The Penrose inequality for perturbations of the Schwarzschild initial data

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Abstract

The Penrose inequality is a lower bound on the mass of the spacetime in terms of surface area of the event horizons of black holes. According to the belief that any solution of the Einstein field equations which describe those objects will eventually settle down to the Kerr metric this relation should also hold on an initial hypersurface. This is a straightforward consequence of laws of black holes thermodynamics and the fact that because of the radiation the mass does not increase during the evolution. If the considered initial data is axisymmetric the similar argument leads to the generalized inequality with the angular momentum. Both versions of this relation have the rigidity statements, according to which it is saturated in Schwarzschild and Kerr spacetimes, respectively. Despite multiple arguments in favor of the Penrose inequality, its proof has only been presented for the special case of initial data which is strictly connected with the time-symmetric hypersurface.

This thesis is devoted to the verification of the Penrose inequality with angular momentum for vacuum axisymmetric perturbations of the Schwarzschild initial data with the spherical inner boundary. They are obtained with the use of conformal method, which allows to prescribe the location of the marginally outer trapped surface in the natural way. The assumption about the axial Killing vector is dictated by the fact that the momentum constraint can be then solved in terms of scalar potentials, which define the relevant quantities and allow for an easy estimate of the difference of their values.

The main result of this thesis is the verification of the Penrose inequality for the maximal and non-CMC perturbations. It is satisfied in the leading order of expansion with respect to the parameter measuring the deviation from Schwarzschild data in both cases, apart from the special (nongeneric) data, where the higher order terms have to be included to yield a proper estimate. Because of the complexity of this endeavour it has been done only for the maximal hypersurface. In this setting the inequality is satisfied up to fourth order of expansion for the generalized Bowen-York extrinsic curvature. This result has also been confirmed with the use of numerical solution of constraint equations with the nonvanishing linear and angular momentum.