Stationary state of a sedimenting suspension for small Reynolds and large Peclet numbers

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Abstract

This thesis adresses the problem of a sedimenting suspension of spherical particles. Considered system is described by vanishing Reynolds number (Re) and divergent Peclet number (Pe). The Reynolds number measures inertial effects in the suspension. The lower it is the quicker flow adjusts to boundary contidions of a system. In the limit of Re=0 the flow becomes Stokesian while in the limit of large Peclet number particles cease to exhibit Brownian motion.

The aim of this thesis is a systematic derivation of equations which govern correlation functions in a uniform stationary state of the sedimenting suspension. It is an important and interesting problem which has been investigated for many years. The system of particles in such regime is hard to tackle since the vanishing Reynolds number results in many body hydrodynamic interactions of infinite range. This leads to a divergent average sedimentation rate and its fluctuations. Additionally the lack of Brownian motion forbids expansions around equilibrium state.

In this thesis there is presented a theoretical foundation for a full statistical description of the uniform stationary state of the sedimenting suspension of spherical particles. Rigorous expressions for the sedimentation velocity flutuations are derived. Systematic analysis of those is performed with diagramatic technique. It allowed for classification of all potentially divergent terms. For each class required conditions are formulated to secure convergence of derived expressions for the fluctuations.

In the following part is presented an analysis of nonequilibrium BBGKY hierarchy for this system. Long range terms in hierarchy were analysed. It was shown that conditions derived for flutuations are not sufficient to avoid nonintegrability of correlation functions.

The next part contains analysis of solutions of Batchelor, Cichocki and Sadlej. It was shown that Batchelors result was not only ambigous result but also contradicted the assumption of existence of expansion in powers of concentration. It was also shown that Cichocki and Sadlej equation is very limited in aplicability and does not solve all problems with their hiererchy.

The last part contains the main result of the thesis which is derivation of a new hierarchy of equations. The new hierarchy does not contain long range terms which was achived due to results of the analysis of Batchelors solution. New hierarchy is based on preventing correltation fucntions to be long ranged by mechanism of a response to a linear flow. This corresponds to the mechanism proposed by Cichocki and Sadlej. Correlations which results from those equations are isotropic and they address the problem of the uniform stationaty state of the sedmienting suspension. It allows to calculate the sedimentation coefficients for mono and polydispers systems. Those results are in a great agreement with experimental results and simulations in periodic boundary conditons and jointly with a scaling analysis they implie that in the uniform case suspension achives the stationary state with short range correlation functions while expressions for velocity flucuations are divergent in the thermodynamic litmit.