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Subject: Report PhD thesis Mr. Maciej Kolanowski.

To whom it may concern,

This thesis is of very high quality, and it is my pleasure to review it. Maciej Kolanowski has written this thesis based on multiple publications and one preprint. These works explore several aspects of extremal black holes, focusing on how these black holes respond under stationary perturbations. The methods used exploit in a creative way the symmetries of extremal black holes; this allows the candidate to obtain quantitative and robust results regarding the stability of black holes. The thesis includes many new results that are timely relative to other developments in the field, where corners of black holes physics, numerical relativity, and holography overlap. Moreover, it manages to balance many technical details and conceptual topics in this challenging and complex subject.

The thesis starts with a brief summary of the state of art, providing a general overview of the developments related to classical aspects of black holes that motivated the candidate's work. An introduction to the mathematical tools used is given in chapter 1, which gives a pedagogical review and basic definitions pertaining black holes. In this chapter, the candidate reviews briefly also concepts of AdS/CFT that connect to some of the technical aspects used later. The material is as well-motivated, well-explained and placed in a modern context; it covers the state of the art that is important to set the stage later on for chapter 2, 3 and 4, which contains the new development on this corner of gravitational physics.

After this introductory chapter, the thesis follows with three chapters which focused on new material, based on the peer-reviewed publications and pre-prints. All chapters share in common extremal black holes, and their response under stationary perturbations. One impressive portion is that despite the specificity of the black hole (that is, being extremal), this thesis illustrates that there are many interesting and novel features regarding the classical behaviour of them. These are important advancements since they will become important as one develops a fundamental understanding of quantum gravity. It is also of great relevance to decoding the principles governing the AdS/CFT correspondence.

The highlights of these chapters are the following:

Chapter 2: This portion focuses on the response of four-dimensional extremal black holes under stationary perturbations in Einstein-Maxwell theory, with and without a cosmological constant. The analysis covers both a numerical and an analytic (linear) component. I will focus on analytic aspects, since it is within my expertise.

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The aim of the chapter is to show that a large class of extremal black holes are singular. The criteria presented is related to the radial behaviour of stationary (time-independent) perturbations: depending on the falloff of the perturbation, this can induce an infinite tidal force for infalling observers. In this sense is the solution deemed singular. One of the main results is that extremal black holes in AdS_4 (i.e., gravitational theories with a negative cosmological constant) are generically singular, with some small exceptions. When the cosmological constant is positive the features are less generic, and in the absence of a cosmological constant the black hole is regular according to the criteria presented.

There is also an analysis about how this singularities arise from the finite temperature (non-extremal) black hole. In particular one of the important outcomes is the response of the heat capacity of the black hole as function of temperature, near to extremality. This is a quantity that is useful to measure and connect within AdS/CFT, since it can give insight on how to model this in the CFT and hence account for it microscopically.

Chapter 3: The focus on this chapter is to investigate stability of $\mathrm{AdS}_2 \times S^n$ for $n > 2$. The overarching aim is to study the stability of the solutions under stationary (or even static) perturbations. This type of analysis gives a way to determine the space of classical configurations: if $\mathrm{AdS}_2 \times S^n$ results to be unstable, then it implies that there is another classical solution in the phase space.

Many of the explicit computations are done for $\mathrm{AdS}_2 \times S^3$ in the context of Einstein-Maxwell theory with a negative cosmological constant. This background is the near-horizon solutions of the charged RN black hole in AdS_5. It is shown that this solution is unstable under general non-spherically symmetric perturbations, i.e., perturbations that do not preserve $SO(4)$. With this, the candidate constructs a new near horizon geometry that is now stable under symmetries that preserve $SO(3)$ perturbations; this construction is done at extremal and at finite temperature. Other topologies are considered, such as $\mathrm{AdS}_2 \times T^3$.

This analysis illustrate very clearly and sharply that the phase space of solutions is rich and interesting. This thesis explores very interesting aspects of this class of solutions and it is important for our advancement of AdS/CFT to be aware of these instabilities.

Chapter 4: This chapter is particularly elegant and of great relevance to my field of work. The thesis discuss how higher derivative corrections have an impact on the properties of near horizon geometries. Along the lines of Chapter 2, here a similar analysis is done when the gravitational theory has terms up to cubic order in the curvature tensor. The chapter discusses carefully the role of field re-definitions and construction of the effective action. Then a very impressive work is done to understand if singularities can occur in the presence of these higher derivative terms. Constraints can be placed, which would be interesting to contrast with the weak gravity conjecture. Personally, I am looking forward to future work in this direction and the impact in our quantum properties of black holes.

One small but important corner to highlight is the final chapter 5. It is short and succinct, still with emphasis on exciting future directions and potential projects that follow this line of research. It displays a more in-depth vision of the impact of the results here, how they interplay with other important developments, and the future of this area of research. I personally would find very interesting the aspects related to the role of higher curvature corrections and the weak gravity conjecture.

I have listed below a series of very small corrections and questions.

1. There is a typo in equation (1.5).
2. There is a typo in text below equation (1.6): "Metrics of the form (2.19) ..." should be replaced by "Metrics of the form (1.6a) ...". This mistake occurs again within that paragraph.

3. Footnote 8 is needs to be slighted corrected. Usually I agree with the intuition that supersymmetry implies extremality, since many known black holes comply with it. But recently this is found to not be always correct, or at least useful. In the context of constructing Euclidean solutions there are explicit examples where supersymmetry does not imply extremality. This is presented in 1810.11442 in regards to supersymmetric black holes in AdS_5. A more exotic example are supersymmetric black holes in higher spin gravity 1512.00073.

4. Equation (1.6b) seems to still support magnetic charges since the gauge field has support on $dx^a \wedge dx^b$. Where is the absence of magnetic charge reflected in equation (1.7)?

5. There is a typo in footnote 5, page 20.

6. On equation (2.21a), should δF be replaced by δC? And is $\delta\mathcal{F} = \delta F$?

7. Clarifications: Δ^0 the laplacian of q^0_{ab}, and not (2.33a), correct? What is γ^0_{ab} in (2.37a)? The expressions (2.38a-g) assume that the solutions are stationary because it uses (2.71), correct?

8. When discussing stability of extremal black holes when $\Lambda > 0$, for example after (2.42) at the end of page 25 or page 29, the emphasis is on the $\mathrm{AdS}_2 \times S^2$ extremal limit of RN black holes in dS_4 (Cold BH). However there are other extremal limits for this class of black holes since the black hole generically contains three horizons: inner, outer and cosmological. Is there an instability when the extremal NHG is $\mathrm{dS}_2 \times S^2$ (Nariai BH) or $\mathrm{Mink}_2 \times S^2$ (Ultracold BH)?

9. It would be useful to connect and contrast the notation around equation (3.13) with the notation used in Chapter 2.

10. I was curious if instead of writing the metric of S^3 as in (3.28), one would cast it as a fibration. That is, to write $d\Omega_3^2 = 1/4(d\psi + \cos\theta d\phi)^2 + 1/4(d\theta^2 + \sin^2\theta d\phi^2)$. Would this lead to a different near-horizon geometry?

11. There is a typo in the third term in equation (4.1).

12. There is a mismatch of notation between equation (4.3) and the notation $c_{1,2}$ used in the sentence below.

13. In relation to the comments below equation (4.6), is it expected that Iyer-Wald charges are sensitive to field redefinitions? For example, would the BMS charges be affected by it? I'm not sure if these type of charges are discussed in the context of higher derivative theories, but I'm curious if the candidate knows how they would respond.

14. What are the field redefinitions considered in deriving equation (4.8)? I assume it is a generalization of (4.4) that includes quadratic terms?

15. What is the resulting size of AdS_2 and S^2 around (4.17)? In equation (4.16) they are equal (both given by r_+) as expected in the two-derivative theory. It will make the text more clear if this data is described more explicitly as the background is changed instead of presenting (4.17). It would be interesting to compare with Ashoke's entropy function, which does something very similar. See, for example, 0708.1270.

16. Chapter 4 has at least two instances where it says "...this paper ...".

These comments are not objections, but simple corrections and minor improvements to the thesis.

Summarising, the most important results here are aimed to understand stability aspects of extremal black holes via the use of modern tools in general relativity that connect to holography. This manuscript reflects how original use of symmetries of the near-horizon geometry of extremal black holes can provide both classical and quantum insight to non-perturbative aspects of black holes. These are challenging topics that are relevant in several corners of theoretical and mathematical physics, and it is of great value to make some progress on them. The PhD candidate has peer-reviewed publications and preprints in this area, which places him in a solid position in a competitive international arena in theoretical physics.

To close, I recommend the approval of this thesis. If further information and comments are needed, please don't hesitate to contact me.

Sincerely yours,

Alejandra Castro