Abstract

This dissertation aims to investigate quantum phase transitions in polarized mixtures of ultracold fermionic atoms with unequal masses. In such systems, a leading instability of the Fermi sea is related to Cooper pairing in the s-wave channel. We will focus on the phase transition at the onset of the uniform superfluid phase. Imbalanced Fermi mixtures possess a wide choice of control parameters, which allow tuning its properties. In particular, they provide a way to modify the characteristics of the phase transition at $T = 0$. In this thesis, we approach the problem of quantum phase transitions from three distinct but complementary perspectives.

Firstly, we consider a mean-field approximation and expand the effective potential in powers of the order parameter ϕ (this is known as the Landau-Ginzburg theory). Of our particular interest is the possibility of suppressing the tricritical point to zero temperature, so that the transition remains continuous down to $T = 0$. Thus the system hosts a quantum critical point (QCP) in the phase diagram. We analytically identify such a possibility for a range of parameters in dimensionality $d = 3$, but on the other hand, we demonstrate that the QCP is excluded in $d = 2$ (at the mean-field level). Moreover, taking into account a gradient term $|\nabla\phi|^2$ in the effective action, we show that it can be tuned to zero at $T \to 0$, which gives a route to realizing a quantum Lifshitz point.

Secondly, we focus on the spectral properties of sound-like collective excitations (known as Anderson-Bogolyubov modes). Such excitations play a prominent role in the description of quantum criticality. We explore the origin of the complex pole of the pair fluctuation propagator $\mathbb{F}_q$, which gives dispersion relations of collective modes and its damping rates. We obtain the temporally non-local contributions to gradient expansion of $\mathbb{F}_q^{-1}$, which correspond to the damping process of collective phonons. These terms allow us to derive analytical conditions under which damping is active. It turns out that Landau damping is exclusively present in the ordered phase for a large enough mismatch of the Fermi surfaces (even at $T = 0$). We subsequently compare this prediction with numerically obtained damping rates.

Lastly, we perform a nonperturbative renormalization group (RG) analysis using its one-particle-irreducible variant. We consider a situation in which the quantum phase transition is continuous at the mean-field level. Then one finds that order-parameter fluctuations, introduced by the Landau-damping term, obstruct the RG flow toward the Wilson-Fisher fixed point at sufficiently low T, which may indicate a first-order transition. Upon increasing T, the impact of damping ceases, and a critical scaling characterized by the dynamical exponent $z = 1$ is observed. In this case, the system shows a quantum-classical crossover at $T > 0$. Moreover, without damping, the QCP is stable with respect to fluctuations and the transition remains second-order.