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CHARLES C. LAURITSEN LABORATORY OF HIGH ENERGY PHYSICS
PASADENA, CALIFORNIA 91125

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Dziekan prof. dr. hab. Dariusz Wasik
Faculty of Physics
University of Warsaw
ul. Pasteura 5, 02-093 Warszawa

Dear prof. dr. hab. Dariusz Wasik,

Thank you very much for inviting me to review the Ph.D. thesis (dissertation) of Mr. Piotr Kucharski. It is a real pleasure, and I wish all dissertations were like that!

Before I get into details of the thesis, let me address one of your questions and say that I do find the thesis of Mr. Piotr Kucharski outstanding and would like to recommend it for the highest encouragements the University can offer.

What do we want to see in a good thesis? New concepts, solutions to long-standing problems, and new directions. Piotr's dissertation has all three components, remarkably in a single volume of only 120 pages.

A good example of a new concept is the "extremal A-polynomial". There is no question about novelty of this concept, or its originality — this term simply did not exist prior to Piotr's work with his supervisor, Piotr Sulkowski, and Stavros Garoufalidis. But is it useful? After all, as influential economist and thinker Fritz Schumacher said, "Any intelligent fool can make things bigger and more complex ... It takes a touch of genius – and a lot of courage to move in the opposite direction."

Yes, the extremal A-polynomial is not just a new concept with little utility: it offers a solution — or, at least, a major step toward a solution — of an old problem. The problem is to find a direct relation, the "shortest bridge" between BPS states and A-polynomial of knots. Both areas play an important role in their own right, as I will try to explain more fully below. And the relation between the two undoubtedly exists, as mountains of evidence in the recent 10-15 years show. They both have integrality, but the problem is that integrality on each side is very different, and the bridges discovered so far connect these two areas in a very complicated and indirect way, in which integral structure is completely lost. I consider the extremal A-polynomials as the best opportunity to make a direct bridge between these two fields that can shed light on the integrality on each side in a deep conceptual way. The fact that such opportunity relies on a newly invented concept is a sweet bonus feature.

Finally, as for the new directions, the dissertation offers a number of them. One that stands out and recently generated a lot of buzz in the field is the material of section 5: a completely unexpected new connection between knots and quivers. In fact, as far as I know, Piotr's dissertation is the only printed account of this exciting development. It is a treasure trove of new ideas that has enormous potential. Why?

One reason, from knot theory viewpoint, is that current methods for studying homological knot invariants are loaded with super-complicated technical details that, frankly, I find hard to teach to newcomers to the field. This is a major challenge, and is not just a technical challenge — much in the spirit of the earlier quote, Sir. Richard Branson says "Complexity is your enemy." In the study of homological knot invariants, it prevents us recognizing simple patterns and new connections, barricaded by technical details and clutter.

The new connection between knots and quivers, on the other hand, is surprisingly simple and elegant. Perhaps one day it can be made entirely combinatorial, thus allowing to strip off unnecessary details and reveal the essential structure. Anyone who worked on homological invariants will certainly vouch that this is something highly desired; not many will dare to hope that something like that exists.

Before I leave the topic of complexity, let me entertain you with a simple calculation. Let's estimate the number of all people in the World who know knot theory, say to the extent they would be comfortable reading that aspect of Piotr's thesis. Then, let us make a similar guess for the number of people that can be equally confident with counting BPS states. According to my estimate, the overlap between these two sets consists of only 20 people or so, which is less than 1% of the number of Siberian tigers left on the Planet, or most other endangered species. Isn't it remarkable that Mr. Piotr Kucharski belongs to this elite group?

What I find even more remarkable is that, already at this stage in his career, he uses this rather unique set of skills not to make things more complex, but rather to find simple truth and beauty in the already-complicated-enough subject.

Now, why is integrality so important? And why will it leave an imprint in science broader than just a mere scope of problems related to knots and BPS states? In math and physics, every time we see an integer number there is usually a good reason for it. It probably counts something. And what it counts is often a mystery, sometimes waiting hundreds or thousands of years to be understood. For example, why coefficients of modular forms turn out to be integer is the subject of the famous Fermat's theorem, the Shimura-Taniyama conjecture proved by Wiles and others. And even this "solved" puzzle answers the question only for a rather limited class of modular forms.

In knot theory the answer to integrality hides in the structure of the so-called homological invariants, or as we now understand, in the spectra of BPS states. At the dawn of the 21-st century, we are only starting to understand this integral structure. It is hard to predict how long it will take, maybe years, maybe hundreds of years. And it is hard to predict what lies ahead. That's why I am delighted to see brave pioneers like Piotr Kucharski venturing in this direction and, instead of being lost in the jungle, coming back with new discoveries, discoveries of the gold mines that he and his future students can fully explore.

The integrality, which is a common theme in all dissertation chapters, also leads to new and exciting connections to number theory. This, too, is quite remarkable and somewhat surprising since physics is usually "continuous"; positions and velocities are all continuous, as well as most other quantities we observe. When some quantity is intrinsically integral, it usually serves as a link to number theory. One general source of such intrinsically integer-valued quantities are charges, electric and magnetic charges, as well as various generalizations thereof, of which BPS states are probably the most sophisticated ones. As a result, the connection to number theory outlined in this dissertation, turns out to be rich and non-trivial, not just superficial: it has something to offer even to pure number theorists.

Reading this dissertation was a real pleasure. The text is well polished (no pun intended) and starts out with a friendly introduction that I hope can be readable to a broad audience of math and physics students. I would suggest Piotr turning it into a survey or review of some sort one day. Then, new results and technical details are gradually introduced. I have to confess I needed a fresh cup of coffee by the time I got to sections 4.1.2 and 4.1.3, but then shrewdly read the rest, especially chapter 5 that, as I already mentioned earlier, appears to be a sacred knowledge.

The only regret I have is that a recent paper of Piotr with Neil Lambert and Miles Owen was not included in this dissertation — it would only make even more remarkable what already is a strong Ph.D. thesis I wish to see from my own Ph.D. students.

Sincerely,

Sergei Gukov
Professor of Theoretical Physics and Mathematics