Needle models of nonequilibrium growth processes

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Abstract

Non-equilibrium growth processes, such as electrodeposition, dielectric breakdown, viscous fingering, or even bacterial colony formation, are often associated with instabilities. The resulting growth patterns are then highly branched, fractal structures. In all these processes the growth may be described in terms of a harmonic scalar field Ψ, the gradient of which sets the velocity of a boundary of a growing phase. A particular interpretation of Ψ varies depending on the problem - it can be e.g. an electrostatic potential, pressure, or nutrient concentration. Such a growth problem is strongly non-linear due to the fact that the boundary conditions are set on an evolving domain. This nonlinearity makes the problem mathematically hard. Consequently, analytical results concerning shapes and growth rates of the fingers are rather scarce.

In the Thesis we considered a simple model of unstable growth in two dimensions, in which the growth takes place only at the tips of long-and-thin fingers. The quantitative analysis of the model was provided by means of the Loewner equation, which can be used to reduce the problem of the interface motion to that of the evolution of the conformal mapping onto the complex plane. Despite its simplicity, the model allows to describe a strong, nonlinear interaction between the fingers and their competition due to the long-range screening.

In the first part of the Thesis we applied this model to describe a particular class of growth processes in which the envelope of the ramified structure grows in a highly regular manner, with the perturbations smoothed out over the course of time. We showed that the regularity of the envelope growth can be connected to small-scale instabilities leading to the tip splitting of the fingers at the advancing front. Whenever the growth velocity becomes too large, the finger splits into two branches. In this way it can absorb an increased flux and thus damp the instability. Hence, somewhat counterintuitively, the instability at a small scale results in a stability at a larger scale. In particular we analyzed the growth in a half-plane geometry, in which case the envelopes form perfect semi-circular shapes with non-uniform intensity of the splitting process along the interface. Interestingly, a similar effect can be observed in the combustion experiments performed in a Hele-Shaw cell.

In the second part we studied patterns formed by viscous fingering in a rectangular network of microfluidic channels. Due to the strong anisotropy of such a system, the emerging patterns have a form of thin needle-like fingers, which interact with each other, competing for an available flow. We developed an upscaled description of this system in which only the fingers are tracked and the effective interactions between them are

introduced, mediated through the evolving pressure field. A complex two-phase flow problem was thus reduced to a much simpler task of tracking evolving shapes in a 2d complex plane. This description, although simplified, turned out to capture all the key features of the system's dynamics and allowed for the effective prediction of the resulting growth patterns.