lee

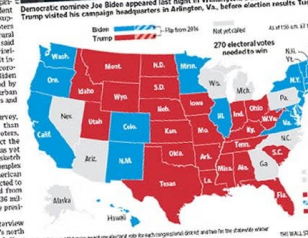
The portrait of America revealed in Thursday's presidential election was one of a deeply divided nation split by three major issues: the economy, the coronavirus pandemic and race. The results showed a clear divide between voters who supported Joe Biden and those who supported Donald Trump, with the latter leaning heavily towards the Republican Party.



Trump, Biden Bullish on Winning

President Trump and former Vice President Joe Biden both expressed confidence in their victory on Wednesday night.

Democratic nominee Joe Biden appeared last night in Wilmington, Del., and President Trump visited his campaign headquarters in Arlington, Va., before election results Tuesday.



ELECTION 2020

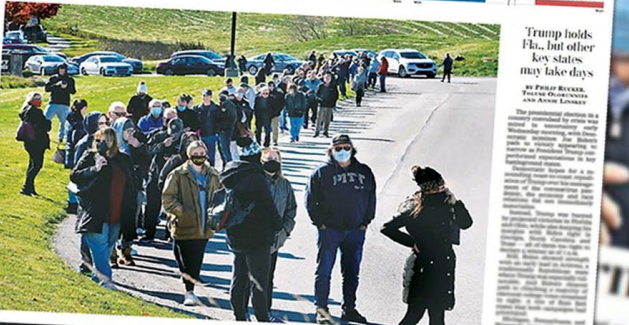
- Go to WSJ.com for the latest results.
- Have Democrats' right to vote snuffed? — A5
- Vote after the vote procedures — A7

The Washington Post

WEDNESDAY, NOVEMBER 4, 2020

A nation divided

TIGHT RACE, HIGH TURNOUT REFLECT PARTISAN SPLIT OVER CRISES



Trump holds fl., but other key states may take days

BY FRANK RICHARD, FRANK BRUNSWICK AND ANNE LISWY



238 BIDEN 50.3% 69,453,553
270 TO WIN
33 LEADING **54 LEADING**
66,547,083 **48.2%** **TRUMP 213**

IN A TIME OF SUSPICION, AMERICANS PUT TRUST IN THE VOTE

As the nation's presidential election unfolded, Americans showed a surprising level of trust in the voting process. Despite concerns about mail-in ballots and election integrity, the vast majority of voters reported confidence in the system.

DIVIDED STATES OF AMERICA



Early mail-in ballots may turn US blue

WASHINGTON: A record number of mail-in ballots were cast before Election Day, totaling more than 100 million. Analysts predict that this surge in mail-in voting could significantly impact the final results, potentially tipping the balance in favor of Joe Biden.

THE IRISH TIMES

WEDNESDAY, NOVEMBER 4, 2020

ELECTION 2020

REPORTS, ANALYSIS & COMMENTARY

BIDEN EDGING TOWARDS VICTORY

America divided

Biden predicts he will win the race once all votes are counted. Trump declares premature victory and launches legal actions. Disappointing night for Democratic congressional races.



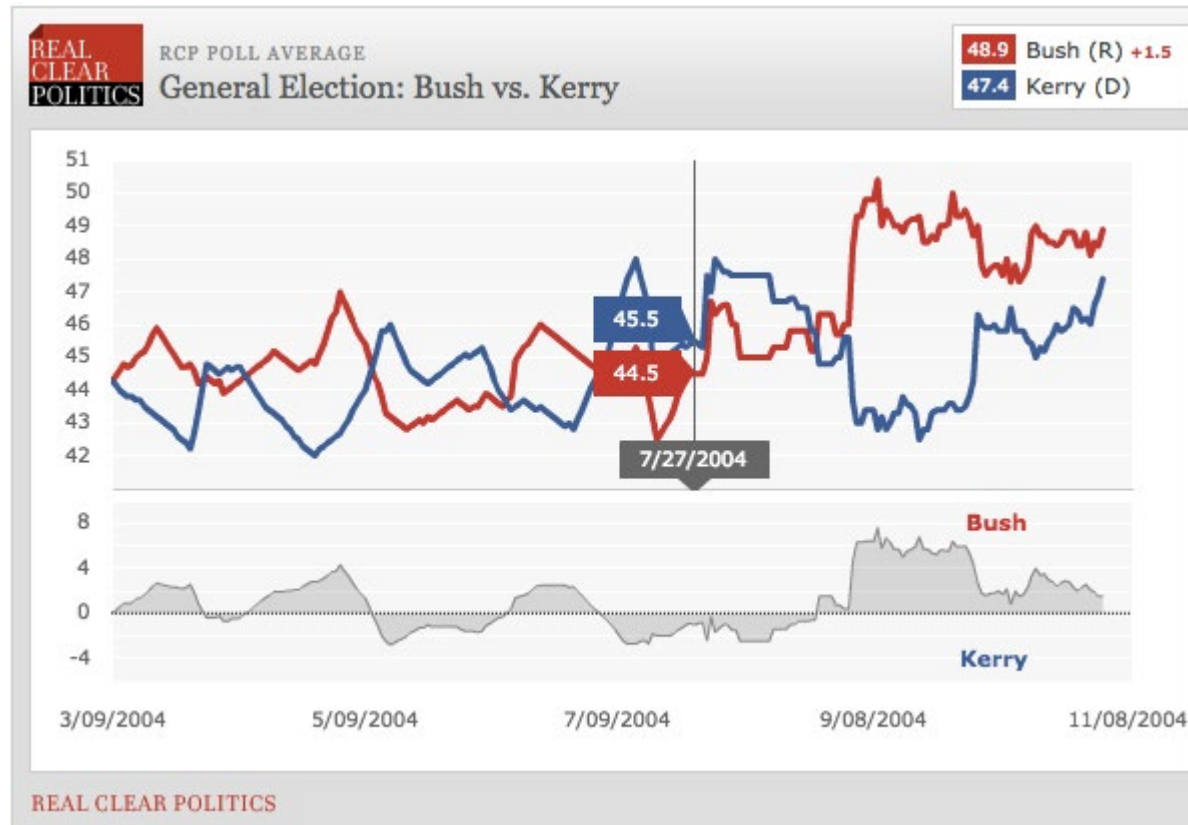
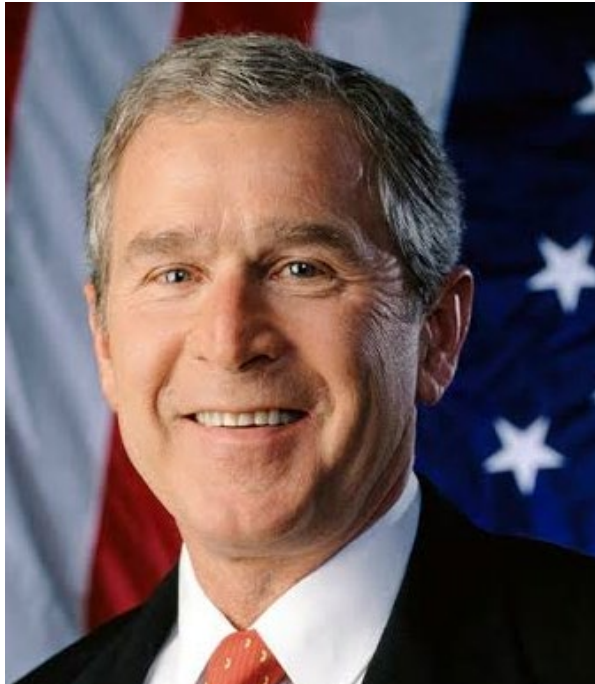
Joe Biden 224

Donald Trump 213

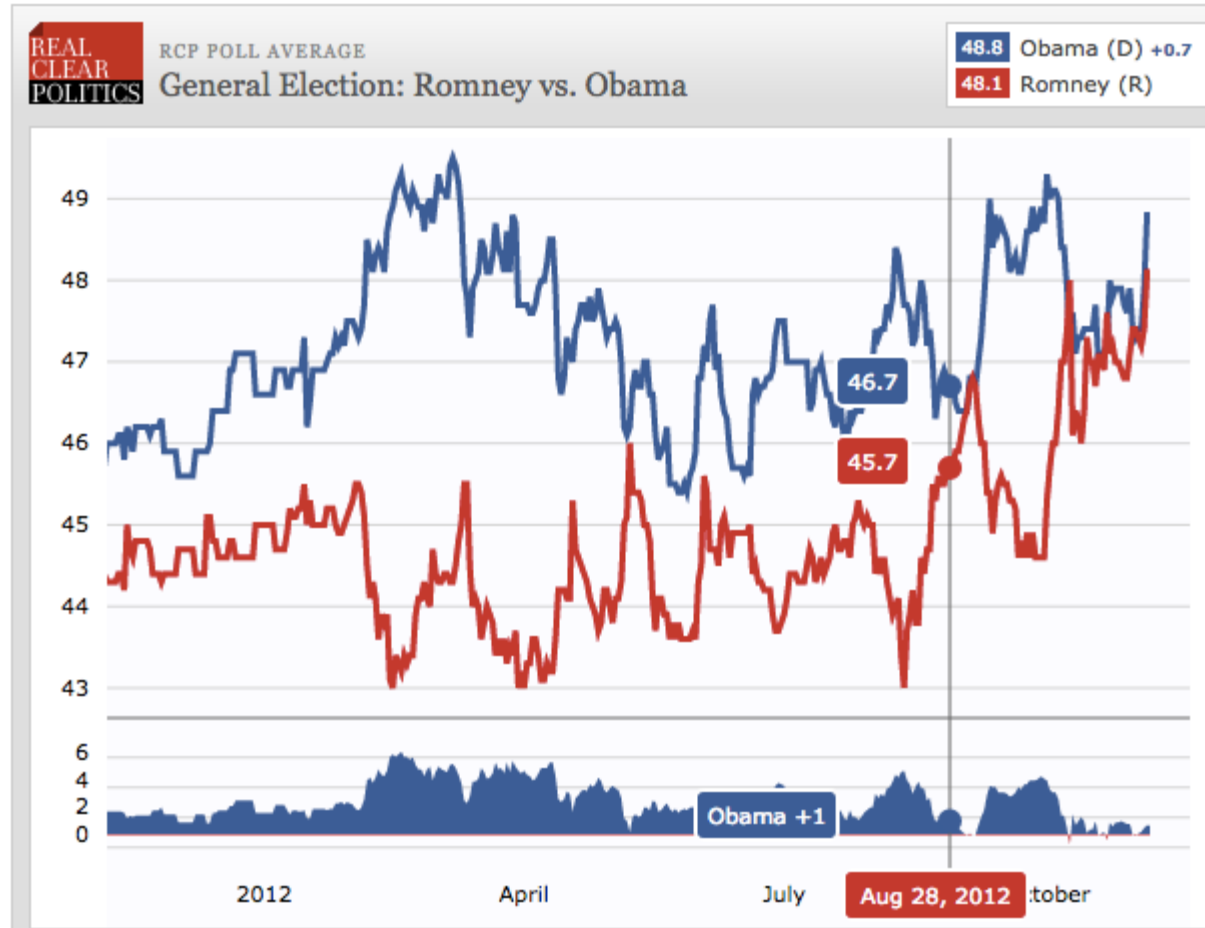
However Prevails, a Nation Remains Divided

The night of the election was marked by a sense of division and uncertainty. While Biden's lead in the electoral college was clear, the results in key swing states remained uncertain, leading to a tense and polarized atmosphere.

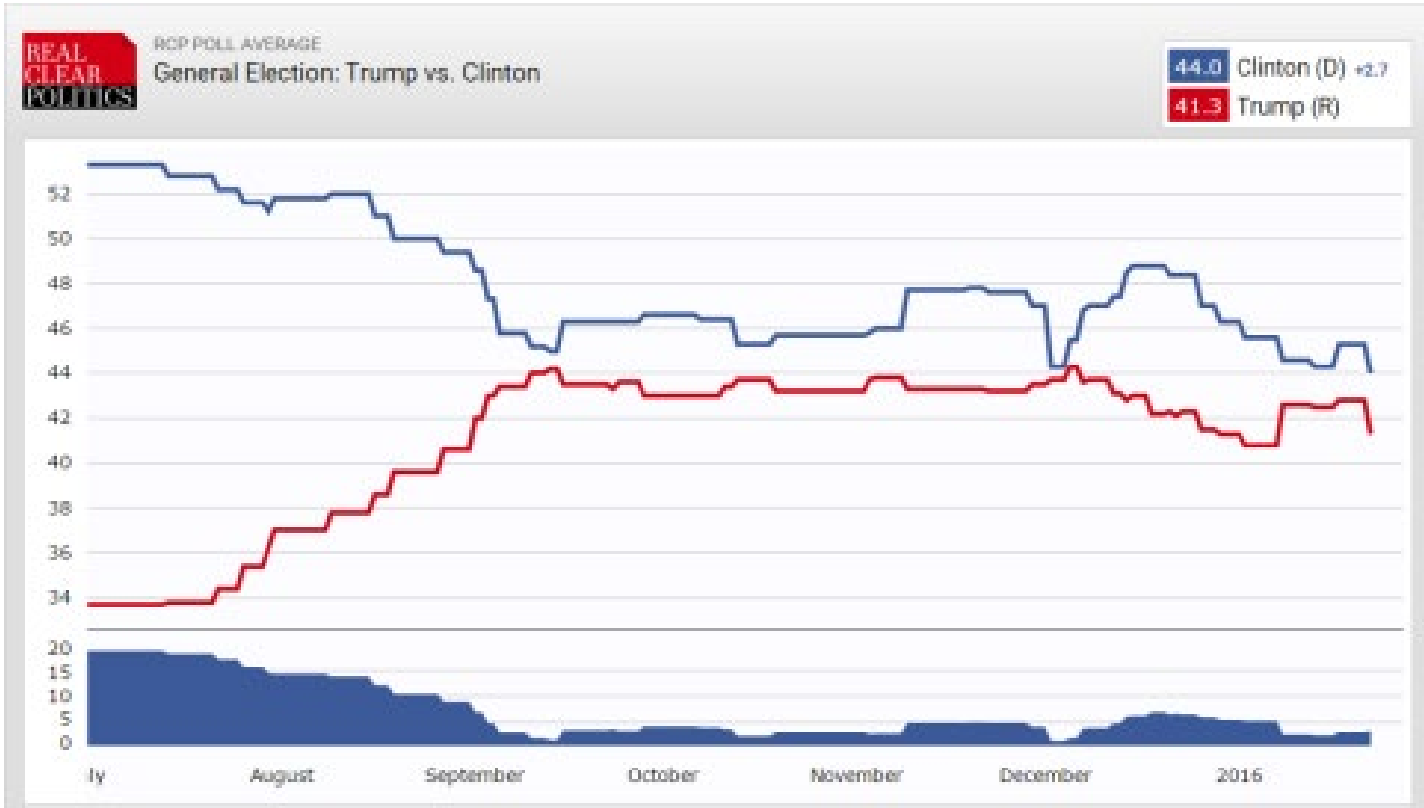
Bush-Kerry 2004



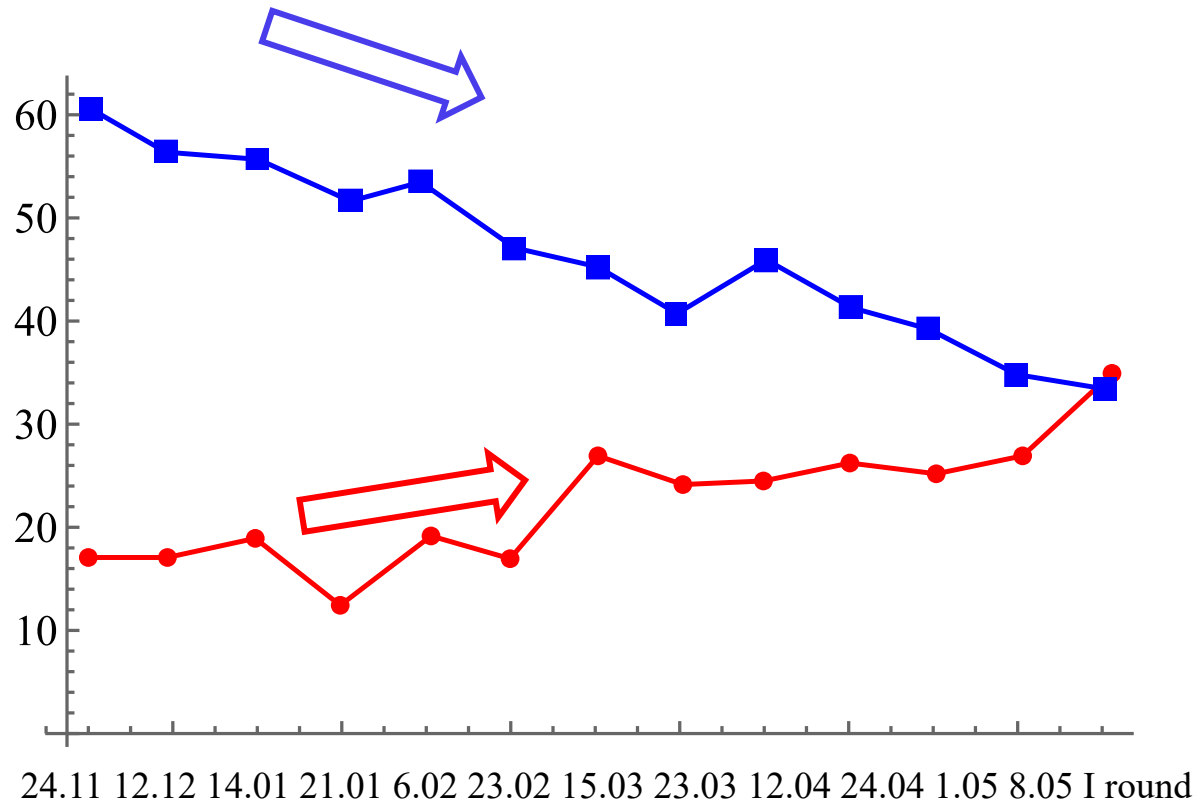
Obama-Romney 2012



Trump-Clinton 2016



Attractor of the dynamics?



Komorowski-Duda 2015



But how the voters know how to stop exactly at 50/50?

Do polls matter?

NA KOGO ZAGŁOSUJE PAN/PANI
W DRUGIEJ TURZE WYBORÓW
PREZYDENCKICH?

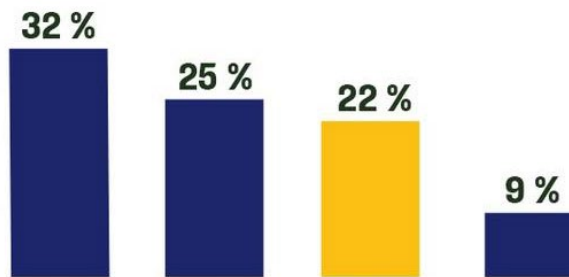
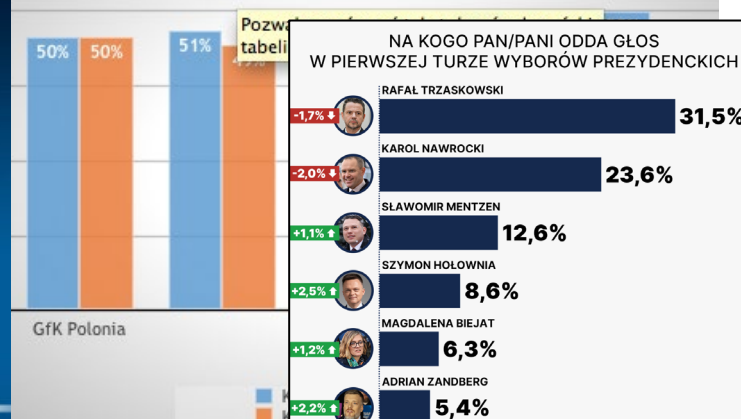


RAFAŁ TRZASKOWSKI
KAROL NAWROCKI

Źródło: Opinia24 dla "Faktów" TVN i TVN24



Sondaże przed II turą wyborów
prezydenckich w 2010 roku

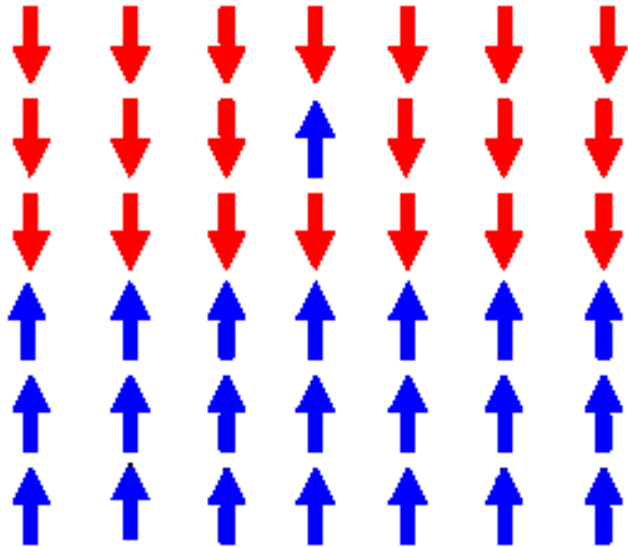


Źródło: IBIS dla Polskiego Radia 24

tvn24

Contrarians

The model



$$s_i = \pm 1, \quad i \in \{0, 1, \dots, N-1\}$$

total Hamiltonian:

$$\mathcal{H} = \sum_i \mathcal{H}_i = -\sum_{i,j} J_{ij} s_i s_j + N \epsilon \bar{s}^2$$

J_{ij} - measures the influence of voter j on voter i

ϵ - sensitivity of a voter to opinion polls

Ising-like Hamiltonian:

$$\mathcal{H}_i = s_i \left(-\sum_j J_{ij} s_j + \epsilon \bar{s} \right)$$

local: sharing the opinion with the people we connect to

polls: mean opinion of the society

$$\bar{s} = \frac{1}{N} \sum_{i=1}^N s_i$$

global: coupling to the polls (dislike of a general opinion)

assume uniform couplings between neighbors: $J_{ij} = 1/2$

(fixing the energy scale)

Glauber dynamics

(Monte Carlo)



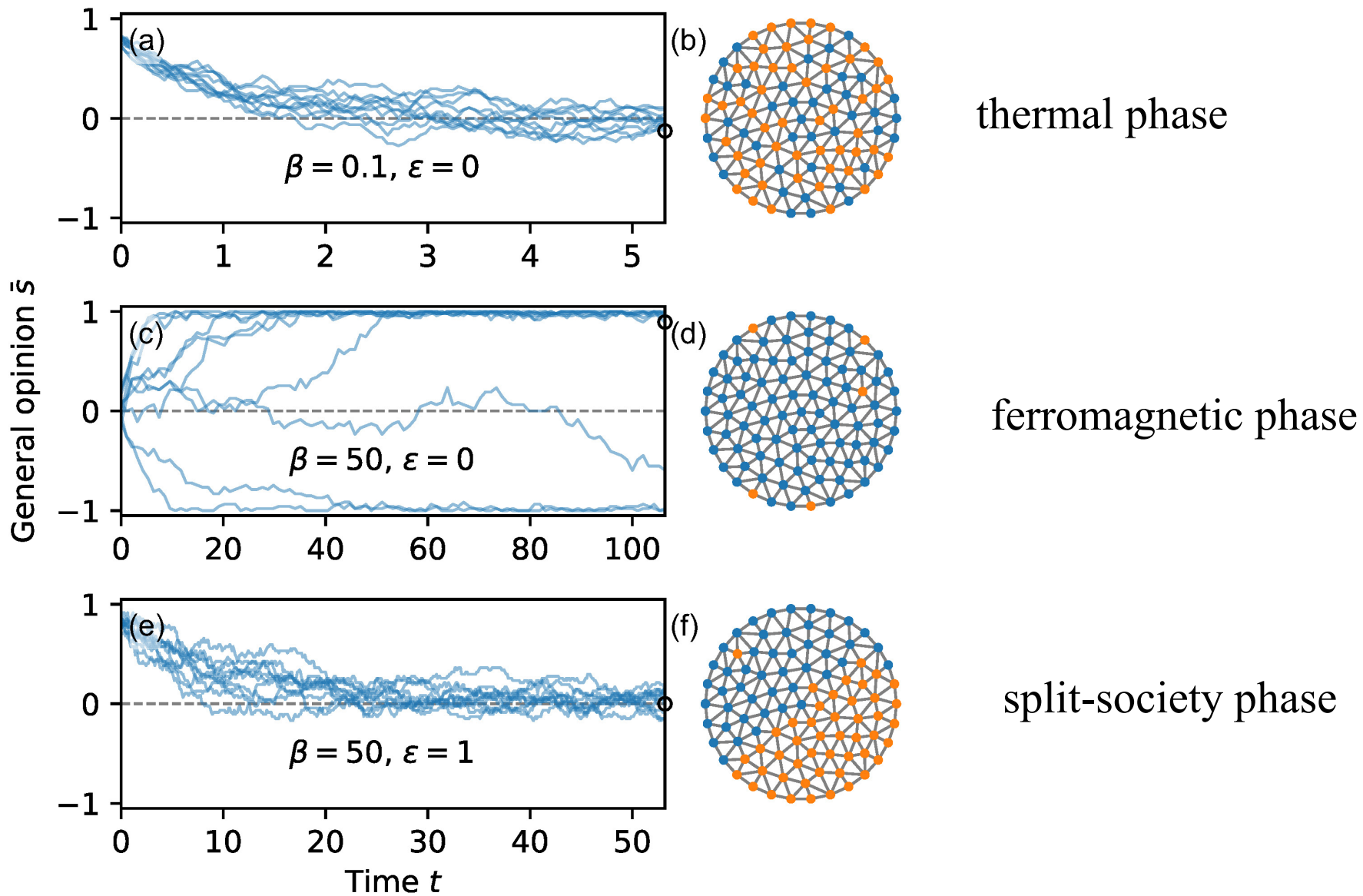
Probability of spin flip: $S_i \rightarrow -S_i$

$$P_{-i} = \frac{e^{-\beta\Delta E_i}}{1 + e^{-\beta\Delta E_i}},$$

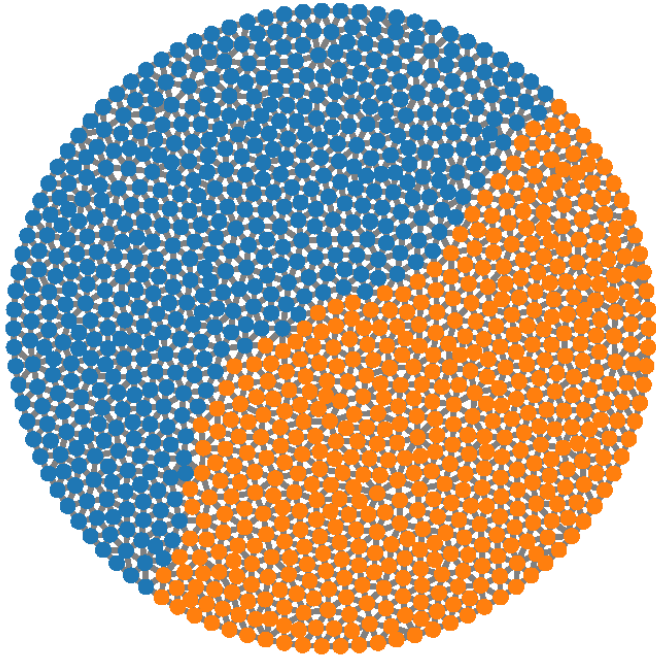
change in energy if
the spin were to flip.

β^{-1} - temperature; sets the amount of randomness (individual autonomy, “free will”, noise, imperfect information)

Evolution



Characterizing the phases



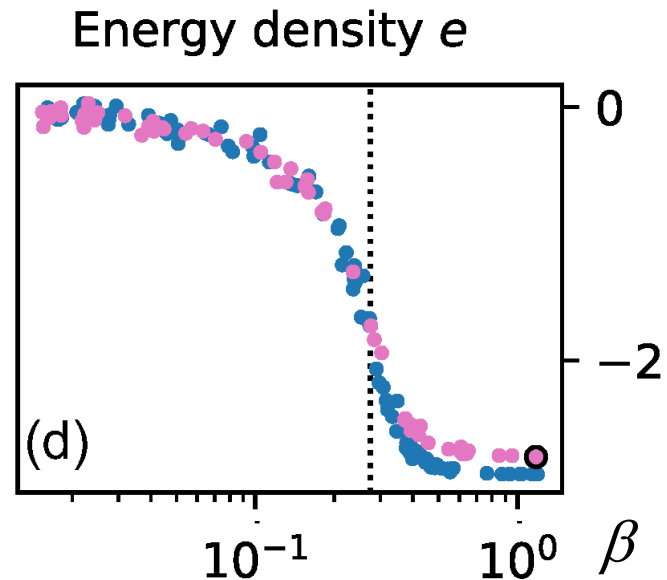
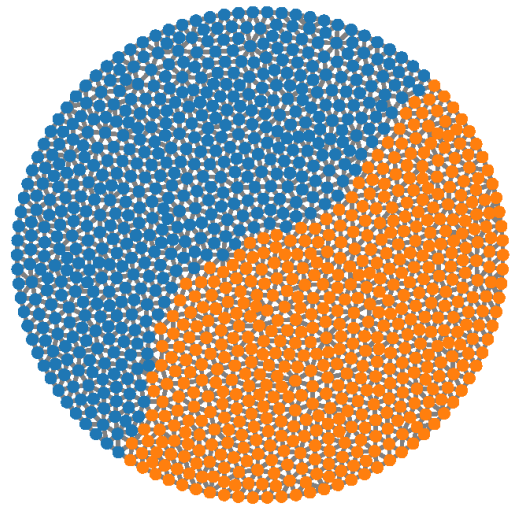
clustering coefficient $c = \frac{1}{zN} \sum_{i,j} \frac{1+s_i s_j}{2}$

average fraction of voter's neighbors that share their opinion

energy density

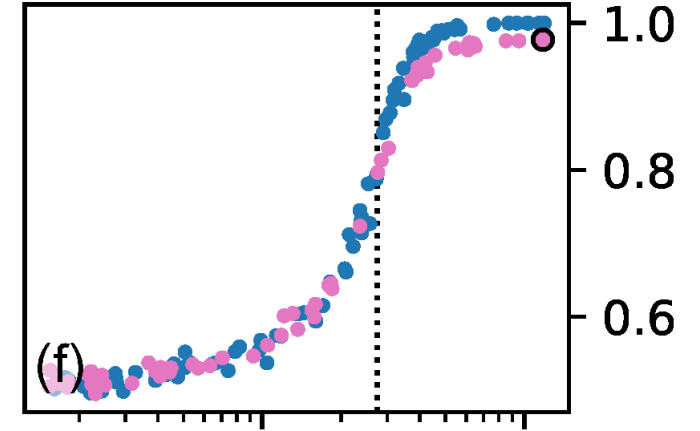
$$e = -\frac{1}{N} \sum_{i,j} J_{ij} s_i s_j + \varepsilon \bar{s}^2$$

Phases

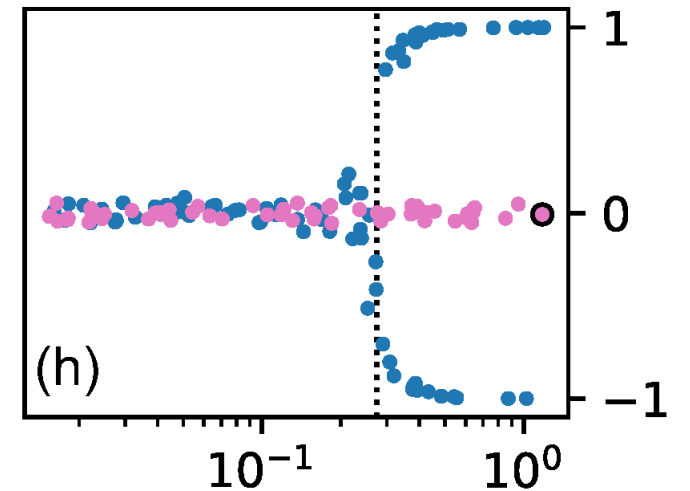


• $\epsilon = 0$
• $\epsilon = 1$

Clustering coefficient c



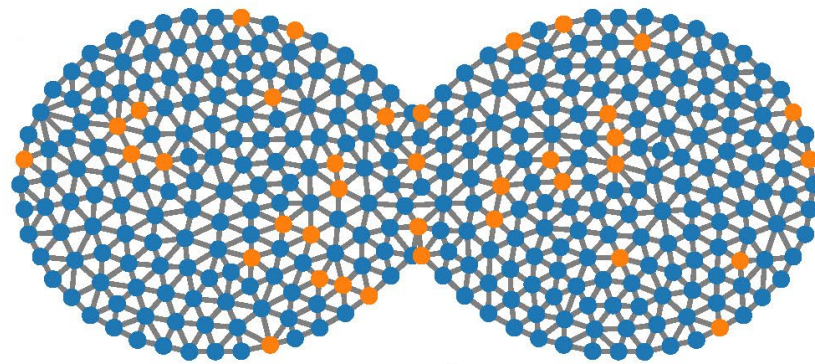
General opinion \bar{s}



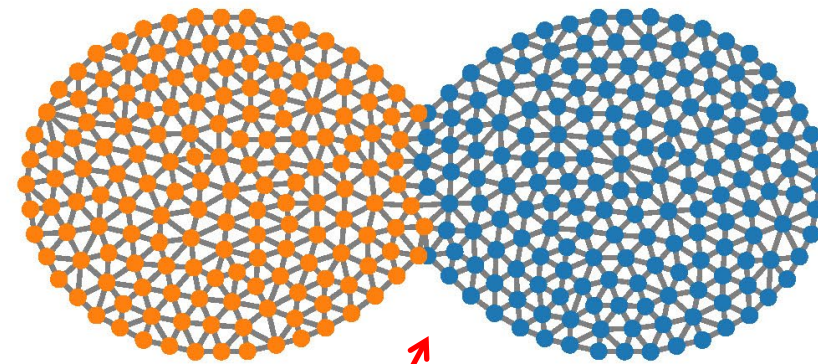
Inverse temperature β

- for $\epsilon=0$, at β_c , the system develops a spontaneous magnetization
- an ordered phase then emerges, with general opinion taking one of two opposed, nonzero values ($\bar{s} \approx \pm 1$)
- for $\epsilon>1$, at β_c , two seemingly ferromagnetic domains appear, at the energetic cost of maintaining an interface between them
- energy density and the clustering coefficient are barely affected by ϵ but \bar{s} remains small at any temperature

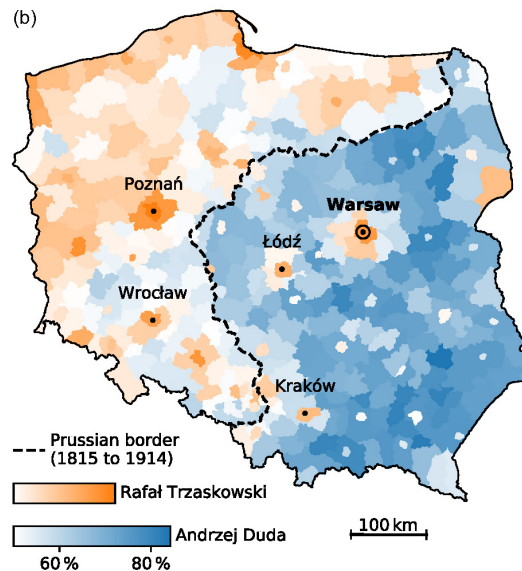
Peanut-shaped Poland



$t=0$



$t=150$



social rift: voters are better connected to their own side of the network than to the opposite

interface between domains tends to align with geographic or historical boundaries, where neighbor coupling is weaker

Cluster model

- assume that electorates form two clusters, with N_+ and $N_- = N - N_+$ voters, respectively
- energy cost of the interface is proportional to the perimeter, $L(N_+)$

$$E = \left(2\alpha \sqrt{\pi N n_+ (1 - n_+)} + N \varepsilon \bar{s}^2 \right)$$

no. of links to opposing voters ($\alpha=2$ for triangular lattice)

$$n_{\pm} = \frac{N_{\pm}}{N}$$

$$\bar{s} = n_+ - n_-$$

$$n_+ = (1 + \bar{s})/2$$

energy density:

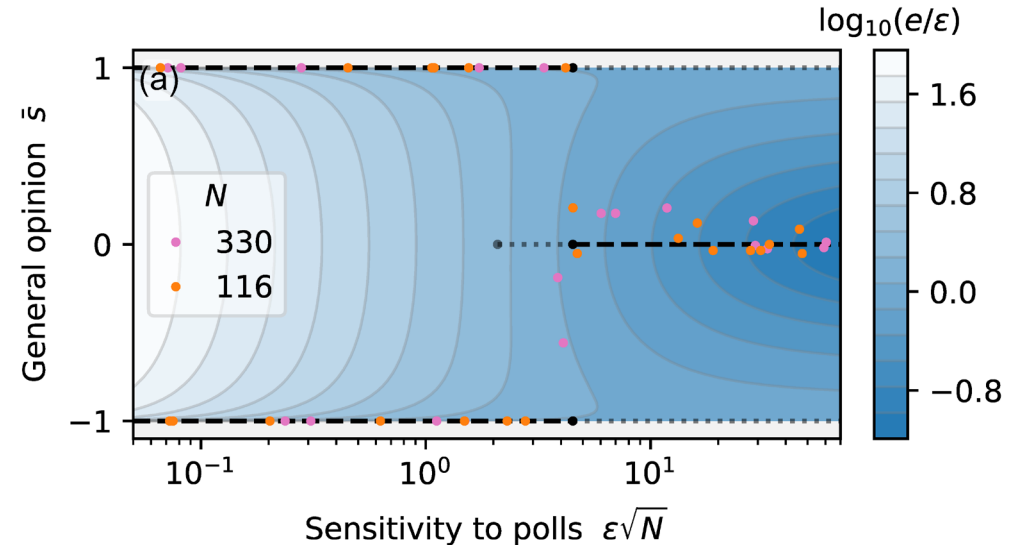
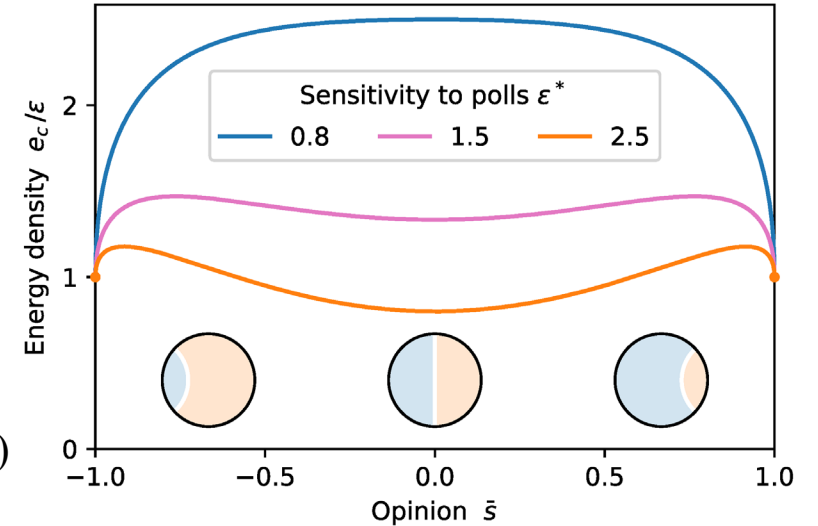
$$e = \sqrt{\frac{\pi}{N}} \left(\alpha \sqrt{1 - \bar{s}^2} + \varepsilon^* \bar{s}^2 \right)$$

$$\varepsilon^* = \varepsilon \sqrt{N/\pi}$$

- local minimum at $\bar{s} = 0$ for $\varepsilon^* > \alpha/2$
- global minimum for $\varepsilon^* > \alpha$

in terms of N:

$$N > \frac{4\pi}{\varepsilon^2}$$



Susceptibility

External field H:

$$e = \sqrt{\frac{\pi}{N}} \left(\alpha \sqrt{1 - \bar{s}^2} + \varepsilon^* \bar{s}^2 \right) - H \bar{s} \approx \alpha \sqrt{\frac{\pi}{N}} - H \bar{s} + \left(-\frac{\alpha}{2} + \varepsilon^* \right) \sqrt{\frac{\pi}{N}} \bar{s}^2 + \dots$$

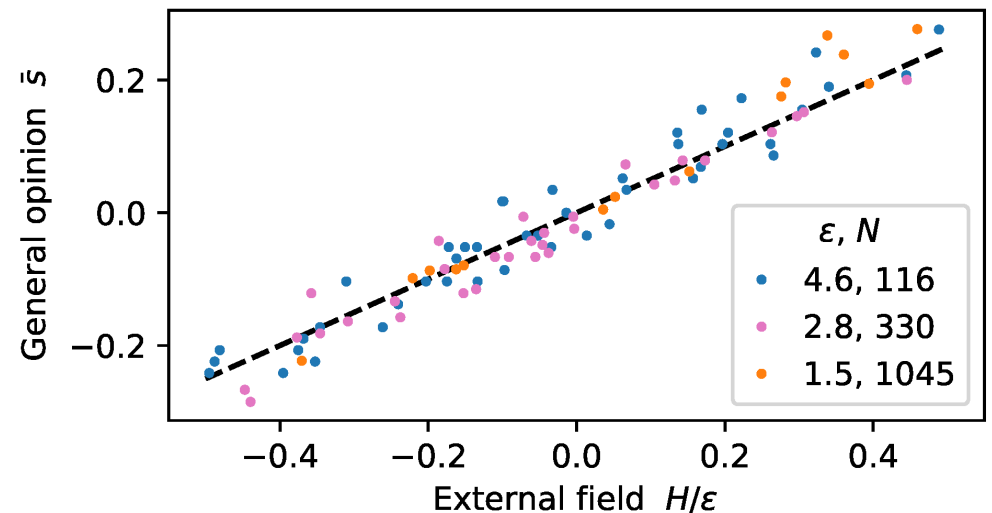
Calculate \bar{s} which minimizes the energy and differentiate with respect to H

$$\chi = \frac{\partial \bar{s}_{\min}}{\partial H} = \frac{1}{2\varepsilon \left(1 - \frac{\alpha}{2\varepsilon^*} \right)} \xrightarrow{\varepsilon \gg 1} \frac{1}{2\varepsilon}$$

Diverges when split society phase disappears ($\varepsilon^* \rightarrow \alpha/2$)

Split-society phase:

- ordered phase similar to a ferromagnetic one, where neighbor interactions dominate thermal fluctuations
- susceptibility remains finite, resembling a paramagnetic rather than a classical ferromagnetic phase.



Active voters

(work in progress)

S. Rakowski, P. S. “Public opinion dynamics in the active voter model”,
APS March Meeting, March 16–21, 2025, Anaheim, USA

watching poll
results



my candidate
is losing...



my candidate
is winning!



I need to act!



I relax...
He will surely win!



Non-reciprocal relations

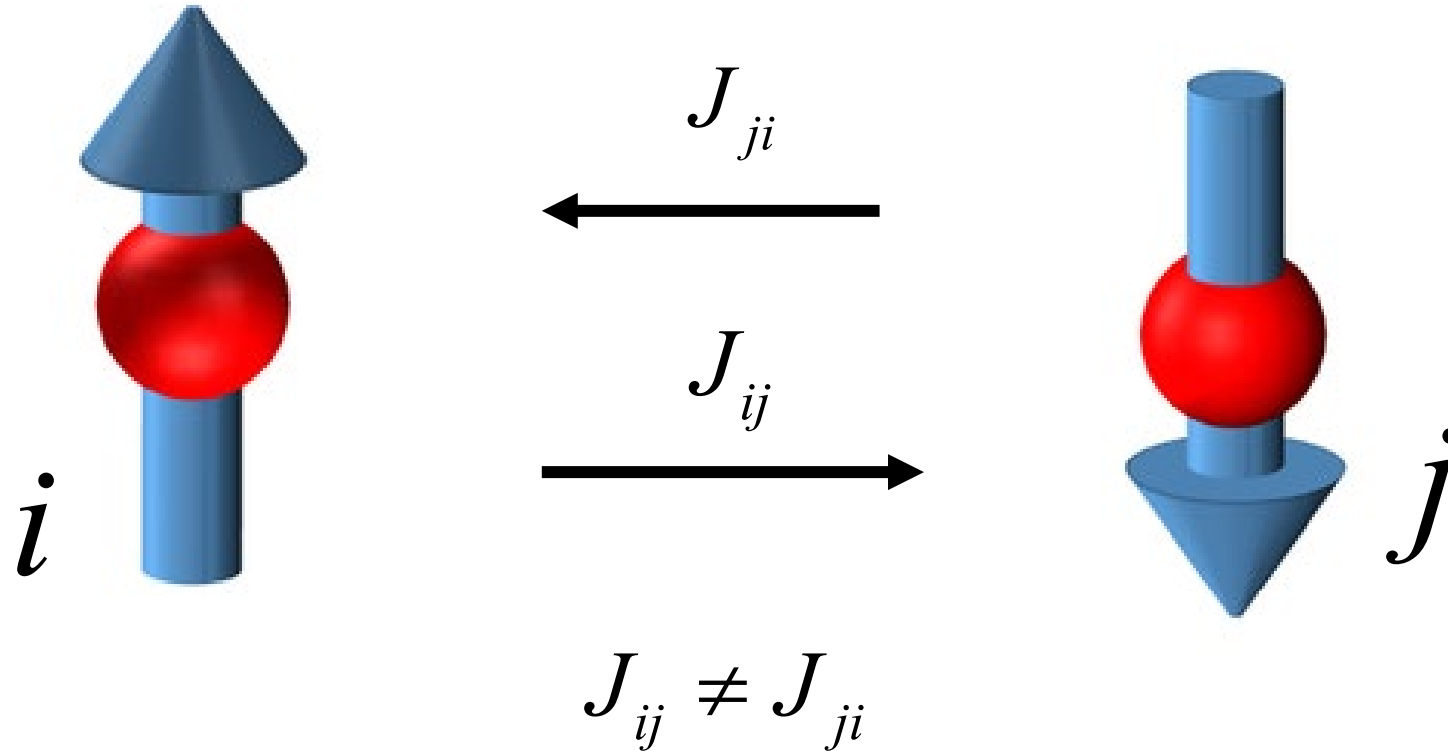


more active



less active

Non-reciprocal Ising model



- Non-reciprocal physics: the third law of dynamics broken $F_{A \rightarrow B} \neq -F_{B \rightarrow A}$
- Hamiltonian does not exist
- No detailed balance

Active voter model

my candidate
is losing...



J_{ji} decreases



J_{ij} increases



my candidate
is winning...

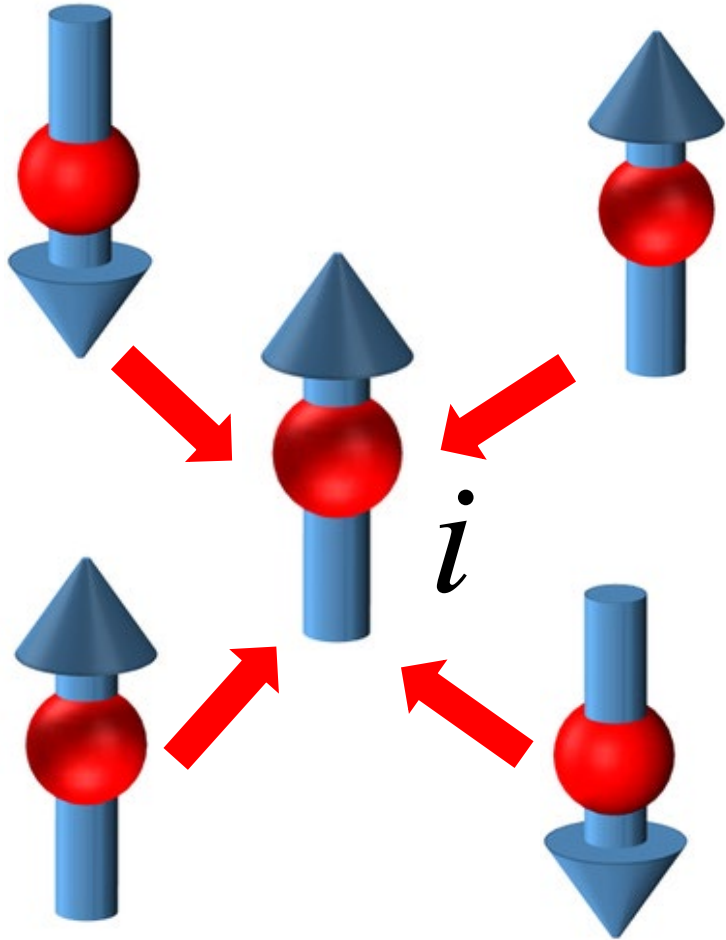


- the couplings between the spins evolve as a result of the polls
- supporters of the losing candidate increase their activity (and hence J_{ij} couplings)
- supporters of the winning candidate get lazy (decrease J_{ji} couplings)

Convincing (spin flipping)



Local field and Glauber dynamics



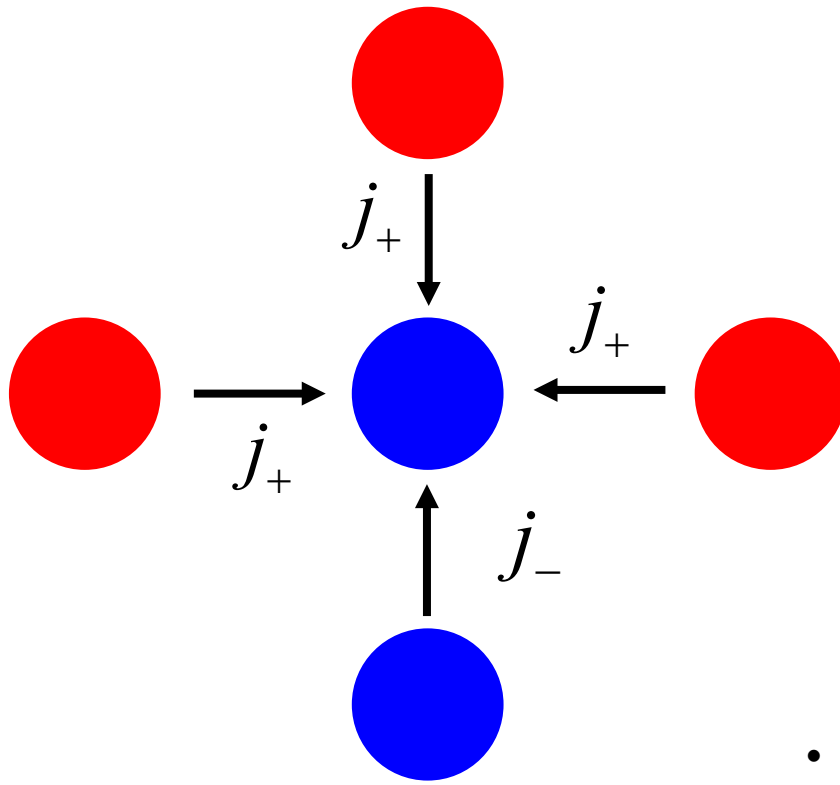
Local field:

$$h_i = \sum_j J_{ij} s_j$$

Probability of spin flip: $s_i \rightarrow -s_i$

$$p_{-i} = \frac{e^{-2\beta h_i s_i}}{1 + e^{-2\beta h_i s_i}} = \frac{1}{1 + e^{2\beta h_i s_i}}$$

The model



- assume there are only two weights—one for voters supporting Candidate A, and one for those supporting Candidate B.

$$J_{ij} = \begin{cases} j_+ & \text{if } s_j = 1 \\ j_- & \text{if } s_j = -1 \end{cases}$$

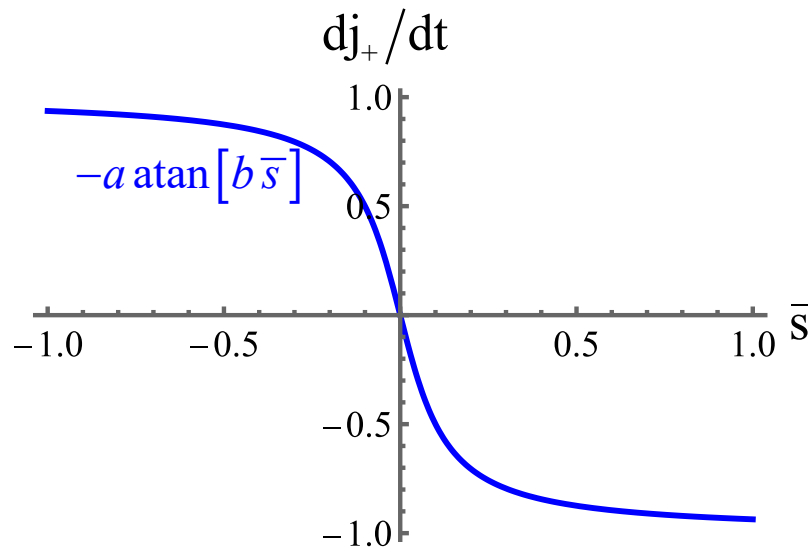
- evolve the weights in response to the polls

$$\partial_t j_+ = -a \operatorname{atan}[b \bar{s}] - c(j_+ - 1/2)$$

$$\partial_t j_- = -a \operatorname{atan}[-b \bar{s}] - c(j_- - 1/2)$$

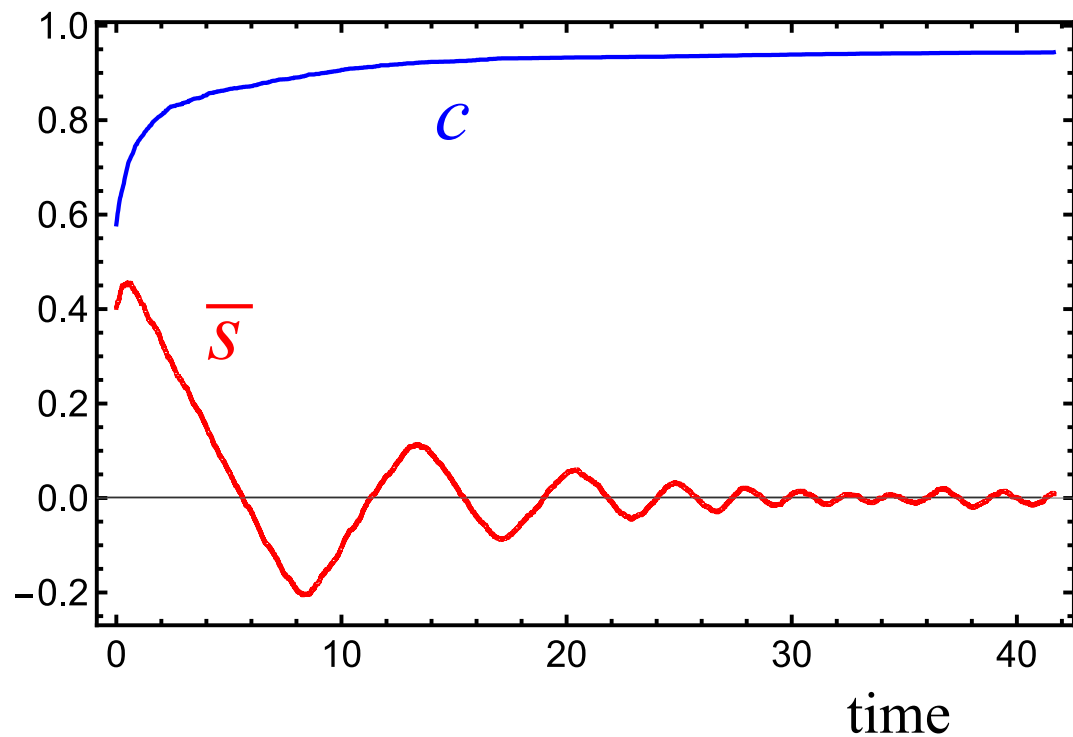
reaction to polls

drift back to a moderate engagement level

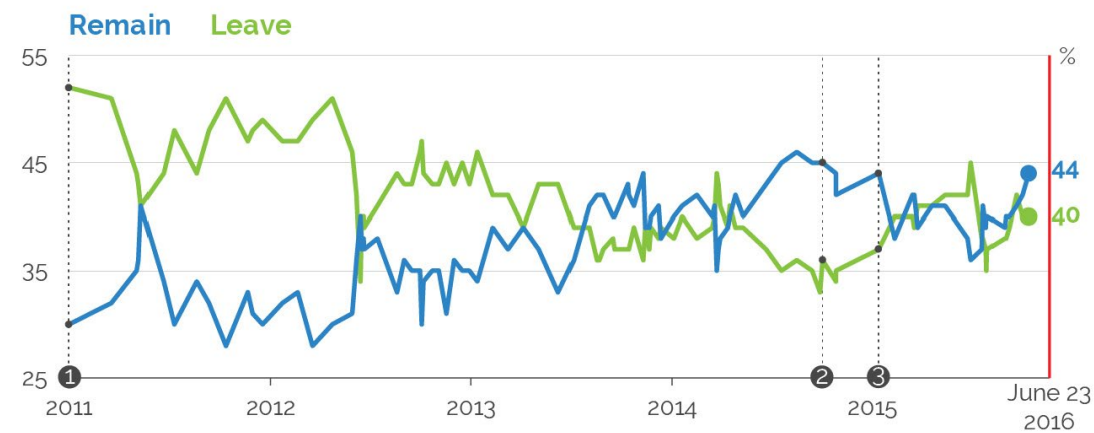


Damped oscillations

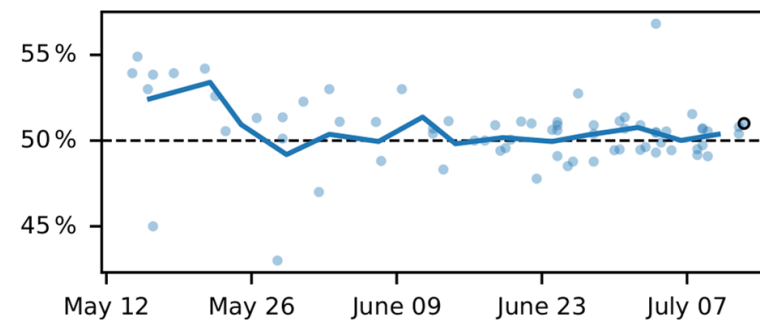
Average opinion (\bar{s}) and clustering coefficient (c) vs time



Not unlike:



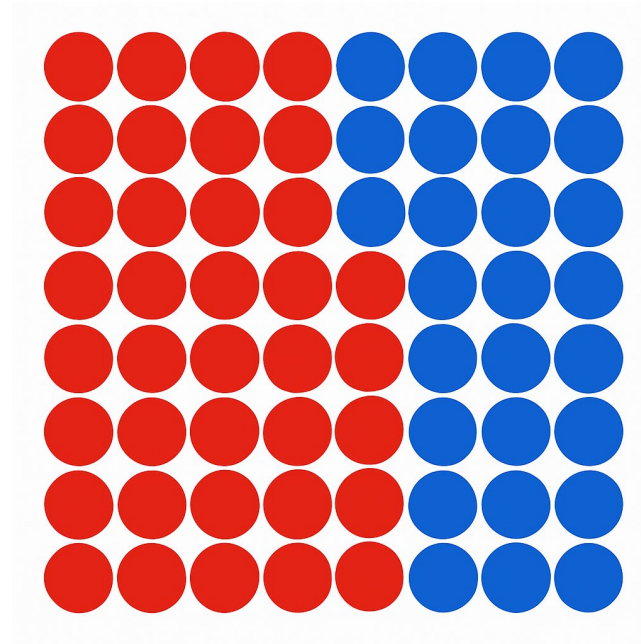
(Brexit polls)



(Duda/Trzaskowski)

Undecided voters

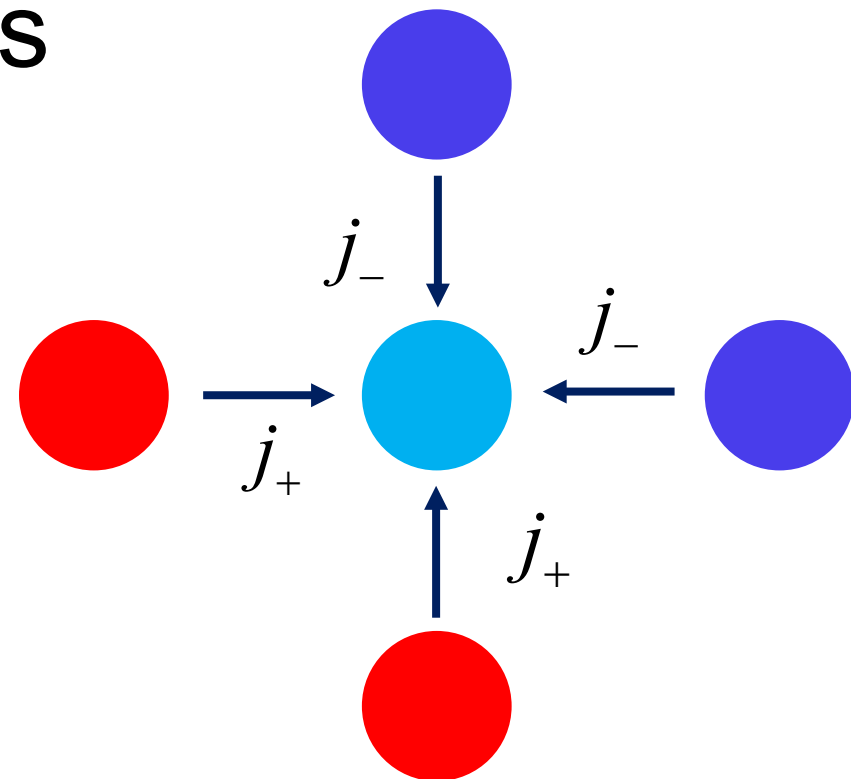
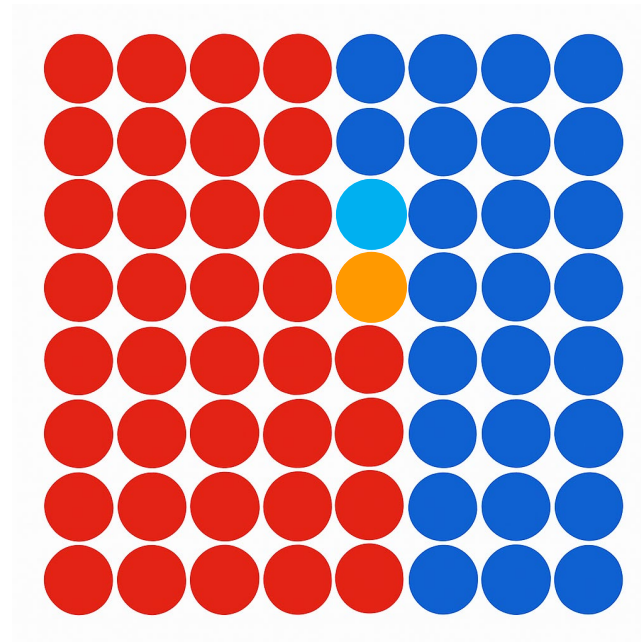
(“kinks”)



- look at the neighbourhood of 50% – 50%
- the pattern consists of two large clusters with the dynamics reduced to spin flips along the perimeter of these clusters
- along the perimeter, the spins with the highest probability of flipping are those which have two ‘+’ and two ‘-’ partners – **undecided voters**



Undecided voters



$$n_{\pm} = \frac{N_{\pm}}{N}$$

$$\bar{s} = n_+ - n_- = 2n_+ - 1$$

$$p_i = \frac{1}{1 + e^{2\beta h_i s_i}}$$

$$\partial_t n_+ = \frac{N_k}{N} (p_{-+} - p_{+-}) = \frac{N_k}{N} \left(\frac{1}{1 + e^{-4\beta(j_+ - j_-)}} - \frac{1}{1 + e^{4\beta(j_+ - j_-)}} \right)$$

Oscillations

$$\partial_t n_+ = \frac{N_k}{N} \left(\frac{1}{1 + e^{-4\beta(2j_+ - 1)}} - \frac{1}{1 + e^{4\beta(2j_+ - 1)}} \right)$$

$$\partial_t j_+ = -a \operatorname{atan} [2b(n_+ - 1/2)] - c(j_+ - 1/2)$$

expanding around $n_+ = 1/2, j_+ = 1/2$

$$\partial_t^2 n_+ = -8 \frac{N_k}{N} \beta ab \left(n_+ - \frac{1}{2} \right) - c \partial_t n_+$$

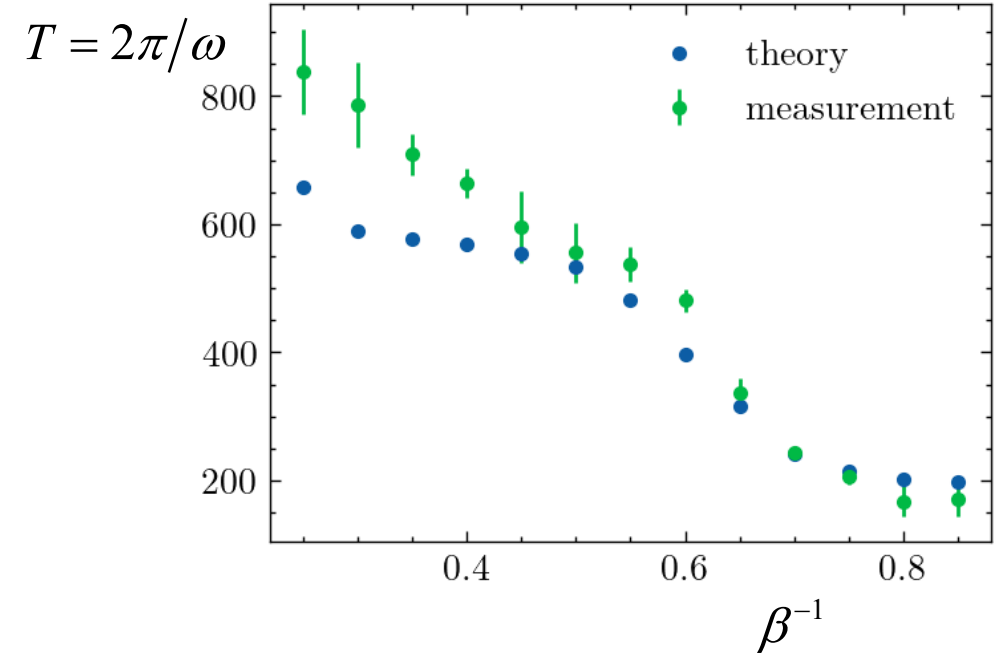
damped harmonic oscillator with

$$\omega_0 = \sqrt{8 \frac{N_k}{N} \beta ab}$$

natural frequency

$$\tau = 2/c$$

relaxation time



$$\omega_d^2 = \omega_0^2 - (c/2)^2$$

Quality factor

$$Q = \omega_0 \tau = \sqrt{32 \frac{N_k \beta a b}{N c}}$$

captures how quickly the system forgets its initial state relative to its natural oscillation period

Number of kinks vs the cluster size :

For a regular 2d lattice: $\gamma = 1/2$

$$N_k \sim N_+^\gamma \approx (N/2)^\gamma$$

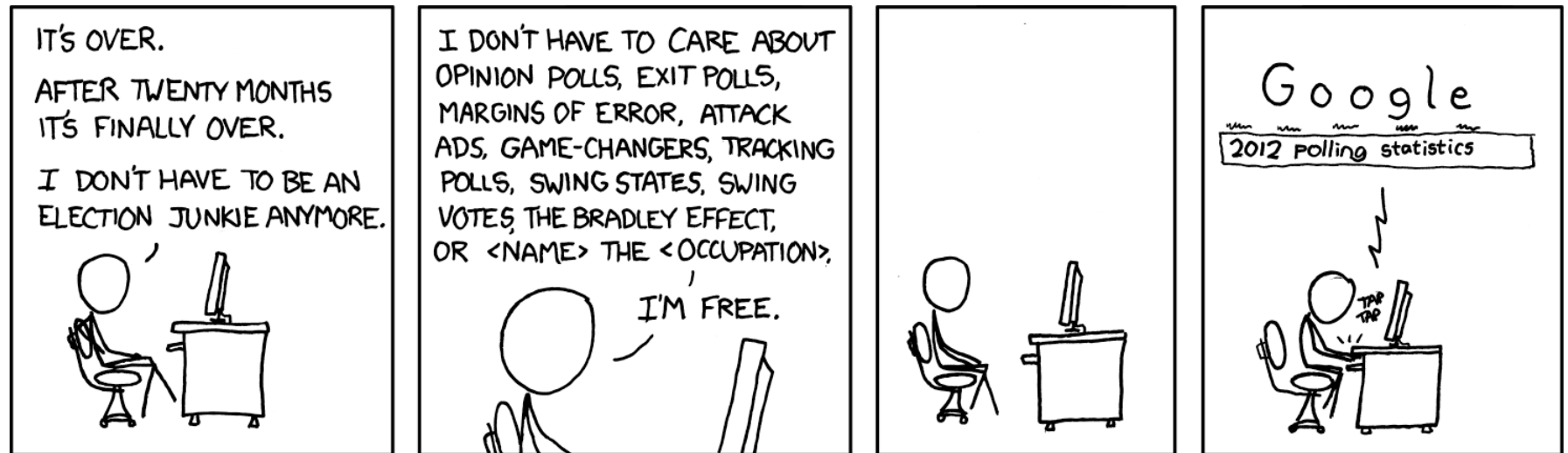
In d dimensions $\gamma = (d-1)/d$

Scaling of the quality factor:

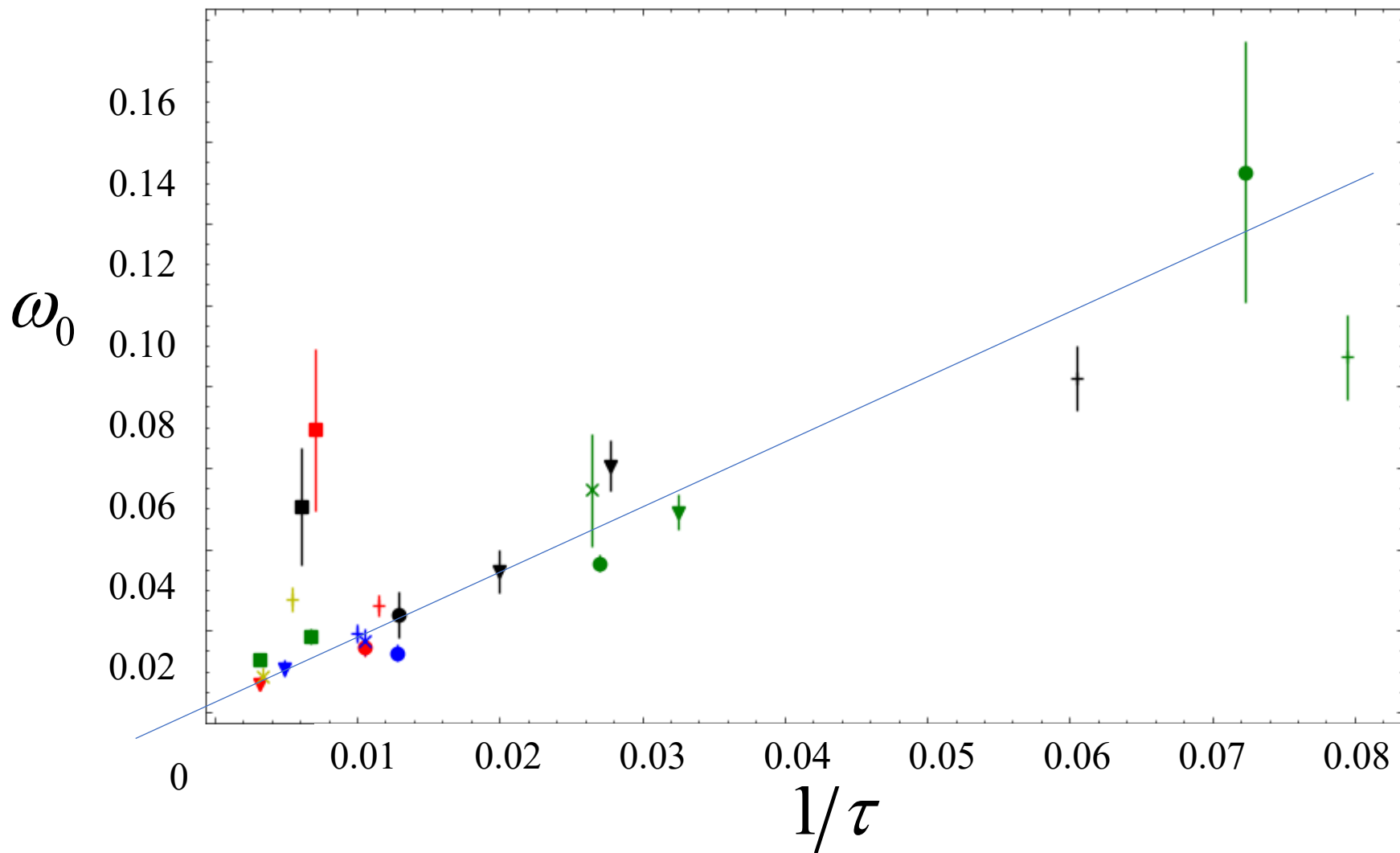
$$Q \sim N^{(\gamma-1)/2}$$

Polling statistics

- Democracies with at least 20 years of continuous functioning
- Direct popular election of the head of state
- Two-round electoral system (second-round data)
- Availability of accessible and reliable electoral data



Polling data

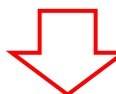


- Austria 2016
- Arizona 2020
- Brazil 2022
- Brazil 2018
- Czechia 2023
- Czechia 2024
- Czech Republi
- Czech Republi
- Ecuador 2021
- ▼ Finland 2024
- ▼ Florida 2020
- ▼ Georgia 2020
- ▼ Georgia 2020
- ▼ France 2017
- ▼ France 2022
- + Michigan 2020
- + North Carolina 2020
- + Poland 2020
- + Pennsylvania 2020
- + Turkey 2023
- × Slovakia 2024
- × Slovenia 2022
- × Wisconsin 2020

$$Q = \omega_0 \tau = \text{const.}$$

theory:

$$Q \sim N^{(\gamma-1)/2}$$


 $\gamma \approx 1$

(alternative explanation in terms of the Ornstein-Uhlenbeck process)

suggests higher dimensionality of the social network

Conclusions

Opinion polls can change the dynamics of the system, leading to the emergence of split society phase

How to prevent it?

- **decouple from poll feedback:** banning polls(?), discouraging the overemphasis of polls in media
- **introduce third attractors:** avoid two-party systems, support nuanced or multi-option discourse (e.g., through centrist candidates, issue-based coalitions etc.) that pulls the system away from strict binary choices
- **promote adaptive engagement:** design incentives so that engagement increases not when one side is "losing" but when mutual understanding or compromise grows. This shifts the dynamics from zero-sum to cooperative.
- **introduce noise or heterogeneity:** external "temperature"—randomness, individual variation, or personal experiences—can disrupt rigid alignment and soften domain walls between opinion clusters

