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Report on the thesis 'Isolated horizons in spacetimes with cosmological constant' by Denis Dobkowski-Rylko.

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The PhD Thesis 'Isolated horizons in spacetimes with cosmological constant' consists of six short chapters and two appendices in a total of 104 pages. It is based on five papers co-authored by the candidate, and published in very reputable journals: Classical and Quantum Gravity, Phys. Rev. D. and Phys Lett B.

A pervading theme is that of *horizons*, which, as far as the Thesis is concerned, are null hypersurfaces in some Lorentzian four–manifold together with intrinsic geometrical structures. While these hypersurfaces share their name with the horizons in General Relativity (like the event horizon, or the Killing horizon), they are studied in their own right, are not necessarily arising in any limiting procedures from the GR horizons. It is important to make this assertion at the start, as otherwise some of the results in the thesis, e.g. those which relate to hypothetical horizons whose spatial sections have non-spherical topology, would be false in view of the Hawking theorem. In fact the analysis of the PDEs governing these spatial sections (like equation (1.131)) forms the central technical part of the Thesis. These PDEs are elliptic, so in essence Dobkowski-Rylko's work is concerned mostly with Riemannian (rather than Lorentzian) geometry. The analysis is applicable to the cases with non–zero cosmological constant, and in fact the most interesting and novel findings correspond to $\Lambda > 0$. This is in agreement with the current observational data, and makes the whole effort (unlike the fashionable theories with $\Lambda < 0$) physically relevant.

The Thesis kicks off with some introductory material about geometry of the null hypersurfaces, and definitions of various horizons. The main result of Chapter 1 is the classification of the Weyl tensor restricted (in an appropriate way) to horizons. It is found that it can only be of Petrov-Penrose type O, II or D. The most interesting, for me, result of this chapter is the analysis of the PDE underlying the type D isolated horizon. This equation is subsequently solved, in Chapter 2, under the additional assumption that the spatial topology is spherical, and that the metric admits a $U(1)$ isometric action. The general solution in this case is parameterised by the cosmological constant, the area and the angular momentum. The resulting horizons can, for each values these parameters, be identified with those arising from the near horizon limits of Kerr/dS or Kerr/AdS. All these solutions have been known previously. Chapter 3 deals with spatial cross sections with higher genus topology, and (essentially) shows that these can not arise in the type D case. This is in agreement with the Hawking theorem. It would be interesting to conduct a similar analysis in higher dimensions (with type D appropriately defined), when the Hawking theorem is known not to hold, and black rings type objects exist. In Chapter 4, the type D equation is considered for horizons which admit a monopole-like topological structure. While the global spatial cross–sections do not exist if the monopole number is non–zero, this analysis is relevant for the Taub–NUT like solutions.

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Finally, in Chapter 5 (which is not directly related to the rest of the thesis) the candidate considers the gravitational radiation emitted by an isolated source, and derives a formula for the radiated energy. This formula is perturbative in the cosmological constant, and reduces to the standard quadrupole formula in the limit when $\Lambda = 0$.

Conclusion. In my opinion Dobkowski-Rylko is one of the emerging leaders in the Polish community of classical general relativity. On several occasions I have had a pleasure to attend the lectures he gave at international meetings. Dobkowski-Rylko is an excellent expositor. In his presentations he skilfully separates the results which are of general interests, from the rather technical aspects of the formalism.

The Thesis submitted by Dobkowski-Rylko, entitled 'Isolated horizons in spacetimes with cosmological constant' meets the requirements applicable to a PhD theses in theoretical physics. It demonstrates that the candidate has more than good understanding and grasp of the existing knowledge in the field, and it contains a mixture of publishable or already published original results. Given the calibre of these original results, and the overall academic standing of the candidate, I suggest that the distinction is also awarded.

Yours sincerely,

Maciej Dunajski