Abstract

One of the most important questions in quantum information theory is the so-called separability problem. It involves characterizing the set of separable (or, equivalently entangled) states among mixed states of a multipartite quantum system. In this thesis we study the generalization of this problem to types of quantum correlations that are defined in a manner analogous to entanglement. We start with the subset of set of all pure states of a given quantum system and call states belonging to the convex hull of this subset "non-correlated" states. Consequently, the states laying outside the convex hull are referred to as "correlated".

In this work we focus on cases when there exist a symmetry group acting on the relevant Hilbert space that preserves the class of "non-correlated" pure states. The presence of symmetries allows to obtain a unified treatment of many types of seemingly unrelated types of correlations. The symmetries also give the possibility to use the powerful methods of differential geometry and representation theory to study various properties of so-defined correlations: What is more, there exist many physically-interesting classes of correlations that posses continuous symmetries. In this work we apply our general results to particular types of correlations: (i) entanglement of distinguishable particles, (ii) particle entanglement of bosons, (iii) "entanglement" of fermions, (iv) non-convex-Gaussian correlations in fermionic systems, (v) genuine multiparty entanglement, and finally (vi) refined notions of bipartite entanglement based on the concept of the Schmidt number.

The thesis is organized as follows. We first present the necessary physical and mathematical background in Chapters 1 and 2. In Chapters 3-6, which form the core of the thesis, we investigate the natural problems and questions concerning the types of correlations defined above. In Chapter 3 we provide exact polynomial criteria for characterization of various types of correlations for pure quantum states. The Chapter 4 deals with cases in which it is possible to use the criteria from Chapter 3 to give a complete analytical characterization of correlated mixed quantum states. In Chapter 5 we derive a variety of polynomial criteria for detection of correlations in mixed states. In Chapter 6 we use the criteria derived in the previous chapter and the technique of measure concentration to study typical properties of correlations on sets of isospectral density matrices. The thesis concludes in Chapter 7 where we summarize the main contributions of the thesis and state the open problems related to results presented in the thesis.