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Spin effects of cavity photons and exciton-polaritons in planar optical resonators

Efekty spinowe fotonów wnękowych i polarytonów ekscytonowych
w planarnych rezonatorach optycznych

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Abstract

This thesis is devoted to the study of spin effects in planar microcavities. Investigated optical resonators were composed of two dielectric mirrors, with the space between them filled with different materials providing polarization dependent effects.

At first the cavity was filled with a layer of nematic liquid crystal. Such birefringent material are characterized with different effective refractive indexes depending on the relative orientation of the linear polarization plane versus the axis of the crystal, which leads to the splitting of the linearly polarized modes of the cavity.

In such system, we investigated a spatial polarization textures arising when polarized light was transmitted through the sample for different orientations of the optical anisotropy of the liquid crystal and observed configurations of second order meron and second order antimeron.

In an analogous system we observed effects resulting from polarization dependent losses. Depending on the non-Hermiticity degree we measured shift in the momentum space and following annihilation of the exceptional points originating from different diabolical points.

Second, by incorporation into the cavity a layer of two-dimensional perovskite, we were able to observe emergence of strong coupling regime between the photonic modes and excitons in the perovskite. By additional orientation of the liquid crystalline layer into a twisted nematic structure, we lifted the degeneracy at the diabolical points around the regime of Rashba-Dresselhaus spin orbit coupling. This lead to a polarized band structure of the cavity modes exhibiting nonzero Berry curvature.

Using the strong coupling regime we obtained a polarized emission from the polariton states in a fully isotropic microcavity incorporating monolayers of tungsten diselenide. To achieve this we used, unique for transition metal dichalcogenides monolayers, spin-valley locking, where an exciton with a given spin orientation is confined within a specific valley in the momentum space with limited possibilities for spin flipping and scattering. Utilizing this property by quasiresonantly exciting excitons with a given spin orientation by circularly polarized pump beam we were able to create exciton polaritons exhibiting nonzero degree of the circular polarization of the emitted light.

Streszczenie

W prezentowanej pracy badano efekty polaryzacyjne występujące w planarnych mikrownękach optycznych. Badane rezonatory składały się z dwóch zwierciadeł dielektrycznych, pomiędzy którymi umieszczano materiały powodujące wyróżnienie różnych polaryzacji światła.

W pierwszej kolejności wewnątrz wnęki umieszczono warstwę ciekłego kryształu. Jako że ten dwójłomny materiał charakteryzuje się różnymi współczynnikami załamania światła w zależności od orientacji polaryzacji liniowej światła względem osi kryształu, powoduje to rozszczepienie liniowo spolaryzowanych modów wnęki. W takim układzie badano tekstury polaryzacyjne światła transmitowanego przez próbkę w zależności od kierunku anizotropii optycznej ciekłego kryształu obserwując tekstury mero-nu i antymeronu drugiego rzędu.

W analogicznym układzie zaobserwowano również efekty związane ze stratami zależnymi od polaryzacji światła. W zależności od stopnia niehermitowskości w układzie zmierzono przemieszczanie się w przestrzeni pędów a następnie anihilację punktów wyjątkowych (ang. exceptional points), powstałych z różnych punktów diabolicznych.

Następnie, dodanie wewnątrz wnęki ciekłokrystalicznej warstwy dwuwymiarowego perowskitu pozwoliło na uzyskanie reżimu silnego sprzężenia pomiędzy modami fotonowymi wnęki a ekscytonami w perowskicie. W układzie tym poprzez zastosowanie orientacji ciekłego kryształu w postaci skręconego nematyka uzyskano zniesienie degeneracji w punktach diabolicznych wokół reżimu sprzężenia spin orbita Rashby Dresselhausa powodujące powstanie polaryzacji pasm modów wnęki wykazujących niezerową krzywiznę Berry'ego.

Wykorzystując silne sprzężenie uzyskano również spolaryzowaną emisję ze stanów polarytonowych z całkowicie izotropowej optycznie mikrownęki zawierającej monowarstwy selenku wolframu. W tym celu wykorzystano unikalne właściwości tego materiału, w którym ekscyton o danej orientacji spinu związany jest z daną doliną w przestrzeni odwrotnej z ograniczoną możliwością do rozpraszania. Dzięki temu kwazirezonansowe kreowanie ekscytonów o danej orientacji spinu przez pobudzającą wiązkę światła o polaryzacji kołowej pozwoliło wykreować polarytony ekscytonowe wykazujące emisję o niezerowym stopniu polaryzacji kołowej.

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Chapter 1

Introduction

The work is structured as follows:

Chapter 1 contains a brief overview of concepts important at all stages of this work, namely: planar optical resonator, polarization of light and topology of vector fields in two dimensions.

Chapter 2 presents more detailed description of the properties of planar resonators. It introduces theoretical methods to calculate optical properties of planar structures and summarizes properties of microcavities based on distributed Bragg reflectors. Afterwards, it demonstrates the experimental setup that allowed mapping of the dispersion relation of such a resonator. The most of the space is dedicated to the properties of the cavities filled with optically anisotropic medium, especially to different regimes of the degeneracy between specific modes.

Chapter 3 is devoted to new results concerning the topology of the spatial polarization textures of polarized light transmitted through a cavity filled with liquid crystal.

Chapter 4 brings non-Hermiticity to the description of cavities filled with liquid crystal, originating from polarization dependent losses in the system. It shows that tunability of the liquid crystals allows for control over the exceptional points: spectral singularities exclusive to non-Hermitian systems. In particular, it demonstrates the possibility of annihilation of the exceptional points originating from different valleys in the momentum space.

Chapter 5 leaves for a moment anisotropic liquid crystals to demonstrate the strong coupling regime. It presents investigations of thin flakes of tungsten diselenide (WSe_2) incorporated within a tunable, open cavity. It shows operation in a strong coupling regime between the cavity mode and the excitonic transition in WSe_2 . Additionally, it uses the valley polarization within

the WSe₂ monolayer to non-resonantly transfer spin state of pumping beam into the polariton state.

Finally chapter 6 goes back to the liquid crystal system in which a strong coupling regime can be obtained by incorporation of 2D perovskites. It also shows how, through changes in the orientation of LCs, the polariton bands can exhibit locally non-trivial geometry quantified by non-zero Berry curvature.

1.1 Optical resonators

Optical cavities are resonators designed to confine the electromagnetic field in the visible range. In an ideal cavity light with at specific wavelengths is confined indefinitely without any losses in distinguished resonant modes. In microcavities, light is confined within a space comparable to wavelength, resulting in a sparser frequency separation between consecutive modes [1].

There are different approaches to trapping light. In so called whispering gallery modes resonators, light is trapped by total internal reflection [2]. It can be also localized in photonic crystal cavities, on a defects introduced in subwavelength periodic structures [3]. Electromagnetic field can be also confined by plasmonic effects in plasmonic nanocavities [4].

One of the most popular type of optical resonator are planar microcavities [5], based on idea of Fabry–Perot interferometer, in which light is confined between two parallel metallic plates. In the variation on this approach a highly reflective, but also introducing losses, metallic planes can be exchanged for a dielectric mirrors, where high reflectivity is achieved by interference between multiple dielectric interfaces.

1.2 Strong light-matter coupling regime

In the vacuum, an atom in the excited state via a dipole transition can transit into the ground state by the emission of photon into a continuum of photonic states. If the same atom is placed within an optical cavity, resonant with the transition, it couples with the vacuum field of the cavity, leading to the energy exchange between the two states. When the Rabi frequency is higher than the dissipation time from the cavity, the system stays in the strong coupling regime.

Such phenomena can be observed e.g. in a planar microcavity incorporating semiconductor quantum well, schematically depicted in Fig. 1.1a. The

system consists of planar resonator made of two distributed Bragg reflectors (DBRs) which localize the photonic field in the center of the cavity. Here, instead of an atomic transition, photons in the quantum well can create excitons, the bound pairs of electrons and holes called.

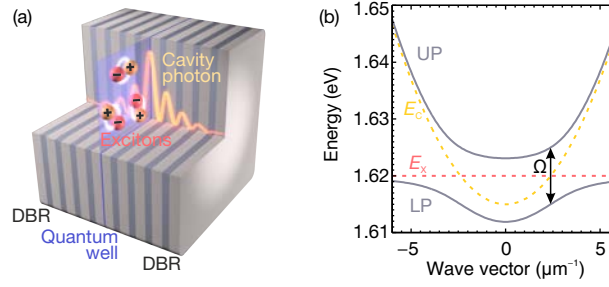


Fig. 1.1 | Strong coupling in planar microcavity. (a) Schematic illustration of a cavity with embedded quantum well. (b) Dispersion relation of microcavity exciton-polaritons.

In such system the strong coupling regime can be described in a coupled oscillator model by 2×2 matrix:

$$\hat{H} = \begin{pmatrix} E_X(k_{\parallel}) & \Omega/2 \\ \Omega/2 & E_C(k_{\parallel}) \end{pmatrix}, \quad (1.1)$$

where excitonic mode with energy E_X couples with the cavity mode resonance E_C with the coupling strength Ω (Rabi energy).

The eigenvalues of Eq. 1.1 describe the dispersion relations of the light-matter mixed states of lower (LP) and upper (UP) polariton modes:

$$E_{LP/UP}(k_{\parallel}) = \frac{1}{2} \left\{ E_X(k_{\parallel}) + E_C(k_{\parallel}) \mp \sqrt{\Omega^2 + [E_X(k_{\parallel}) - E_C(k_{\parallel})]^2} \right\}, \quad (1.2)$$

plotted as a function of the photon wave vector $k_{\parallel}$ in Fig. 1.1b. When the energy of the cavity mode crosses the exciton resonance a clear anticrossing behavior can be observed, with the minimum energy splitting between the polariton branches equal to the Rabi energy Ω .

1.3 Polarization of light and two level system

Light in the form of a propagating plane wave possesses a degree of freedom in the form of orientation of the electric field called polarization.

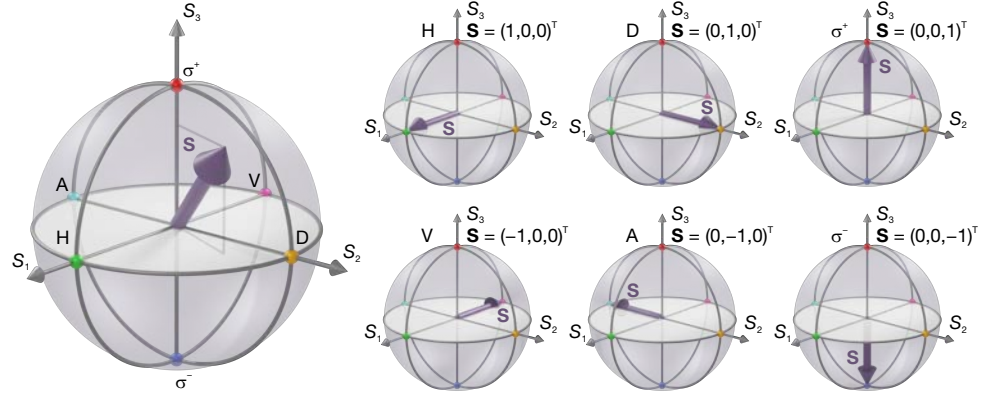


Fig. 1.2 | Bloch/Poincaré sphere.

Based on the intensities of light detected in linear: horizontal (I_H), vertical (I_V), diagonal (I_D), antidiagonal (I_A), and circular: right (I_{σ^+}) and left (I_{σ^-}) one can determine the polarization state of light by calculation of Stokes parameters:

$$S_1 = \frac{I_H - I_V}{I_H + I_V}, \quad (1.3)$$

$$S_2 = \frac{I_D - I_A}{I_D + I_A}, \quad (1.4)$$

$$S_3 = \frac{I_{\sigma^+} - I_{\sigma^-}}{I_{\sigma^+} + I_{\sigma^-}}. \quad (1.5)$$

Additionally, to describe the total intensity of light, the zero Stokes parameter is defined based on the total intensity :

$$S_0 = I_H + I_V = I_D + I_A = I_{\sigma^+} + I_{\sigma^-}. \quad (1.6)$$

Based on the Stokes parameters one can construct a 4-element Stokes vector $\mathbf{S} = (S_0, S_1, S_2, S_3)^T$, which e.g. is used in analysis of the light polarization by Mueller matrices. Alternatively a 3-element Stokes vector $\mathbf{S} = (S_1, S_2, S_3)^T$ can be used to represent given polarization state by a point within so-called Poincaré sphere (Fig. 1.2).

$\sigma^\pm$ polarized photons carry $\pm\hbar$ angular momentum. This intrinsic angular momentum connected with $\sigma^\pm$ polarization is referred to as spin angular momentum (SAM), or simpler spin of the photon, to differentiate additional orbital angular momentum (OAM) dependent on the field spatial distribution.

Due to the angular momentum conservation rule, light can only create

within a medium an excitonic state with the same spin state as the incident photon. Thus the Poincaré sphere is equally useful for the description of the spin state of an optically active exciton, but from historical reasons such representation is known as Bloch sphere. The same conservation of the angular momentum rule applies for photoemission from an excitonic state, thus the photons eventually emitted from the cavity within its polarization carry full information about the spin state of the exciton or exciton-polariton.

1.4 Topology: winding number

Topology is a branch of mathematics, which investigates properties of the systems which are invariant to smooth deformations. Its goal is to divide different geometrical objects into so called homotopy classes, within which each member can be transformed into one another by a continuous deformation without cutting and/or binding.

Within this work topological classification will be useful for the analysis of three-dimensional vector fields in two-dimensional space. Here the vector would describe spin or a polarization state described by a Stokes vector in two dimensional spatial or momentum coordinates. The topological charge classifying such fields will be described in Sec. 3.1, but very useful intuitions can be gained by the introduction of a winding number classifying 2D vector fields in 2D space.

Meaning of the winding number can be easily illustrated on a specific example [6]. Let's consider a texture given by two-dimensional vectors marked by the orange arrows arranged around the core represented by the red sphere in Fig. 1.3a. We can think about it as an in-plane projection of a spin of a photon in spatial x - y coordinates.

The winding number of a given field can be determined based on the mapping of the orientation of the spin while encircling the core. Here we encircle the core counterclockwise starting from position 1. (white), which can be chosen arbitrary and does not influence the final result. Spin at position 1. is oriented upwards, which corresponds to $S_2 = 1$ polarization. We then trace the orientation of the spins while encircling the core, where the progress along the loop is marked by pseudocolor white-blue-black-green scale. At position 2. (blue) the spin orientation undergo a smooth transition to $S_1 = -1$ tracing a quarter of a unit circle. Moving forwards through positions 3. (black), 4. (green) and back to the starting position the traced spin orientation also performs a one full loop around the origin. Additionally the loop at the mapping side followed counterclockwise direction as

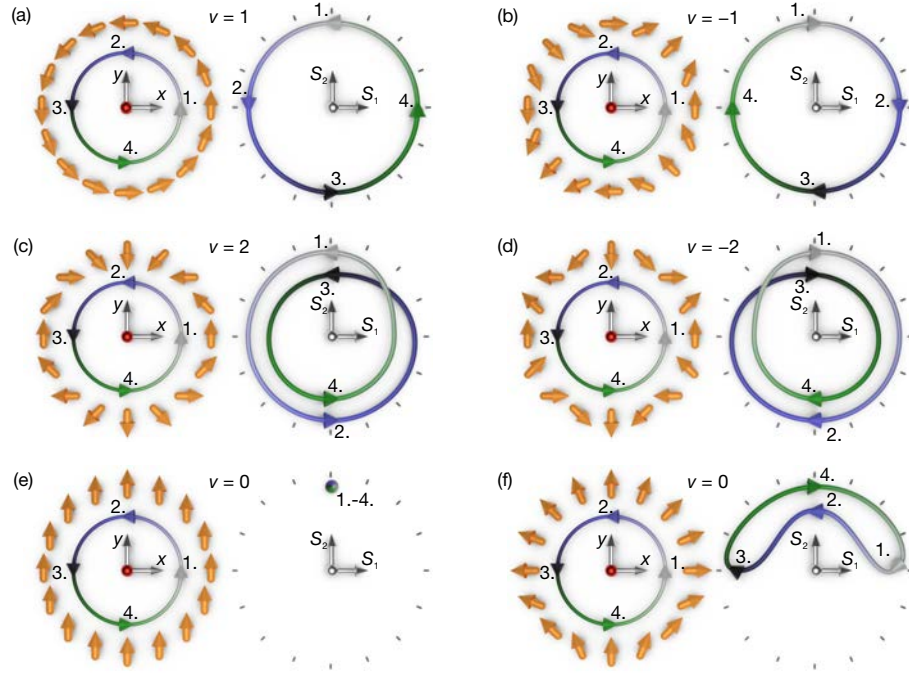


Fig. 1.3 | Winding number. Examples of 2D spin textures with different winding numbers v : (a) $v = 1$, (b) $v = -1$, (c) $v = 2$, (d) $v = -2$ and (e)–(f) $v = 0$.

marked with direction of the arrows in Fig. 1.3a. As the direction of the encircling loop and the mapping is the same, the resulting winding number v is equal to $+1$.

In the same fashion we can consider different configurations of the arrows like the one presented in Fig. 1.3b. Here during the same procedure of mapping of the spin orientation while encircling the core counterclockwise the spin undergoes clockwise rotation. As it is opposite to the looping direction the resulting winding number is equal to -1 .

For different spin configurations encircling the core can correspond to more than one rotation of the spin. One of such configurations is shown in Fig. 1.3c. During one loop around the core in counterclockwise direction spin undergoes two full rotations in the same direction leading to winding number $+2$. Correspondingly, an example of a vector field in which spin undergoes two full rotations in the opposite direction to the encircling, resulting in the winding number -2 , is shown in Fig. 1.3d.

In the presented examples the winding number took integer numbers, in particular it can be also equal to zero. One of such cases is trivial, homogeneous vector field shown in Fig. 1.3e. Here all spins point upwards,

so mapping the spin orientation during encircling an arbitrary core leads to single point, which does not loop around the origin. In a slightly more complex configuration depicted in Fig. 1.3f, the spin orientation is not constant around the core. Nevertheless, when encircling the center of the vector field, the spin does not undergo the full rotation, thus the winding number is also equal to zero.

Fig. 1.3 presented only a single example corresponding to different winding numbers, but multiple distinct spin textures are characterized with the same winding number.

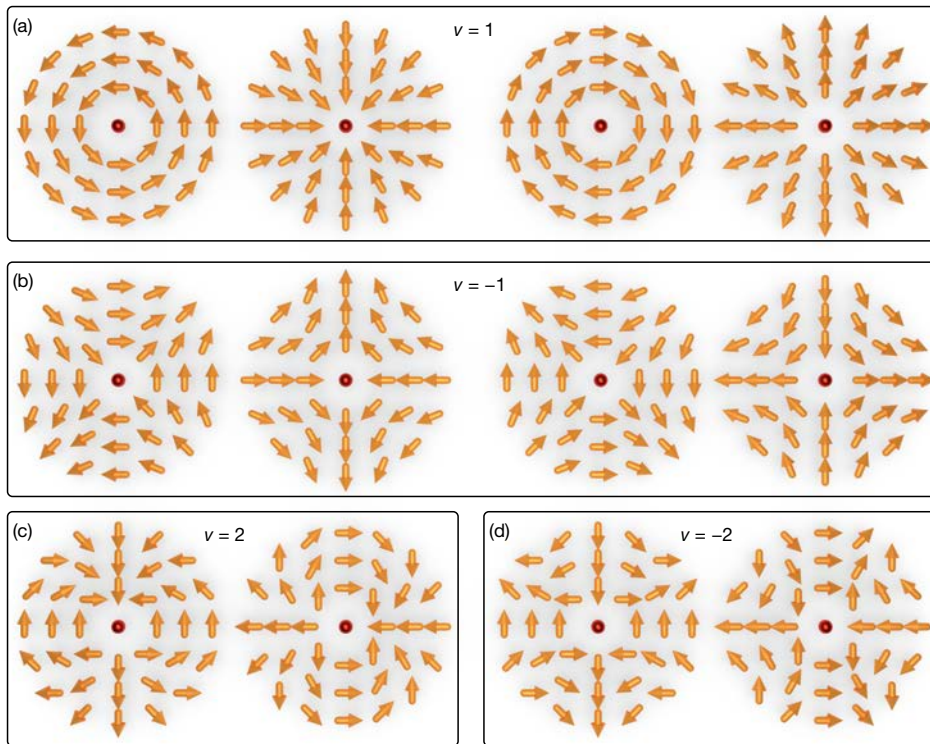


Fig. 1.4 | Different vector fields with the same winding number. (a) $v = 1$, (b) $v = -1$, (c) $v = 2$ and (d) $v = -2$.

Fig. 1.4 presents more variations of the spin configuration with equivalent winding numbers. Fig. 1.4a presents four configurations which each corresponds to the $v = 1$. All of them were obtained by a smooth deformation performed as rotation of all of the spins by 90° counterclockwise, so all belong to the same homotopy class.

Similarly, starting from texture presented in Fig. 1.3b and rotating all spins by 90° we obtain several new representatives of the much wider group characterized by winding number -1 , as shown in Fig. 1.4b.

For $v = \pm 2$ rotation of all of the spins by a constant value results in the rotation of the whole texture as presented in Fig. 1.4c,d.

3-dimensional vector fields in two dimensions are topologically classified in terms of topological charge. This quantity will be described in Sec. 3.1, but for a core polarized in S_3 direction the topological charge Q is the product of S_3 at the position of the core, winding number of the in-plane components and additional positive factor depending on the rotation range of the S_3 component. Thus the topological charge can be reversed by reversing the polarity of the core or the winding number of the in-plane field.

Chapter 2

Planar optical microcavities

This chapter introduces the physics of planar microcavities.

The first section (Sec. 2.1) starts with the description of two theoretical approaches to simulate optical properties of such layered structures: the transfer matrix method and the Berreman matrix method. Both methods are further on used to present the basic properties of a dielectric cavity. If possible, the numerical results are compared with the experimental data.

The second section (Sec. 2.2) concentrates on cavities that are filled with birefringent medium, specifically with nematic liquid crystals. Based on the Berreman matrix simulations it shows tunability of the cavity modes and we discuss various regimes of degeneracies between the different modes.

2.1 Optical planar microcavities

2.1.1 Theoretical simulations of light propagation in planar structures

Transfer matrix method

One of the most useful description of light propagation through homogeneous layered structure is based on so called Transfer Matrix method. It can be used to find transmission, reflection coefficients and spatial distribution of the electric field of any stack of layers with given thicknesses and refractive indices. In this section summary on the transfer matrix method follows formulation described in Ref. [7].

In transfer matrix method electric field amplitudes at the left- and right-hand side of a layered structure (Fig. 2.1) can be connected by a 2×2 matrix

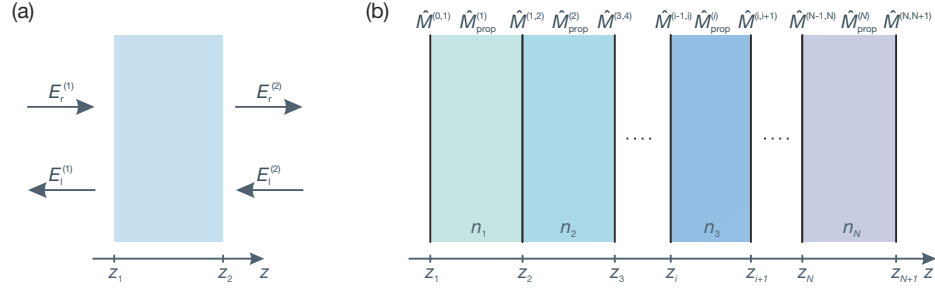


Fig. 2.1 | Transfer matrix method. (a) Schematic representation for electric field amplitudes in transfer matrix method. (b) Description of a multilayer structure with corresponding transition $\hat{M}^{(i,j)}$ and propagation $\hat{M}_{\text{prop}}^{(i)}$ matrices.

$\hat{M}$ according to equation:

$$\begin{pmatrix} E_r^{(2)} \\ E_l^{(2)} \end{pmatrix} = \hat{M} \begin{pmatrix} E_r^{(1)} \\ E_l^{(1)} \end{pmatrix}. \quad (2.1)$$

The matrix $\hat{M}$ for a given layered structure can be obtained by matrix multiplication of the partial matrices corresponding to the propagation of the electric field within i -th layer in the distance between z_i and z_j :

$$\hat{M}_{\text{prop}}^{(i)} = \begin{pmatrix} \exp[ik_z(z_j - z_i)] & 0 \\ 0 & \exp[-ik_z(z_j - z_i)] \end{pmatrix}, \quad (2.2)$$

where k_z wave vector component in i -th layer is equal to:

$$k_z^{(i)} = \sqrt{\frac{\omega^2}{c^2} n_i^2 - k_{\parallel}^2}. \quad (2.3)$$

Here ω is the frequency of the incident wave, c the speed of light in vacuum, n_i refractive index in a given layer and $k_{\parallel}$ wave vector component of the incident light for parallel to the interface plane.

Additionally, every interface between two layers i -th and j -th of different refractive indexes has to be taken into account by multiplication by a proper transition matrix $\hat{M}^{(i,j)}$. As Fresnel reflectance coefficients are different for electromagnetic waves with transverse electric (TE, s polarization) or transverse magnetic (TM, p polarization) polarization the form of this matrix depends on the polarization of the incident field. For TE polariza-

tion the transition matrix takes the form:

$$\hat{M}_{\text{TE}}^{(i,j)} = \begin{pmatrix} \frac{k_z^{(j)} + k_z^{(i)}}{2k_z^{(j)}} & \frac{k_z^{(j)} - k_z^{(i)}}{2k_z^{(j)}} \\ \frac{k_z^{(j)} - k_z^{(i)}}{2k_z^{(j)}} & \frac{k_z^{(j)} + k_z^{(i)}}{2k_z^{(j)}} \end{pmatrix}, \quad (2.4)$$

when for the perpendicular TM polarization the transition matrix reads

$$\hat{M}_{\text{TM}}^{(i,j)} = \begin{pmatrix} \frac{n_j^2 k_z^{(j)} + n_i^2 k_z^{(i)}}{2n_i n_j k_z^{(j)}} & \frac{n_j^2 k_z^{(j)} - n_i^2 k_z^{(i)}}{2n_i n_j k_z^{(j)}} \\ \frac{n_j^2 k_z^{(j)} - n_i^2 k_z^{(i)}}{2n_i n_j k_z^{(j)}} & \frac{n_j^2 k_z^{(j)} + n_i^2 k_z^{(i)}}{2n_i n_j k_z^{(j)}} \end{pmatrix}. \quad (2.5)$$

Total transfer matrix independent for TE and TM polarization can be written as:

$$\hat{M}_{\text{TE/TM}} = \left(\prod_{i=1}^N \hat{M}_{\text{TE/TM}}^{(i-1,i)} \hat{M}_{\text{prop}}^{(i)} \right) \hat{M}_{\text{TE/TM}}^{(N,N+1)}. \quad (2.6)$$

For known total transfer matrix $\hat{M}$ assuming all of the light is incident from the left side with normalized amplitude $E_r^{(1)} = 1$ and introducing reflectance coefficient r and transmission coefficient t can be found based on the matrix equation:

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix}. \quad (2.7)$$

This set of two linear equations can be solved in terms of elements of matrix $\mathbf{M}$ to obtain transmittance (t) and reflectance (r) coefficients as:

$$t = -\frac{M_{21}}{M_{22}}, \quad (2.8)$$

$$r = \frac{M_{11}M_{22} - M_{12}M_{21}}{M_{22}}. \quad (2.9)$$

The total transfer matrix of the whole structure $\hat{M}$ can be used to obtain reflectance and transmission coefficients of the entire system, based on which it is possible to calculate the electric field distribution within the investigated sample. Based on Eq. (2.6) we can construct partial transfer matrix $\hat{M}_{\text{part}}(z_p)$ up to certain distance z_p within i -th layer. Then calculation of the

electric field amplitudes of left ($E_l^{(i)}$) and right ($E_r^{(i)}$) propagating waves are given by the solution of a matrix equation:

$$\begin{pmatrix} E_r^{(i)}(z_p) \\ E_l^{(i)}(z_p) \end{pmatrix} = \hat{M}_{\text{part}}(z_p) \begin{pmatrix} 1 \\ \frac{M_{11}M_{22}-M_{12}M_{21}}{M_{22}} \end{pmatrix}, \quad (2.10)$$

which gives electric field at position z_p as $E(z_p) = \text{Re}(E_r^{(i)}(z_p) - E_l^{(i)}(z_p))$.

Berreman matrix method

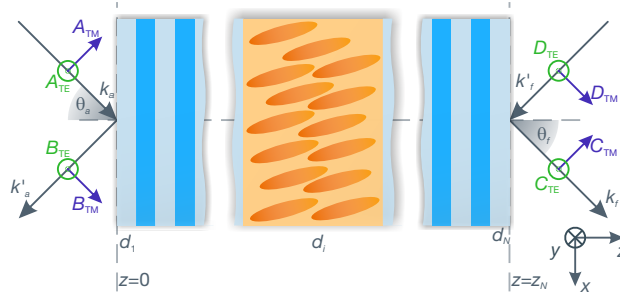


Fig. 2.2 | Schematic representation for electric field amplitudes in Berreman method.

In the transfer matrix method all of the layers in the investigated stack have to be optically isotropic and characterized by a specific refractive index at a given wavelength. An extension to this approach for layered structures is so called Berreman method, named after the author of initial paper [8], also referred to as 4×4 matrix method [9]. Berreman method can be used for layered structures with optical anisotropy, which can be described by 3×3 dielectric tensor $\hat{\epsilon}$.

Description of this method and later results of numerical simulations are following formulation given by Schubert [10]. Broader description including calculations of electric field distribution inside the structures can be found e.g in Ref. [11].

In Berreman method the amplitudes of incident (A), reflected (B), transmitted (C) and back-incident (D) light in both TE and TM linear polarizations are connected by 4×4 matrix $\hat{T}$:

$$\begin{pmatrix} A_{\text{TE}} \\ B_{\text{TE}} \\ A_{\text{TM}} \\ B_{\text{TM}} \end{pmatrix} = \hat{T} \begin{pmatrix} C_{\text{TE}} \\ D_{\text{TE}} \\ C_{\text{TM}} \\ D_{\text{TM}} \end{pmatrix}. \quad (2.11)$$

Investigated amplitudes are schematically drawn in Fig. 2.2. Especially in approach described by Schubert the incident plane lays within x - z plane (i.e. $k_y = 0$). Simulation of light propagation/reflection from investigated structure at any angle can be obtained by rotation of the structure along z axis. Such operation is equivalent to the rotation of dielectric tensors of each layer according to:

$$\hat{\varepsilon}' = \hat{A} \hat{\varepsilon} \hat{A}^{-1}, \quad (2.12)$$

where $\hat{A}$ is rotation matrix by the angle ϕ , which reads:

$$\hat{A} = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.13)$$

Each layer in this approach is described by the partial transfer matrix $\mathbf{T}_i$:

$$\hat{T}_i = \exp\left(i \frac{2\pi}{\lambda} \hat{\Delta}_i d_i\right), \quad (2.14)$$

where d_i is the thickness of a given layer, λ is wavelength of investigated light and $\hat{\Delta}_i$ is a 4×4 matrix given by:

$$\hat{\Delta}_i = \begin{pmatrix} -k_x \frac{\varepsilon_{31}}{\varepsilon_{33}} & -k_x \frac{\varepsilon_{32}}{\varepsilon_{33}} & 0 & 1 - \frac{k_x^2}{\varepsilon_{33}} \\ 0 & 0 & -1 & 0 \\ \varepsilon_{23} \frac{\varepsilon_{31}}{\varepsilon_{33}} - \varepsilon_{21} & k_x^2 - \varepsilon_{22} - \varepsilon_{23} \frac{\varepsilon_{32}}{\varepsilon_{33}} & 0 & k_x \frac{\varepsilon_{23}}{\varepsilon_{33}} \\ \varepsilon_{11} - \varepsilon_{13} \frac{\varepsilon_{31}}{\varepsilon_{33}} & \varepsilon_{12} - \varepsilon_{13} \frac{\varepsilon_{32}}{\varepsilon_{33}} & 0 & -k_x \frac{\varepsilon_{13}}{\varepsilon_{33}} \end{pmatrix}, \quad (2.15)$$

where

$$k_x = n_a \sin \theta_a, \quad (2.16)$$

and θ_a is the incidence angle and n_a is refractive index of the incident me-

dium.

Transfer matrix $\mathbf{T}$ of the total system is a product of partial transfer matrices $\mathbf{T}_i$:

$$\hat{T} = \hat{L}_a^{-1} \left(\prod_{i=1}^N \hat{T}_i \right) \hat{L}_f, \quad (2.17)$$

and input and output matrices $\hat{L}_a^{-1}$ and $\hat{L}_f$, which are given by:

$$\hat{L}_a^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 1 & -1/n_a \cos \theta_a & 0 \\ 0 & 1 & 1/n_a \cos \theta_a & 0 \\ 1/\cos \theta_a & 0 & 0 & 1/n_a \\ -1/\cos \theta_a & 0 & 0 & 1/n_a \end{pmatrix}, \quad (2.18)$$

$$\hat{L}_f = \begin{pmatrix} 0 & 1 & -n_f \cos \theta_f & 0 \\ 0 & 0 & 0 & 0 \\ \cos \theta_f & 0 & 0 & n_f \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (2.19)$$

θ_f is the angle at which light exits the structure. In typical conditions where both incidence and exit medium is vacuum with $n_a = n_f = 1$ it is equal to the incidence angle θ_a .

When total transfer matrix $\hat{T}$ is calculated, assumption that there is no back-incident light (i.e. $D_{\text{TM}} = D_{\text{TE}} = 0$) and for known incident light fields (A_{TE} and A_{TM}) (2.11):

$$\begin{pmatrix} A_{\text{TE}} \\ B_{\text{TE}} \\ A_{\text{TM}} \\ B_{\text{TM}} \end{pmatrix} = \hat{T} \begin{pmatrix} C_{\text{TE}} \\ 0 \\ C_{\text{TM}} \\ 0 \end{pmatrix}, \quad (2.20)$$

reduces to set of 4 linear equations for 4 reflection/transmission fields am-

plitudes $B_{TE}, B_{TM} / C_{TE}, C_{TM}$. Results can be written as:

$$B_{TE} = -\frac{A_{TM}(T_{13}T_{21} - T_{11}T_{23}) + A_{TE}(T_{23}T_{31} - T_{21}T_{33})}{-T_{13}T_{31} + T_{11}T_{33}}, \quad (2.21)$$

$$B_{TM} = -\frac{A_{TM}(T_{13}T_{41} - T_{11}T_{43}) + A_{TE}(T_{31}T_{43} - T_{33}T_{41})}{-T_{13}T_{31} + T_{11}T_{33}}, \quad (2.22)$$

$$C_{TE} = -\frac{-A_{TM}T_{13} + A_{TE}T_{33}}{T_{13}T_{31} - T_{11}T_{33}}, \quad (2.23)$$

$$C_{TM} = -\frac{-A_{TM}T_{11} + A_{TE}T_{31}}{-T_{13}T_{31} + T_{11}T_{33}}. \quad (2.24)$$

Finally, for known incident light amplitudes A_{TE} and A_{TM} , Eq. (2.21) allows for calculation of all reflected and transmitted light electric fields amplitudes. As in general the elements of the matrix $\hat{T}$ are complex, calculated fields carry information about the amplitude as well as about the phase of all of the investigated fields.

2.1.2 Distributed Bragg reflector

Typical planar microcavity consists of two distributed Bragg reflectors (DBRs). A DBR is a structure comprised of repeated layers made of two materials characterized by different refractive indexes: higher n_{high} and lower n_{low} . The thickness of each layer are designed for specific central wavelength λ_0 equal:

$$d_i = \frac{\lambda_0}{4n_i}. \quad (2.25)$$

For this condition the optical length of each layer is equal exactly one fourth of the wavelength. For such conditions Fresnel reflections at each interface between the consecutive layers interfere destructively at the transmission side of the DBR resulting in high reflectance coefficient. Exact reflectance properties of the DBRs can be easily simulated using transfer matrix, or Berreman matrix method.

Fig. 2.3a presents simulated reflectance spectra for increasing number of pairs of layers characterized by refractive indexes: $n_{\text{high}} = 2.2$ and lower $n_{\text{low}} = 1.5$ (corresponding roughly to dielectrics like SiO_2 and TiO_2). With the increasing number of pairs at the central wavelength $\lambda_0 = 700$ nm the reflectance coefficient increases, exceeding 91% for 5 DBR pairs and reaching 99.8% for 10 DBR pairs. Additionally for high number of DBR pairs flat and high value of the reflectance is preserved over a relatively broad spectral range, often referred to as photonic stopband region. In presented simulations for 10 DBR pairs reflectance stays above 95% between 620 nm and

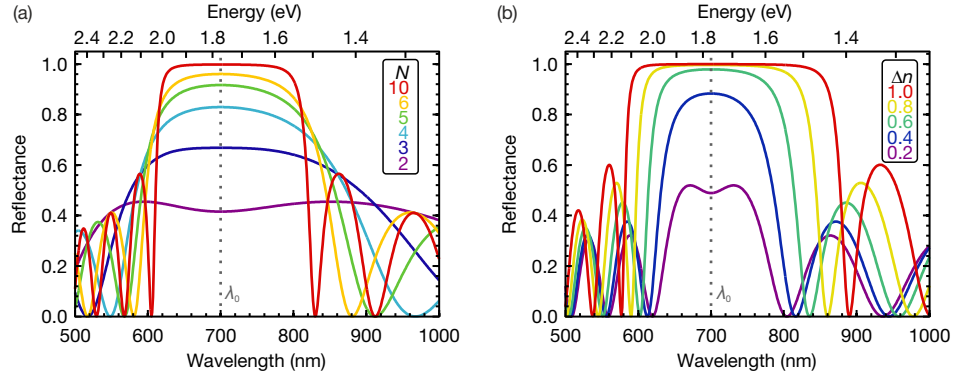


Fig. 2.3 | Distributed Bragg reflector (DBR). (a) Simulated reflectance spectra for a DBR made of different number N of pairs materials with refractive indexes $n_{\text{hig}} = 2.2$ and $n_{\text{low}} = 1.5$. (b) Reflectance spectra for increasing refractive index difference $\Delta n = n_{\text{hig}} - n_{\text{low}}$ performed for 6 pairs and $0.5(n_{\text{hig}} + n_{\text{low}}) = 1.85$. Simulations performed for central wavelength $\lambda_0 = 700$ nm.

795 nm.

For a given number of DBR layers the reflectance can be increased by increasing the refractive index contrast between the layers $\Delta n = n_{\text{hig}} - n_{\text{low}}$. Fig. 2.3b shows how simulated reflectance spectra changes upon increase of the Δn between 0.2 and 1 keeping constant mean refractive index $0.5(n_{\text{hig}} + n_{\text{low}}) = 1.85$. With 6 DBR pairs and the lowest investigated $\Delta n = 0.2$ the reflectance of the structure does not exceed 50%. With refractive index contrast above 0.6 the reflectance at λ_0 approaches 1 with simultaneous broadening of the photonic stopband range.

Structure of a realistic dielectric DBR is presented in Fig. 2.4a. It shows scheme of DBR made of 8 pairs of dielectrics: silicon dioxide (SiO_2) and titanium dioxide (TiO_2), which are characterized with high refractive index contrast. Structure was designed to provide maximum reflectance at the wavelength of 520 nm, which for refractive indexes $n_{\text{SiO}_2} = 1.46$ [12] and $n_{\text{TiO}_2} = 2.22$ [13] leads to the thicknesses of the layers: $d_{\text{SiO}_2} = 89.0$ nm and $d_{\text{TiO}_2} = 58.7$ nm. Experimentally measured reflectance spectrum at normal incidence to the DBR is shown by gray dots in Fig. 2.4b with clearly visible photonic stopband region of high reflectance between approx. 480 nm and 600 nm. The experimental results are in a good agreement with simulations performed with transfer matrix method, presented in orange line in Fig. 2.4b. Simulations well recreate not only the stopband region, but also the oscillations of the reflectance coefficient with wavelength outside of this range, especially spectral positions of minima of the reflectance.

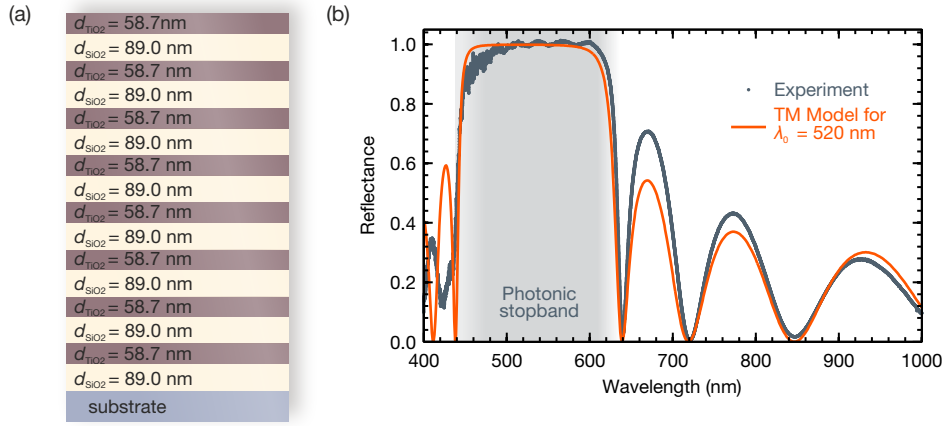


Fig. 2.4 | Distributed Bragg reflector (DBR). (a) Scheme of a $\text{SiO}_2/\text{TiO}_2$ stack of layer forming a DBR centred at $\lambda_0 = 520$ nm. (b) Experimentally measured reflectance spectrum of a DBR (gray dots) compared with transfer matrix simulation (orange line).

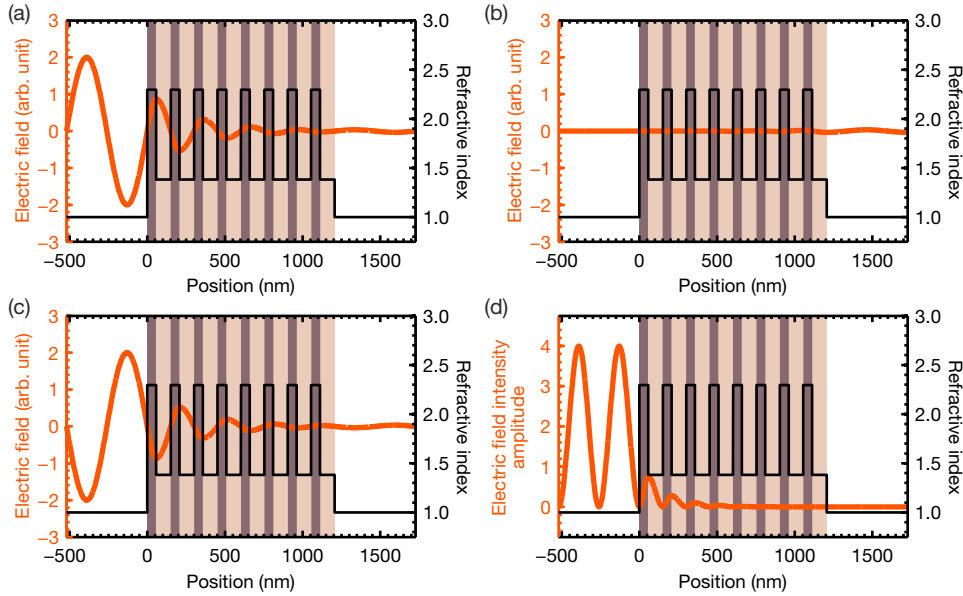


Fig. 2.5 | Electric field distribution in DBR. Electric field distribution with initial (a) $\frac{3\pi}{2}$, (b) π and (c) $\frac{\pi}{2}$ phase shift. (d) Spatial distribution of mean value of electric amplitude ($|E|^2$). Incident light is a plane wave with wavelength $\lambda_0 = 520$ nm.

Transfer matrix method can be also used to calculate the distribution of the electric field across the DBR. Fig. 2.5a–c present simulated electric field within the structure at different initial phases of the incident plane wave

with $\lambda_0 = 520$ nm. As the DBR was designed to provide high reflectance at λ_0 almost all electric field is reflected leading to maximum electric field at the incident side two times higher than the incident amplitude of 1 for $\frac{3\pi}{2}$ initial phase Fig. 2.5a (constructive interference of forward propagating incident field and traveling backwards reflected wave). Consequently at the transmission side amplitude of the electric field is minimal. Within the DBR the amplitude of the electric field gradually decreases. With changing initial phase of the incident light, electric field distribution behaves as a standing wave, and for π incident phase there is destructive interference between the incident and reflected field Fig. 2.5b. A time averaged electric field can be obtained by averaging electric field intensity over incident phase factors from 0 to 2π range, what is plotted in Fig. 2.5d. Electric field partially penetrates the structure of the DBR, but with amplitude decreasing the deeper into the structure.

2.1.3 Planar microcavity

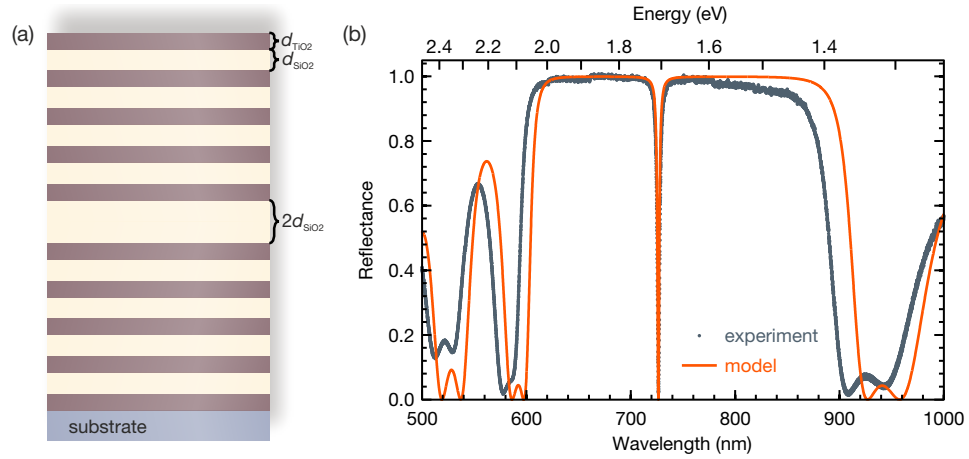


Fig. 2.6 | Dielectric microcavity. (a) Scheme of a $\text{SiO}_2/\text{TiO}_2$ stack of layer forming a microcavity. (b) Experimentally measured reflectance spectrum of a microcavity (gray dots) compared with transfer matrix simulation (orange line).

Two DBRs separated by a layer with optical thickness of $\lambda_0/2$ schematically show in Fig. 2.6a acts as an optical cavity. Such structure made of dielectric $\text{SiO}_2/\text{TiO}_2$ layers was fabricated for central wavelength of $\lambda_0 = 725$ nm. Measured reflectance spectra of such structure in Fig. 2.6b shows spectrally narrow cavity mode of high transmission within the photonic stopband region. Measured spectrum is in a good agreement with transfer

matrix simulations performed for $\lambda_0 = 724.8$ nm. Cavity modes in reflectance or transmittance spectra are characterized with Lorentzian lineshapes and their spectral width determines quality factor Q of the cavity defined as

$$Q = \frac{E_0}{\delta E}, \quad (2.26)$$

where E_0 is the resonance energy and δE linewidth (full width at half maximum, FWHM). Fitting the reflectance spectra to the experimental results gives the resonance linewidth of 6.7 nm with Q -factor of 250. The value reasonably well matches the simulated predictions of 350 quality factor for such structure based on two DBRs containing only 9 dielectric layers.

For the state of the art planar microcavities based on the DBRs the Q factors exceed 1×10^5 for both semiconductor [14] and dielectric [15] materials. The number of Bragg pairs exceeds 14 for dielectric and 32 for GaAlAs-based structures.

The cavity resonance is characterized by strong localization the electric field corresponding to the cavity mode. This can be shown using transfer matrix simulations of the electric field distribution within the investigated dielectric structure.

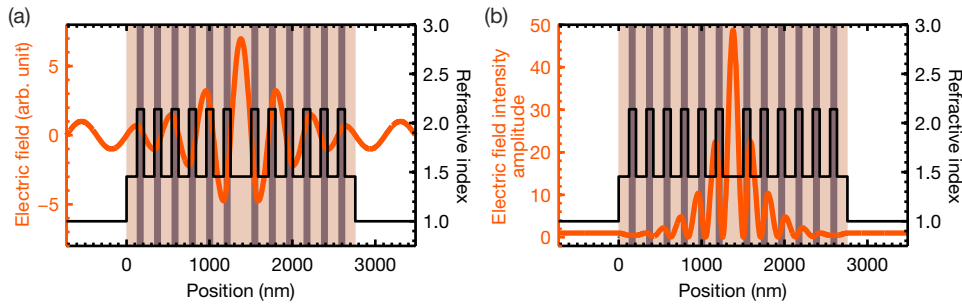


Fig. 2.7 | Electric field distribution in dielectric microcavity. (a) Electric field distribution with initial $\frac{3\pi}{2}$ phase shift. (b) Space distribution of mean value of electric amplitude ($|E|^2$). Incident light is a plane wave with wavelength $\lambda_0 = 724.8$ nm.

Fig. 2.7a presents temporal distribution of the electric field within the cavity. The highest amplitude of the electric field is located exactly at the center of the structure in the middle of the $\lambda/2$ -thick layer. The corresponding time-averaged electric field intensity distribution within this cavity is presented in Fig. 2.7b, which shows that at the center the electric field intensity amplitude is almost $50\times$ higher than the incident light. Electric field spans the entire structure, but it is mostly confined within the center of the cavity.

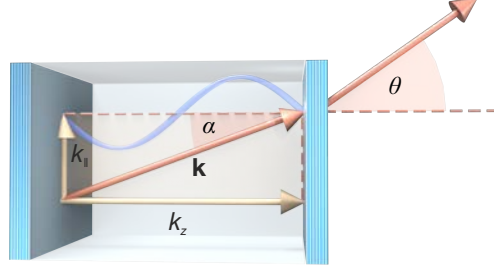


Fig. 2.8 | Cavity photon wave vector. Schematic illustration of a cavity with marked wave vector of cavity photon $\mathbf{k}$ separated into components in cavity growth direction k_z and in-plane component $k_{\parallel}$.

The resonance condition of a planar cavity changes slightly for the electromagnetic waves with nonzero wave vector component in the plane of the cavity $k_{\parallel}$, corresponding to the light incident/exiting the structure at nonzero angle. Dispersion relation describing the dependence of the cavity mode energy on wave vector of the cavity photon is given by:

$$E_{ph} = \frac{\hbar c}{n_c} \sqrt{k_z^2 + k_{\parallel}^2}. \quad (2.27)$$

Wave vector components in growth direction of the cavity k_z and in-plane component $k_{\parallel}^2 = k_x^2 + k_y^2$ are schematically depicted in Fig. 2.8.

As k_z wave vector component is a constant determined by the cavity optical length for small in-plane wave vector $k_{\parallel}$ values relation (2.27) can be expanded in Fourier series, which up to the second order leads to quadratic dependence in the form:

$$E_{ph} = \frac{\hbar c}{n_c} \sqrt{k_z^2 + k_{\parallel}^2} \approx E_0 + \frac{\hbar^2}{2m_{\text{eff}}} k_{\parallel}^2, \quad (2.28)$$

where m_{eff} is effective mass of the photon.

Value of in-plane wave vector is directly connected with the angle θ at which it exits the cavity structure (Fig. 2.8) by equation:

$$k_{\parallel} = \frac{E_{ph}}{\hbar c} \sin \theta. \quad (2.29)$$

This direct dependence grants easy experimental access to the dispersion relation of the cavity mode for angle resolved emission/reflectance measurements.

2.1.4 Fourier space imaging and tomography

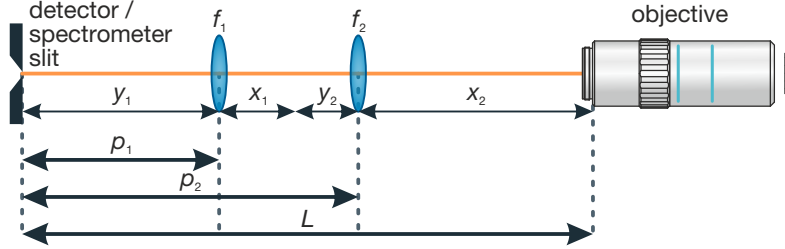


Fig. 2.9 | Scheme of the setup: imaging the Fourier plane at the entrance of a monochromator.

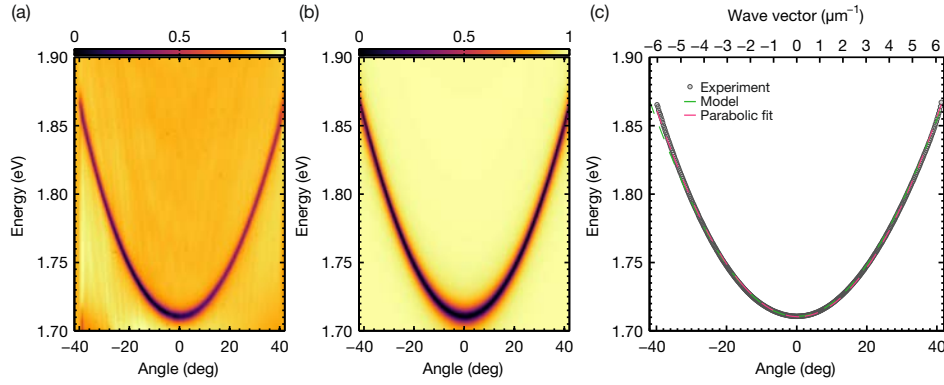


Fig. 2.10 | **Dispersion of a cavity mode.** (a) Angle-resolved reflectance spectra from $\text{SiO}_2/\text{TiO}_2$ microcavity. (b) Simulated by Berreman method angle-resolved reflectance spectra of dielectric microcavity. (c) Comparison between experimental cavity mode energy dependence on detection angle (gray circles), results of Berreman model and parabolic fit (pink dashed line).

An extremely useful experimental technique allowing to get access to the emission angle is Fourier plane imaging, also known as back focal plane imaging. A positive (converging) lens at the focal length distance performs Fourier transform of object plane. At this plane all rays emitted from an object at a given angle θ are focused at a certain distance r from the optical axis given by equation:

$$\sin \theta = r \frac{\text{NA}}{r_{\text{max}}}, \quad (2.30)$$

where r_{max} is radius of the image and NA is numerical aperture of the lens/objective.

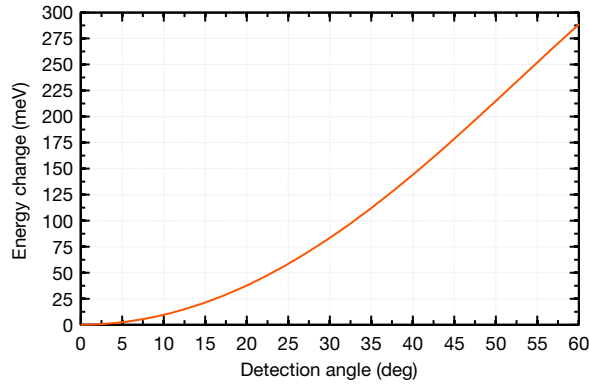


Fig. 2.11 | Dependence of the cavity mode energy on detection angle. Calculated for $\lambda/2$ SiO₂/TiO₂ dielectric microcavity with the resonant wavelength of 720 nm.

In the experiment to maintain high numerical aperture and long working distances lens performing Fourier transform can be replaced with a microscope objective. Such more complex optical system can also be used to optically perform Fourier transform, but the distance of the Fourier plane is not directly connected with effective focal length of the objective. Some manufacturers specify the exact position of back focal plane of their objectives (e.g. [16]), but for typical objectives with long working distance the position of the Fourier plane is close to the rear aperture of the objective.

Fourier transform performed optically by the objective can be further imaged on a desired detector. Fig. 2.9 shows a specific optical setup in which two positive lenses can be used to obtain the Fourier plane image on a detector or, as shown in the image, on the entrance slit of the spectrometer. In this configuration f_2 lens produces image of the Fourier plane at position y_2 , which is further on imaged at the position of the detector. The use of two lenses allows to optimize magnification M of the Fourier space to fully use the vertical direction of the detector, especially a CCD matrix connected to a spectrometer. Spatial size of the Fourier image projected by a microscopic objective in the first approximation is equal to the objectives' pupil diameter D , which can be calculated as

$$D = 2NAf, \quad (2.31)$$

where f is the effective focal length of the objective and NA its numerical aperture.

For configuration presented in Fig. 2.9 positions of the lenses $p_{1,2}$, their

focal lengths $f_{1,2}$ and the magnification of the final image M at the distance L from the objective entrance is connected by set of three linear equations:

$$\begin{cases} \frac{1}{f_2} = \frac{1}{x_2} + \frac{1}{y_2} = \frac{1}{L-p_2} + \frac{1}{y_2} \\ \frac{1}{f_1} = \frac{1}{x_1} + \frac{1}{y_2} = \frac{1}{p_2-p_1-y_1} + \frac{1}{p_2} \\ M = \frac{y_2}{x_2} \cdot \frac{y_1}{x_1} = \frac{y_2}{L-p_2} \cdot \frac{p_1}{p_2-p_1-y_2} \end{cases} \quad (2.32)$$

This set of equations (2.32) for known focal lengths $f_{1,2}$, desired magnification M and the distance between objective and spectrometer entrance slit can be numerically solved to obtain positions of the lenses $p_{1,2}$. In general, this set of equations is mathematically fulfilled by two set of positions of the lenses $p_{1,2}$, but not every solution is physical as calculated positions of the lenses can be negative or imaginary. Typically trials with different experimentally available focal lengths allows to find solution which can physically fit within the experimental setup. In practice it is typically easier to use lenses with relatively long focal lengths (above 500 mm), as the final image they project is often less sensitive to their exact positioning.

Calibration of the Fourier plane imaging is the easiest to preform with broadband light illumination of a sample with a cavity mode. Starting from the calculated position of the lenses, their further position can be adjusted to obtain the highest possible energy depth of the parabolic dispersion relation of the cavity mode which preserves narrow spectral linewidth of the mode.

Fig. 2.10a presents angle-resolved reflectance spectrum of the dielectric cavity. A scheme of the experimental setup is presented in Fig. 2.12. Spectrally broadband illumination from a halogen lamp through a non-polarizing beam-splitter is directed towards a microscopic objective, which focuses the incident lamp on the surface of the sample. Two lenses $f_{1,2}$ projects the image of the Fourier plane of the microscope objective at the entrance slit of a monochromator equipped with a CCD camera. In this specific configuration for the total distance between the spectrometer and the microscopic objective $L = 168$ cm for focal length of the lenses equal to $f_1 = 500$ mm $f_2 = 750$ mm and magnification $M = -1.1$, calculated positions of the lenses according to the Eq. (2.32) are equal to: $p_1 = 72.3$ cm, $p_2 = 124.4$ cm.

Vertical slit of the monochromator selects from the 2D image in (k_x, k_y) directions a cross section along k_y (Fig. 2.12b). Inside the monochromator this signal is spectrally resolved by a grating and produces a 2D image in wavelength- y space on the CCD camera, as schematically shown in Fig. 2.12b. Following Eq. (2.30) the distance along y direction on the camera can be recalculated to the reflection angle to obtain angle-resolved spec-

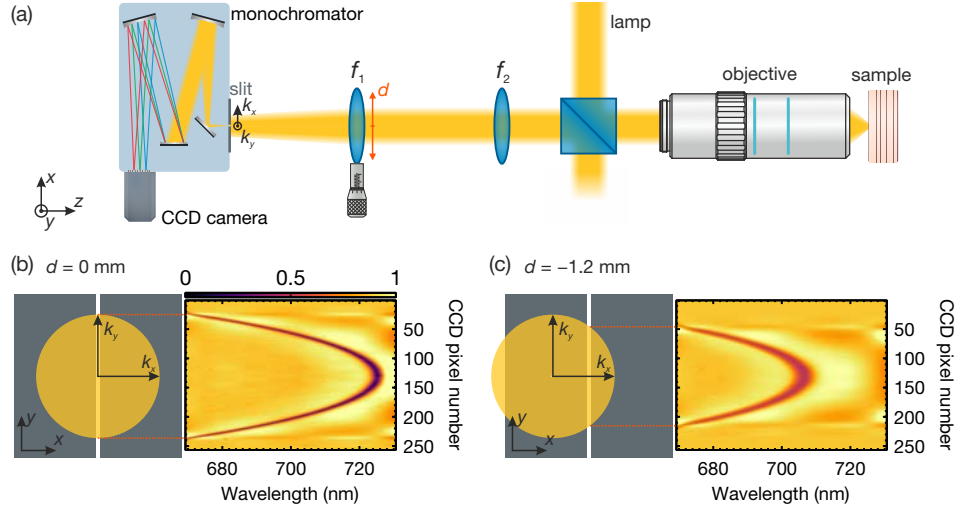


Fig. 2.12 | Experimental setup for angle-resolved tomography. (a) Scheme of the experimental setup in reflectance configuration. (b) Image recorded when f_1 lens is at the optical axis, when the measured signal corresponds to k_y wave vector direction. (c) When lens f_1 is moved along x direction by a distance $d = 1.2$ mm, the monochromator slit selects slice of the dispersion relation at nonzero k_x .

trum as shown in Fig. 2.10a. As predicted by Eq. (2.29) the spectrum shows parabolic-like dependence of the cavity mode energy from the incidence or reflectance angle. Experimental results are also consistent with the transfer matrix simulations performed for light incident at different angles. Results of simulations can be also used to precisely calibrate the angle scale in the experiment. Using Eq. (2.30) to calibrate the angle scale assumes that full numerical aperture of the microscopic objective is fully used. It is a useful approximation, which can be improved by comparison with the simulated spectra in which $NA/r_{\max}$ ratio from Eq. (2.30) can be used as a fitting parameter to obtain the best fit between the experiment and the theoretical predictions. Such comparison is shown in Fig. 2.10c presenting fitted position of the cavity mode as a function of the reflection angle. Additionally, the parabolic fit presents that in the investigated angle/wave vector range the first parabolic expansion of the Eq. (2.28) is enough to properly describe the dispersion relation of the cavity mode.

In this figure to obtain the best agreement between the experimental results and the transfer matrix simulations $NA/r_{\max}$ ratio was set to 0.0031 pix^{-1} . As a result, the maximum reflection angle observable in

Fig. 2.10a is equal to 40.6 deg, which is less than maximum value expected from observation with an objective with numerical aperture $NA = 0.75$, which corresponds to maximum angle of 48.6 deg.

Fig. 2.11 presents simulated dependence of the energy shift of the cavity mode in $\text{SiO}_2/\text{TiO}_2$ cavity as a function of the observation angle, which can be used for calibration of setups with objectives of varying numerical apertures.

Presented earlier experimental scheme allows for easy angle-resolved spectroscopic measurements within cross section along specific wave vector directions (k_y). To measure a dispersion relation for all wave vector directions in 2D space a angle-resolved tomography setup can be used. As shown in Fig. 2.12 one of the imaging lenses can be mounted on a stage allowing for movement of the lens in x direction. Such small movement away from the optical axis results in a shift of the Fourier plane image in x direction at the position of the spectrometers' entrance slit, as schematically shown in Fig. 2.12c. In this case vertical slit of the spectrometer still selects cross section of the Fourier plane along k_y , but at nonzero k_x component, dependent on the position of the stage d . Motorized movement of the stage synchronized with the readout of the CCD camera of the spectrometer allows to collect data for all k_x values contained within the Fourier plane and obtain a 3D set of data in k_x , k_y and wavelength (photon energy).

Results of such tomography in reflectance for investigated dielectric cavity are shown in Fig. 2.13, presenting cross sections along different wave vector directions and photon energies. The cross sections along all presented: k_x , diagonal ($k_x = k_y$) and k_y wave vector directions show parabolic like dispersion relation of the cavity mode.

As expected from fully isotropic sample, the results show parabolic dispersion relation of the cavity mode in all $k_{\parallel}$ wave vector directions.

2.2 Cavity filled with birefringent medium

Cavities described in this work up to now were fabricated from optically isotropic materials. Introduction of optically anisotropic materials brings completely new phenomena, especially convenient to study in a liquid crystal systems, where the direction of the optical anisotropy can be dynamically tuned by external fields.

Following especially brief introduction of the properties of liquid crystal, especially nematics, based on [17, 18, 19, 20] this section concentrates on dielectric cavities filled with nematic liquid crystal.

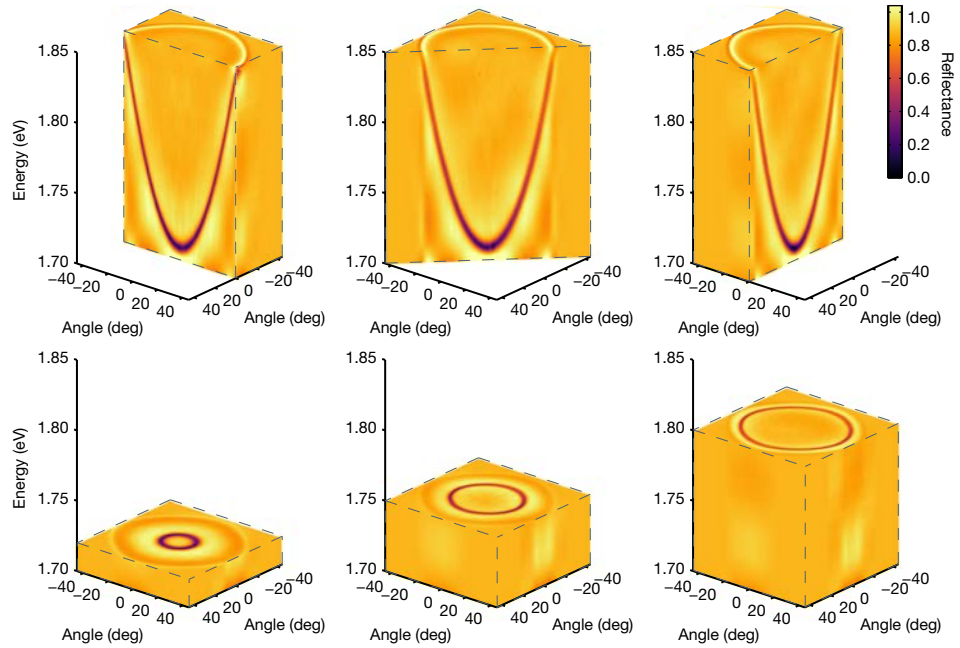


Fig. 2.13 | Angle resolved tomography in reflectance. Cross sections across different wave vector directions and at different photon energies.

2.2.1 Nematic liquid crystals

Liquid crystals (LC) exhibit various different mesophases: thermodynamically stable phases with properties combining characteristics of the solid crystal with features typical for isotropic liquid. Substances demonstrating such phases staying in between crystal and liquid are referred to as mesogens.

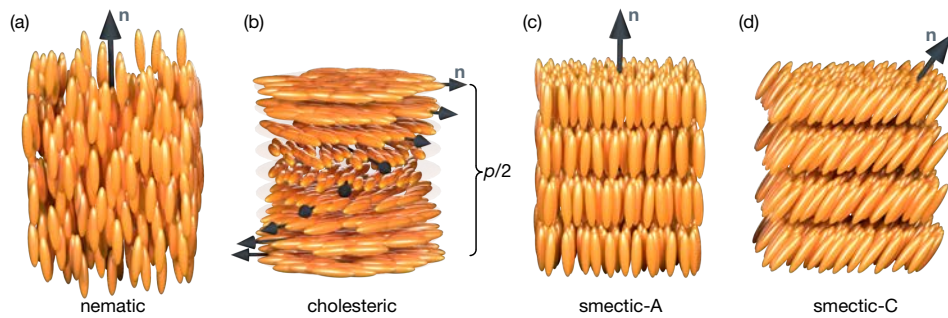


Fig. 2.14 | Mesophases of liquid crystals. (a) nematic, (b) smectic-A (c) smectic-C and (d) cholesteric (chiral nematic).

Depending on how mesophases are achieved in a material one can di-

stinguish two types of liquid crystals. Lyotropic liquid crystals are obtained when a sufficient amount of material is dissolved in a solvent. In them the critical parameter governing over the existence of different mesophases is the concentration. Examples of lyotropic liquid crystals are soaps and phospholipids. In the other type called thermotropic, liquid crystalline properties are obtained by melting a crystalline solid. Here the most important parameter to control between different mesophases are temperature or pressure. There are also mesogens exhibit both lyotropic and thermotropic phases, which are sometimes referred to as amphotropic.

All of those types can be further distinguished based on their symmetry, differentiating three major classes: nematics, cholesterics and smectics. Order in those classes is typically depicted using various configuration of an elongated rods representing their anisotropic molecules, as is presented in Fig. 2.14.

In the nematic phase (Fig. 2.14a) position of the rod-like molecules is random, but their orientation is aligned along a specific direction represented by a so-called director $\mathbf{n}$ vector, which can be regarded as a crystal axis. Their orientation can be described using a unit vector $\mathbf{n}$ called director.

In the nematic phase exhibit long range orientational order, as schematically shown in Fig. 2.14a. All of the molecules within a nematic mesophase are preferentially aligned along specific direction. Their orientation can be described using a unit vector $\mathbf{n}$ called director, which can vary within the sample. Molecules forming nematic phase are free to rotate around their long axis, and have no preferential arrangement of their ends, thus the sign of the director has no physical meaning. In nematics there is no long-range positional order, and the locations of the centers of mass of the molecules is not correlated between them. As a result nematics are one of the least ordered liquid crystalline phase. Optically the nematic phase is birefringent and behaves as a uniaxial medium with ordinary (n_o) and extraordinary (n_e) refractive indexes and the anisotropy axis pointing in the direction defined by the director.

Without specific anchoring provided by the boundaries liquid crystals take on a milky appearance due to scattering of the light at random fluctuations within its volume. They become crystal clear with consistent director within the whole LC volume only after specific direction is favored by the interfaces.

In the LC cells consisting of the LC imposed between two glass surfaces orientation of the director is defined by the treatment of the glass substrates. One can distinguish two main nematic orientation types in such geometry. In homeotropic alignment (HT) the director is oriented perpendicular to

the plane of the substrate, as shown schematically in Fig. 2.15a. Such configuration can be achieved for example by covering both glass substrates with thin layer of surfactant such as HTAB (hexadecyl trimethyl ammonium bromide). This soap-like material tends to orient themselves perpendicularly to the surface and impose homeotropic orientation of the liquid crystal.

In the other planar or homogeneous alignment (HG) LC director is oriented parallel to the glass substrates, as shown schematically in Fig. 2.15b. To obtain such orientation both glass substrates can be covered by a thin layer of a polymer such as PVA (polyvinyl alcohol). To favor a desired orientation within the plane surface of the polymer layer can be rubbed in this specific direction. Due to strain and/or stress imposed during the rubbing process long axis of the liquid crystal molecules tend to align along the rubbing direction.

Apart from this two main alignment types there are multiple other variations like hybrid, twisted or fingerprint with various ordering techniques suitable for specific situations like photoorientation possible for photosensitive materials.

Chemically molecules exhibiting nematic mesophases have typically anisotropic shape often based on rigid core. Often this core consists of connected aromatic rings with various terminal groups which strongly affects properties of the material.

Another distinguished LC phase is so called chiral nematic phase, historically also referred to as cholesteric. Similarly to the nematics, in cholesteric phase lacks long range positional ordering, as positions of the centers of mass of the molecules is fully random. Cholesterics exhibit long range orientation order, but the director rather than being constant within the whole volume assumes a helical form as in Fig. 2.14b. Helical structure appears due to admixture of chiral molecules into the liquid crystal. Within the cholesteric director rotates around a specific axis and the distance within which the director exhibit full 360° rotation is referred to as a pitch p . Nematic phase can be thought of as a specific case of chiral nematic in which the pitch length is infinite. Specific helicity of the cholesterics combined with periodically varying refractive index leads to specific optical properties of this phase resulting in selective reflection of circularly polarized light of specific handedness at wavelength dependent of the birefringence and the pitch length.

In specific conditions cholesteric liquid crystals can organize into a more complex 3D photonic crystal structures referred to as Blue phases.

In the smectic phase in addition to the orientation order, there is also positional order, which defines multiple subphases. In smectic-A presented in

Fig. 2.14c molecules are arranged into layers with director normal to the plane of the layer. In smectic-C the molecules also form layers, but with the director $\mathbf{n}$ tilted versus the normal direction as shown in Fig. 2.14d. Smectic subphases C, C*, G, H, I, F ... Q have been identified.

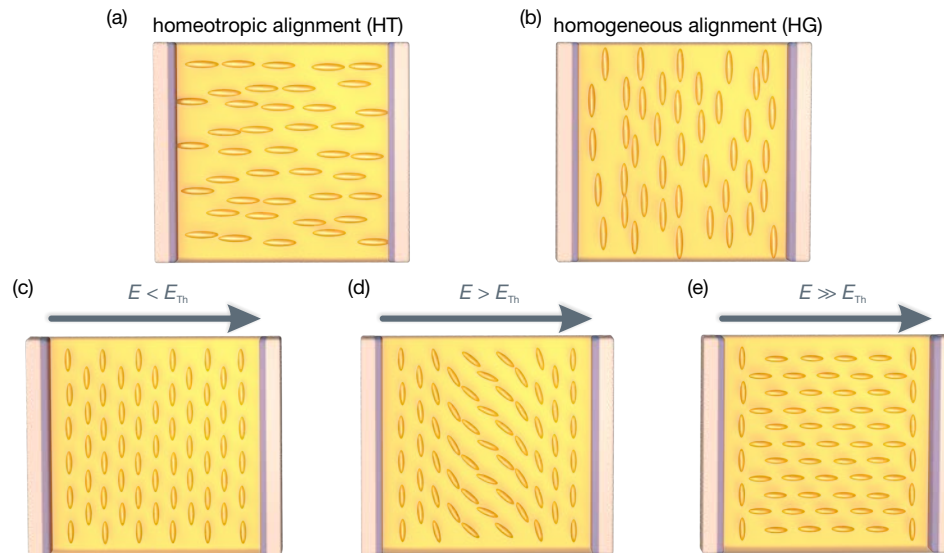


Fig. 2.15 | Liquid crystal cells. (a) Nematic LC cell with homeotropic (HT) alignment. (b) Nematic LC cell with homogeneous (HG) alignment. (c) In HT oriented cell external electric field E does not induce reorientation of the LC as long as the field is smaller than threshold value E_{Th} . (d) When external field E is higher than the threshold value LC molecules within the cell reorient with the long axis towards the electric field direction. (e) At high electric fields bulk of the material is oriented along the external field.

In addition to the preparation of the boundaries structure of the liquid crystals can be affected by external electric or magnetic fields. In the nematic LC cells such fields can induce rotation of the director, which is known as Frederiks transition (Fréedericksz transition). For positive dielectric anisotropy $\Delta\epsilon = \epsilon_{\parallel} - \epsilon_{\perp}$ orientation of the LC molecules parallel to the external electric field E is energetically favorable. Nevertheless to cause actual reorientation the influence of the external field has to at first overcome the energy required for such deformation. For this reason starting from the planar orientation of the LC with the electric field applied perpendicular to the cavity plane (Fig. 2.15c) there is no disorientation as long as the electric field E is lower than the threshold field which for the simplest one elastic

constant approximation is equal to [17]:

$$E_{\text{Th}} = \frac{\pi}{d} \sqrt{\frac{4\pi K}{\Delta\epsilon}}, \quad (2.33)$$

where K is the elastic constant and d is the thickness of the cell.

When the electric field is above the threshold value the LC molecules start to reorient, with the reorientation angle dependent on the strength of the external field Fig. 2.15d, but it is typically assumed that the molecules located at the vicinity of the orienting layers are stationary.

The same effect can be achieved applying external magnetic field to the cells filled with material exhibiting anisotropy of diamagnetic susceptibility $\Delta\chi$.

Ability to affect the optical anisotropy of the nematic liquid crystals by means of externally applied electric field makes them extremely interesting medium to consider within an optical resonator.

2.2.2 Cavity filled with liquid crystal

Fig. 2.16a presents a scheme of a cavity filled with liquid crystal investigated in this work. The cavity consists of two dielectric DBRs which are separated by a nematic liquid crystal layer. Initial orientation of the liquid crystal anisotropy layer is determined by polymer orienting layers located at the top of the each DBR. The polymer layers are relatively thin (approx. 50 nm) and subjected to a rubbing process, which determines the preferential direction of the LC molecules at the polymer interface. For nematic liquid crystals, depending on the choice of the liquid crystal mixture and processing of the orienting layer there are two main configurations of the LC molecules within the cell.

In so called homogeneously oriented cells without external electric field the director of the LC molecules is oriented parallel to the cavity plane, as schematically shown in Fig. 2.16b. In this configuration for linearly polarized light along vertical (V, y) direction effective refractive index of the LC layer is equal to n_o . For light with perpendicular linear polarization along horizontal (H, x) direction effective refractive index is equal to the extraordinary refractive index n_e .

The opposite configuration of the LC molecules is referred to as homeotropic (HT) alignment, schematically depicted in Fig. 2.16c. In this case the director is oriented along z axis, perpendicular to the cavity plane. As a result effective refractive index for a plane wave incident perpendicularly onto

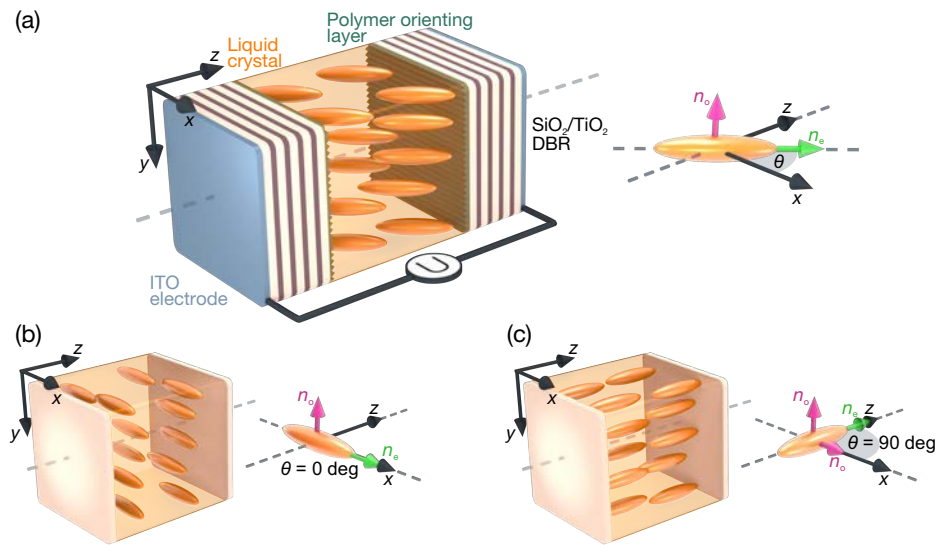


Fig. 2.16 | Microcavity filled with nematic liquid crystal. (a) Scheme of microcavity filled with liquid crystal. Liquid crystal fills space between two dielectric DBRs. To initially orient LC molecules DBRs are finished by polymer layer. LC molecules orientation angle θ can be smoothly controlled by external voltage applied to transparent ITO electrodes. (b) When molecules rotation angle $\theta = 0$ extraordinary direction of LC is oriented along x axis. (c) At $\theta = 90$ deg extraordinary direction is oriented along z direction, leading to equal refractive index (n_o) along x and y directions.

the cavity plane is characterized by the same effective refractive index equal to n_o , regardless on the polarization of the incident light.

One of the biggest advantages of the liquid crystals due to their liquid nature is ability to steer their optical properties by external fields. A phenomenon utilized in investigated LC microcavities is a reorientation of the director of the LC molecules under external electric field, which is also referred to as Fréedericksz transition. To achieve such tunability DBRs forming the cavity are grown directly on top of a indium-tin oxide (ITO). ITO is transparent in the visible range and is electrically conductive. Applying external voltage to the ITO electrodes at the opposite DBRs results in electric field to the LC layer, which can induce reorientation of the LC molecules.

Fig. 2.17a presents a photo of such device fabricated at the Military University of Technology in the group of prof. Wiktor Piecek using a highly birefringent nematic liquid crystal synthesized in the group of prof. Przemysław Kula.

The cavity is formed between two 5 mm thick glass substrates. DBRs

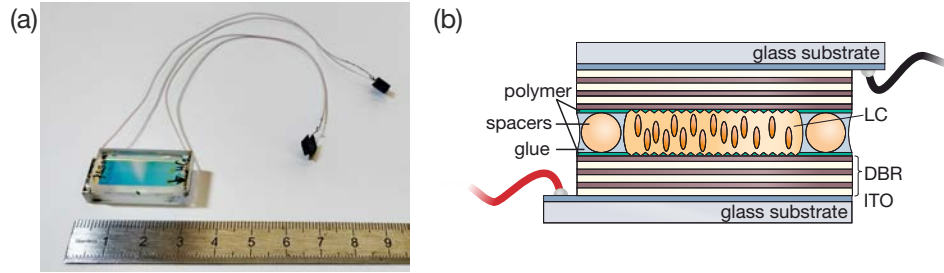


Fig. 2.17 | LC microcavity image. (a) Photo of the investigated LC microcavity compared with a centimetre scale. (b) Schematic cross section through the device.

centered at 745 nm can be visible in this picture by a blue hue between the glasses. Both glass substrates are finished with ITO layers which are connected to the wires to allow application of the electric signal. The actual cavity is formed between the DBRs, with the space filled with the liquid crystal. Spacing between the DBRs for presented cavity is around $3.5 \mu\text{m}$. The spacing is set during the fabrication process by the use of silica spacers with calibrated diameter. The spacers are mixed with the glue which is distributed at the edges of the cavity to set the cavity width.

Configuration of the LC molecules without the applied voltage is determined by processing of orienting polymer layers. In presented device rubbing along longer edge of the cavity leads to orientation of LC molecules as presented in Fig. 2.16b. External voltage applied to the ITO electrodes introduces reorientation of the director around the y axis by an angle θ dependent on the amplitude of the signal. At sufficiently high voltages the LC layer can be almost completely to HT orientation shown in Fig. 2.16c.

During rotation of the molecules the effective refractive index for vertically polarized light remains constant and equal to n_o , but the effective refractive index for horizontally polarized light changes between n_e at $\theta = 0$ deg to n_o at $\theta = 90$ deg. Berreman matrix simulations presenting how such change affects the cavity mode of the cavity is presented in Fig. 2.18. The reflectance spectra at normal incidence within the photonic stopband spectral region was calculated independently for horizontal and vertical linear polarization of incident light. Existence of the cavity mode resonance for specific energy and incident light polarization appear as a minimum of the reflectance. Due to significant thickness of the liquid crystal layer multiple cavity modes can be observed within the stopband region. They can be numbered by the integer mode number N describing how many antinodes of the standing wave are confined within the cavity. For the investigated cavity

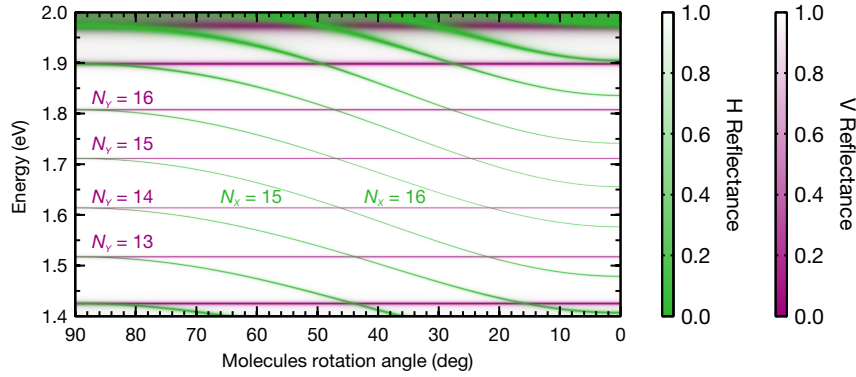


Fig. 2.18 | LC microcavity tuning at normal incidence. Simulated by Berreman method normal incidence reflection from LC microcavity at different LC molecules orientation angles.

consecutive resonances with mode numbers between $N = 12$ and $N = 17$ are confined within the photonic stopband region.

At molecules rotation angle $\theta = 90$ deg corresponding to high voltage applied to the LC microcavity cell, modes for linear H and V polarizations are degenerate, when director is oriented perpendicularly to the cavity plane. When the molecules rotation angle decreases effective refractive index of H polarized mode starts to increase resulting in the decrease of the cavity mode energy. As the energy of V polarized mode remains unaffected it introduces splitting between H and V polarized modes with the same mode numbers.

As the molecules rotation advances the energy of H polarized modes can decrease enough to intersect with V polarized mode characterized with lower mode number. E.g. tracing vertically polarized mode with $N_V = 14$ at $\theta = 45.7$ deg it becomes degenerate with H polarized mode with $N_H = 15$.

Depending on the thickness and the birefringence of the LC layer multiple crossings of perpendicularly polarized mode can occur. In the investigated cavity only one additional degeneracy condition can be found: for $N_V = 14$ at $\theta = 22.8$ deg.

Finally when the molecules are aligned parallel to the cavity plane ($\theta = 0$) the cavity modes with perpendicular linear polarizations are energy-split.

Based on the Fig. 2.18 three main regimes appearing in LC cavity can be distinguished based on which of the modes characterized with different mode numbers (N_x, N_y) are close to the degeneracy.

In the first regime when in which molecules are aligned along the z axis

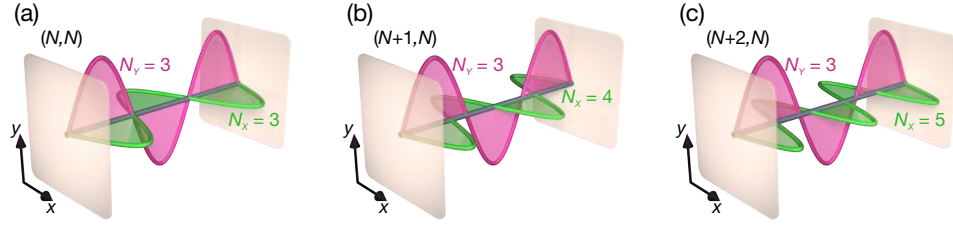


Fig. 2.19 | Different regimes of mode degeneracy in LC microcavity. (a) In typical isotropic multimode cavity mode numbers for linearly polarized modes along x and y directions are the same i.e. $(N_x, N_y) = (N, N)$. When refractive index along x direction is higher than for y direction modes of different mode numbers can be degenerate: (b) $(N + 1, N)$ when two modes have opposite parity, (c) $(N + 2, N)$ when two modes have the same parity.

at normal incidence, modes of the same mode number are degenerate. As $N_x = N_y$ this regime will be marked as (N, N) regime. In this regime both modes of orthogonal linear polarizations exhibit the same number of antinodes within the cavity, as schematically shown in Fig. 2.19a which sketches distribution of the electric field inside the cavity. In the described LC microcavity such situation occurs at high voltages applied to the electrodes, but it is typical for most of the regular cavities made of optically isotropic materials.

Another regime corresponds to the molecules rotation angle at which mode number of the horizontally polarized cavity mode is one greater than the vertically polarized one $(N + 1, N)$. At normal incidence both modes polarized horizontally and vertically are degenerate but due to different effective refractive indexes within the cavity both exhibit different number of electric fields antinodes inside the cavity (Fig. 2.19a). As the mode numbers vary by one electric field distribution are of different parities, with one being symmetric and the other antisymmetric versus the cavity center. Within this regime the LC layer acts as a half-wave plate with π retardance.

The last degeneracy regime available in the cavity occurs when two modes of the same parity but mode numbers different by two $(N + 2, N)$ are close to the resonance Fig. 2.19b.

Simulated reflectance maps at normal incidence in Fig. 2.18 show possible degeneracy conditions in LC microcavity, but the differences between those different regimes can be observed investigating light incident at non-zero angle in y - z plane. Simulations performed for incidence angle of 15 deg are shown in Fig. 2.20.

As expected for observation at nonzero angle all cavity modes are blu-

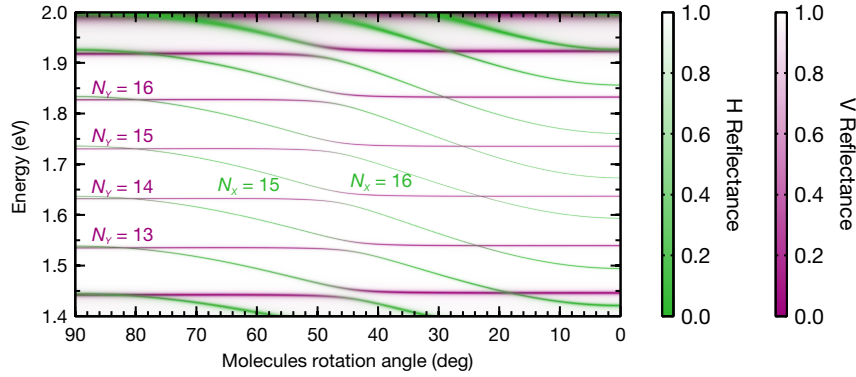


Fig. 2.20 | LC microcavity tuning at 15 deg incidence angle. Simulated by Berreman method normal incidence reflection from LC microcavity at different LC molecules orientation angles.

shifted. Starting from 90 deg molecules rotation angle degeneracy at normal incidence is lifted due to TE-TM splitting which is significantly higher in cavities filled with anisotropic medium, as it will be analyzed in more detail in Sec. 3.2.1. As at $\theta = 90$ deg horizontally polarized cavity mode is of higher energy with rotation of the director the both H/V modes with $N_x = N_y = 14$ cross at $\theta = 81$ deg. This crossing point corresponds to one of the diabolical points which will be described in Sec. 2.2.3.

The biggest difference between Fig. 2.20 and the simulations at normal incidence emerges close at the next $(N + 1, N)$ regime. At nonzero incidence angle the modes do not cross but exhibit typical anticrossing behavior which is a result of Rashba-Dresselhaus like spin orbit coupling observed for photonic modes of different parity, which we observed in [21].

No anticrossing appears for lower LC molecules orientation angles, where close to $\theta = 22$ deg corresponding to $(N + 2, N)$ modes degeneracy regime.

The differences between the regimes can be observed in Fig. 2.21, which presents angle resolved (in y - z plane) reflectance spectra simulated at different molecules rotation angles compared with experimental results obtained at different voltages applied to the LC microcavity.

Fig. 2.21a corresponds to (N, N) regime. Here according to simulations for increasing incidence angles the cavity modes should be split due to TE-TM splitting with higher energy TE (H for y - z incidence plane). This splitting is less visible in the experiment. Driving the investigated device to this specific regime required high voltage which resulted in spectral broadening of the photonic modes. In the $(N + 1, N)$ regime presented in Fig. 2.21b the dispersion relation is dominated by the Rashba-Dresselhaus SOC term,

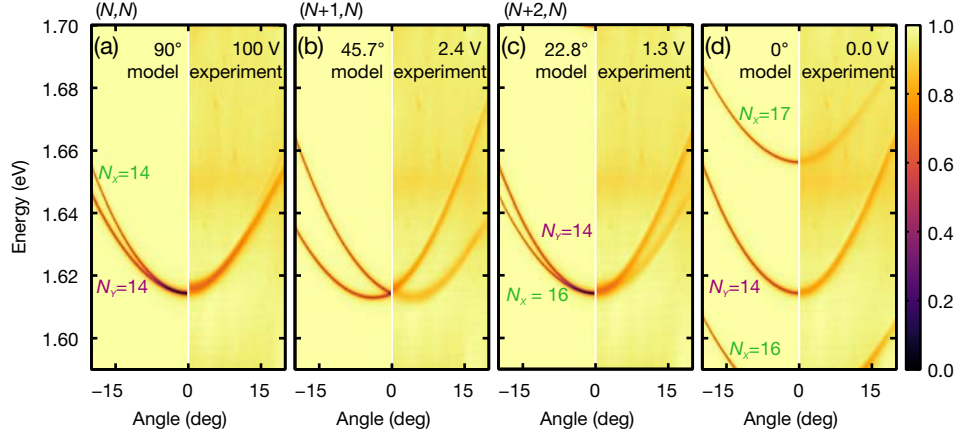


Fig. 2.21 | LC microcavity in different regimes. Simulated and measured angle-resolved reflectance spectra from LC microcavity in regimes: (a) (N,N) degeneracy, (b) $(N+1,N)$ degeneracy, (c) $(N+2,N)$ degeneracy and (d) sample without applied voltage.

with perfect agreement between the simulations and the experiment. The dispersion relation consists of two separated in wave vector parabolas with circularly polarized eigenstates. For lower molecules rotation angle and voltage another degeneracy in $(N+2,N)$ regime can be observed. Along y - z incidence plane modes with linear H/V polarizations are clearly separated in both simulations and the experiment. It is important to mention that although the dispersion relation consisting of two centered parabolas with energy splitting quadratically increasing with observation angle closely resembles this of (N,N) it is characterized by at least the opposite polarization of the modes with more specific differences discussed in Sec. 2.2.5 and Sec. 3.2.3. Finally without external voltage H and V polarized modes are significantly separated in energy for both: theoretical and experimental results Fig. 2.21b.

Overall Fig. 2.21 Berreman matrix simulations present perfect agreement with the experimental results in all of the presented regimes depending on voltage/LC molecules angle. It is however important to note that in order to obtain such qualitative and quantitative agreement cavity assumed in Berreman matrix model is not fully filled with LC which reorients.

For example for the data presented in Fig. 2.21 it is assumed in the model that the total cavity width is equal to 3508 nm. It is separated into three layers. At the interfaces with the both dielectric mirrors it is assumed that the LC is well oriented by the orienting layers thus such layers are characterized by a dielectric tensor $\hat{\epsilon} = \text{Diag}(n_e^2, n_o^2, n_o^2)$ corresponding to no electric bias

and rotated around the y axis to take into account reorientation in external electric field. Both such layers are separated by additional layer, here assumed to be isotropic and characterized by refractive index n_o . In Fig. 2.21 thickness of both interface layers is equal to 1159 nm whereas the central isotropic layer is 1190 nm thick.

Such approach can be justified based on the experimental results that at very high voltages the cavity modes are degenerate (Fig. 2.21a) thus no birefringence in x - y plane must be zero. On the other hand total energy shift of the H polarized mode when voltage is decreased to zero determines the length of the LC which follows reorientation with electric field. To fulfill those conditions either the LC is homogeneous along all the cavity length but with lower birefringence, or does not exhibit long order across the whole cavity length, as is assumed in this case.

To analyze all of this regimes the next section concentrates on more precise description of the whole dispersion relation of the cavity modes.

2.2.3 (N, N) regime

In typical planar microcavities fabricated from the optically isotropic materials at normal incidence there is a total degeneracy of the electromagnetic field with any orientation of the polarization plane. Such configuration corresponds to Fig. 2.19a in which mode number for horizontally polarized light trapped in the cavity is equal to the mode number of vertically polarized light, as both are equivalent.

Such perfect equivalence is however lifted when the light is incident at an nonzero angle. Such configuration distinguishes TE and TM polarized modes as due to different Fresnel reflectance coefficients for TE and TM polarized waves leads to different effective cavity lengths.

Such behavior in planar microcavities is described by TE-TM splitting [22, 23]. With introduction of the TE-TM splitting dispersion relation of the cavity modes can be described by 2×2 effective Hamiltonian, which in circular polarization basis reads [24]:

$$\begin{aligned} \hat{H}_{\text{TE-TM}} &= \left(\frac{\hbar^2 \mathbf{k}^2}{2m} \right) \hat{\sigma}_0 + \beta(k_x^2 - k_y^2) \hat{\sigma}_x + 2\beta k_x k_y \hat{\sigma}_y \\ &= \begin{pmatrix} \frac{\hbar^2 \mathbf{k}^2}{2m} & \beta(k_x - ik_y)^2 \\ \beta(k_x + ik_y)^2 & \frac{\hbar^2 \mathbf{k}^2}{2m} \end{pmatrix}. \end{aligned} \quad (2.34)$$

Here spin-independent parabolic dispersion of the cavity modes is described by an effective mass m and TE-TM splitting strength is quantified by

parameter β .

Additional contribution from birefringence in x direction originating from small reorientation of the LC molecules in the cavity or XY splitting in semiconductor cavities can be added to the Eq. (2.34) [25]:

$$\hat{H}_{\text{TE-TM}} = \left(\frac{\hbar^2 \mathbf{k}^2}{2m} \right) \hat{\sigma}_0 + \beta(k_x^2 - k_y^2) \hat{\sigma}_x + 2\beta k_x k_y \hat{\sigma}_y + \frac{1}{2} \Delta_{\text{HV}} \hat{\sigma}_z, \quad (2.35)$$

where Δ_{HV} describes energy detuning between horizontally and vertically polarized cavity modes.

Expansion of Eq. (2.35) in terms of Pauli matrices $\hat{\sigma}_i$ allows by following analogy to spin 1/2 in magnetic field find eigenstates of the Hamiltonian describing polarization of the modes in terms of Stokes parameters S_i as antiparallel to the effective magnetic field.

Such predictions of the model can be compared by exact numerical solutions of Maxwell equations derived in Berreman matrix form for an investigated planar structure. Energy position of the cavity modes can be determined by transmission maxima or minima of reflectance. Additionally in this approach polarization of any given mode can be determined by finding polarization of light (in terms of Stokes parameters) which is fully transmitted through the cavity.

In Fig. 2.22c Polarization of the mode is plotted as Stokes vector $\mathbf{S} = (S_1, S_2, S_3)$. In plane components $\mathbf{S}_{\parallel} = (S_1, S_2)$ of the Stokes vector $\mathbf{S}$ are plotted by arrows. It is important to stress that directions of the arrows mark in-plane component of Stokes vector, not the direction of the polarization plane. I.e. horizontal polarization of light corresponding to $S_1 = 1$ leads to arrow pointing in right (x) direction, but vertical polarization of light $S_1 = -1$ leads to $\mathbf{S}_{\parallel}$ pointing leftwards. Diagonal $S_2 = 1$ (antidiagonal $S_2 = -1$) light polarization corresponds to upwards (downwards) orientation of $\mathbf{S}_{\parallel}$. As a result Stokes vector rotates in x - y plane two times faster than orientation angle of the polarization plane of linearly polarized light. The remaining S_3 component of the Stokes vector in Fig. 2.22c is marked with pseudocolor scale.

When the director of nematic LC layer is oriented perpendicularly to the cavity plane the cavity modes are degenerate at normal incidence, what constitutes (N, N) resonance in LC cavity. Dispersion relation of the cavity modes obtained by the Berreman matrix method at such conditions is plotted in Fig. 2.22a. The dispersion relation consists of two paraboloids degenerate at a single point $k = 0$ in reciprocal space. Degeneracy point can be easier to observe when the energy (E) of each of the cavity modes is subtracted

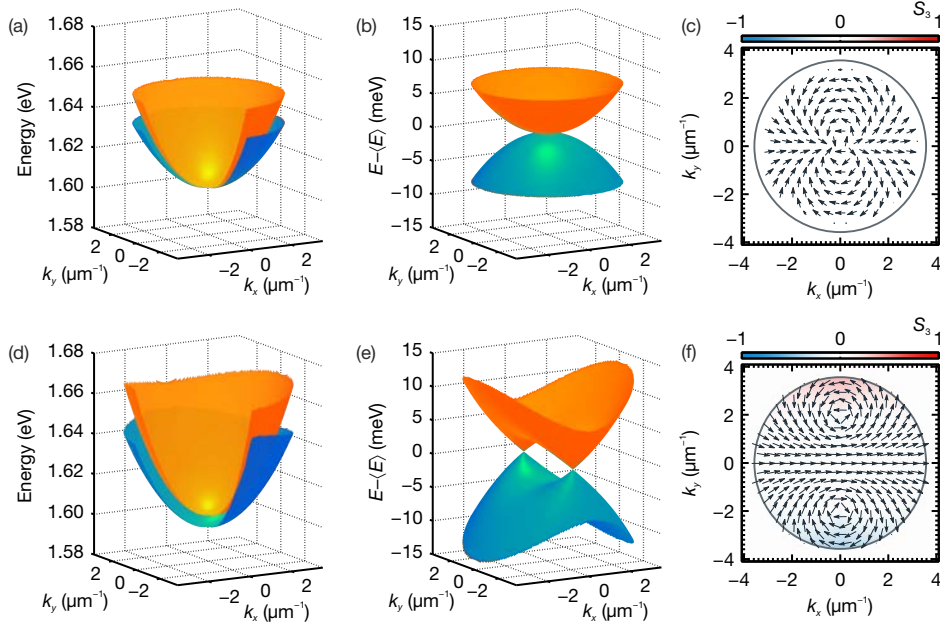


Fig. 2.22 | Cavity modes dispersion in (N,N) regime based on Berreman matrix simulations. (N,N) resonance: (a) calculated dispersion of the cavity modes. (b) Energy of the modes subtracted by their mean value $\langle E \rangle$. (c) Polarization state of the lower energy mode. Out of (N,N) resonance for $\theta = 81$ deg: (d) calculated dispersion of the cavity modes. (e) Energy of the modes subtracted by their mean value $\langle E \rangle$. (f) Polarization state of the lower energy mode.

by their mean value [$\langle E \rangle = (E_1 + E_2) / 2$], as it is shown in Fig. 2.22b. Observed splitting between the cavity modes can be described by TE-TM term. Polarization of the lower energy branch obtained by the Berreman method shows pseudospin structure with +2 winding around the $k = 0$ point. The lower energy mode is TM polarized as the polarization state in Fig. 2.22c follows the incidence plane. At $k_y = 0$, thus with linear polarization plane within x - z plane, the cavity mode is horizontally polarized. As we follow to another wave vector direction, at diagonal direction of the incidence plane ($k_x = k_y$) cavity mode becomes diagonally polarized, and for $k_x = 0$ polarization of the mode changes smoothly to vertical one. As along $k_y = 0$ for this structure we observe horizontal polarization according to the Berreman method it corresponds to negative value of TE-TM splitting strength β in Eq. (2.35).

The dispersion of the cavity modes changes when the LC molecules filling the cavity start to rotate. In Fig. 2.22d the molecules rotation angle of

$\theta = 81$ deg results in splitting of the cavity modes. As it introduces an increase of the effective refractive index for light polarized in x direction, horizontally polarized mode at normal incidence decreases resonance energy, what can be described by the HV detuning term ΔE_{HV} which becomes negative. At nonzero observation angle combined effect of HV detuning and TE-TM splitting leads to specific dispersion relation exhibiting two degeneracy points, best visible in Fig. 2.22e. They constitute so-called diabolical points (DP) [26, 27] where around a degeneracy point in a parameter space the energy dispersion constitute Dirac-cone like dependence, thus they are also referred to as conical intersections [28].

Appearance of the diabolical points can be easily explained by analysis of the Hamiltonian Eq. (2.35). Along $k_y = 0$ due to negative TE-TM splitting strength β , factors before the first Pauli matrix $(\beta k_x^2 + \frac{1}{2}\Delta E_{HV}) \hat{\sigma}_x$ become more negative, which increases the energy splitting between the modes. Along the perpendicular wave vector direction $k_x = 0$ the situation is different. Here $-\beta k_y^2 \sigma_x$ contribution is positive and increases with the wave vector. Especially it can cancel out negative HV detuning described by the ΔE_{HV} term where degeneracy points appear $k_y^{DP} = \pm \sqrt{\frac{\Delta E_{HV}}{2\beta}}$. Those are two only possible degeneracy points as for any other wave vector direction terms multiplying the first $[(\beta k_x^2 - \beta k_y^2 + \frac{1}{2}\Delta E_{HV}) \hat{\sigma}_x]$ and the second $(2\beta k_x k_y \hat{\sigma}_y)$ Pauli matrix in Eq. (2.35) are never simultaneously equal to zero.

Comparing polarization state of the lower energy mode in Fig. 2.22f around each diabolical point the pseudospin exhibit a +1 winding within S_1 - S_2 plane. The total topological charge between the (N, N) resonance (Fig. 2.22a-c) and small negative HV detuning (Fig. 2.22d-f) is conserved as single degeneracy point with +2 winding is spit into two DP with +1 winding each.

As mentioned earlier (N, N) regime with TE-TM splitting is typical for most planar microcavities in [23] and without strong coupling with excitonic resonance [29]. In regular semiconductor microcavities TE-TM splitting is at the order of tens of $\mu\text{eV } \mu\text{m}^2$ [30, 31, 29] and depends on the design of the cavity [22] like or intentional designing the energy of the cavity mode far away from the stopband center [32]. It can be also significantly increased in open cavities due to high refractive index contrast interfaces [33]. It is also significantly higher in cavities incorporating birefringent media like perylene [34] or liquid crystals, where TE-TM splitting can be used to describe impacts of optical anisotropy of the medium as it will be described in Sec. 3.2.1.

The emergence of the diabolical points was predicted theoretically in Ref. [35]. It was observed experimentally in microcavities with birefringent perovskites [36, 37, 38] and in semiconductor microcavities based on AlGa-As system [39, 40]. External magnetic field can lift the degeneracy at those diabolical points, which leads to two valleys in momentum space exhibiting nonzero Berry curvature and nonzero Chern number of the bands [39, 38].

In the presence of polarization dependent losses diabolical points split into pairs of exceptional points as it was at first predicted theoretically [41] and demonstrated experimentally in microcavities based on ZnO [42] and with perovskites [43].

2.2.4 ($N + 1, N$) regime and Rashba-Dresselhaus spin orbit coupling term

In the ($N + 1, N$) regime two cavity modes close to the resonance are of different parity, which corresponds to a half-wave plate situated inside an optical cavity. For such configuration based on the waveguide approximation developed by prof. W. Bardyszewski the eigenvalue problem for the cavity modes $\mathbf{P}$ in circular polarization basis can be written as [21]:

$$\left(\frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} \right) \mathbf{P} + (\delta_x k_x^2 + \delta_y k_y^2) \hat{\sigma}_x \mathbf{P} - 2\alpha k_y \hat{\sigma}_z \mathbf{P} + \frac{1}{2}(E_H - E_V) \hat{\sigma}_x \mathbf{P} = E \mathbf{P}, \quad (2.36)$$

equivalent to 2×2 Hamiltonian:

$$\hat{H}_{RD} = \left(E_0 + \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} \right) \hat{\sigma}_0 + \frac{1}{2} \Delta E_{HV} \hat{\sigma}_x + (\delta_x k_x^2 + \delta_y k_y^2) \hat{\sigma}_x - 2\alpha k_y \hat{\sigma}_z. \quad (2.37)$$

ΔE_{HV} term describes detuning of the modes from the RD resonance. It is equal to zero when two modes undergoing RD coupling are exactly degenerate at normal incidence. In experimental realization of the LC cavity this term is controllable by the amplitude of the external voltage applied to the electrodes.

Apart from spin-independent effective mass anisotropy in k_x and k_y wave vector directions described by parameters m_x and m_y terms $\delta_i k_i^2 \hat{\sigma}_x$ introduce splitting of H/V polarized modes increasing quadratically with wave vector.

The strongest effect originating in ($N + 1, N$) regime is given by the last term $2\alpha k_y \hat{\sigma}_z$ referred to as Rashba-Dresselhaus like spin orbit coupling. For LC cavities with high birefringence it is typically the strongest contribution. Its presence leads to the dispersion relation of two paraboloids separated

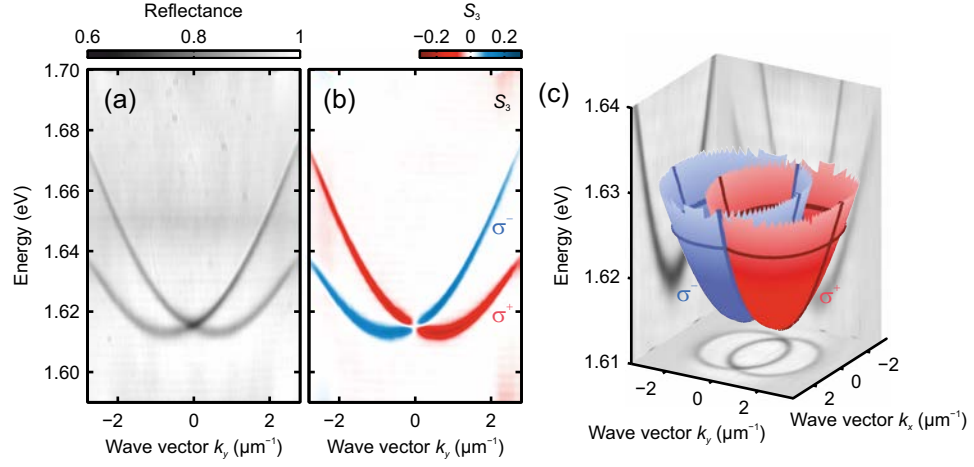


Fig. 2.23 | Rashba-Dresselhaus regime in LC microcavity. (a) Angle-resolved reflectance spectrum of LC microcavity at RD resonance. (b) Corresponding S_3 parameter of reflected light. (c) Polarization-resolved tomography in reciprocal space.

along k_y wave vector direction, as shown in experimental angle-resolved spectra in Fig. 2.23a. As the term is proportional to the third Pauli matrix in circular polarization basis resulting shifted parabolas are circularly polarized with opposite helicities, as can be demonstrated in polarization resolved reflectance spectra with S_3 parameter of the reflected light is shown in Fig. 2.23b.

In LC cavities Rashba-Dresselhaus term is proportional to wave vector direction perpendicular to the plane which contains extraordinary direction of the LC filling the cavity. For LC molecules oriented initially along x axis, the SOC term is proportional to k_y wave vector direction. The dispersion relation in both wave vector directions can be measured by reciprocal space tomography. Such result is plotted in Fig. 2.23 presenting two, circularly polarized paraboloids separated in k_y direction.

Name of the Rashba-Dresselhaus spin orbit coupling comes from the analogy with the solid state physics. There the most general linear spin-orbit coupling Hamiltonian for a two-dimensional electron gas takes the form [44]:

$$\hat{H} = \frac{\hbar^2 \mathbf{k}^2}{2m} + \alpha (k_y \hat{\sigma}_x - k_x \hat{\sigma}_y) + \beta (k_x \hat{\sigma}_x + k_y \hat{\sigma}_y), \quad (2.38)$$

where α is the strength of Rashba and β strength of Dresselhaus contributions. In specific situation when both contributions are of equal strengths

2.2 ($N + 1, N$) regime and Rashba-Dresselhaus spin orbit coupling term

i.e. $\alpha = \beta$ Bernevig et al. demonstrated that under specific rotation of spatial and spin coordinates the same Hamiltonian can be written as [44]:

$$\hat{H} = \frac{\hbar^2}{2m} (k_+^2 + k_-^2) + 2\alpha k_+ \hat{\sigma}_z, \quad (2.39)$$

which is mathematically equivalent with the term found in description of LC cavities in ($N + 1, N$) regime Eq. (2.36). The condition of equal Rashba and Dresselhaus contributions are of special interests in the solid state systems as due to resulting symmetry the system is robust against spin-independent disorder and Coulomb interactions [44]. This leads to significant increase in spin coherence time for charge carriers in 2D electron gas [45], which also leads to emergence of spatial spin texture known as Persistent Spin Helix with proposed multiple spintronic applications (see Ref. [46] for a review).

According to the waveguide approximation the Rashba-Dresselhaus coupling strength α is equal to:

$$\alpha = -\frac{\hbar^2}{m_o' L} \frac{MN}{M^2 - N^2} \left(\frac{n_e^2 - n_o^2}{n_e n_o^2} \right) \frac{\sin \theta \cos \theta}{\sqrt{n_o^2 \sin^2 \theta + n_e^2 \cos^2 \theta}}, \quad (2.40)$$

where L is length of the cavity and M and N are mode numbers of the resonances close to degeneracy. RD term appears for any pair of the modes in which M and N are of different parity but due to $\frac{MN}{M^2 - N^2}$ the highest coupling strength is typically observed when M and N differ by one. As it is not directly visible from the formula, Eq. (2.40) applies only for the modes of different parity thus RD coupling is equal to zero when both N and M are simultaneously odd or even.

On the other hand coupling strength depends also on the molecules orientation angle θ . Term $\sin \theta \cos \theta$ is maximal for the molecules reoriented by 45 deg angle. Especially this term leads to zero RD coupling strength when $\theta = 0$ deg or $\theta = 90$ deg. For this reason RD SOC cannot be observed when ordinary/extraordinary directions are parallel/perpendicular to the cavity, which for LC cell corresponds to requirement to apply external voltage.

Additionally the RD coupling strength increases for higher birefringence of the LC used due to $n_e^2 - n_o^2$ term in the numerator of Eq. (2.40) and decreases for higher cavity widths L .

Value of the RD coupling strength α in the experimental system can be determined by direct fitting of the cavity modes dispersion by theoretical prediction given by eigenvalues of the Eq. (2.37). It can be also extracted

from measurements at different voltages applied to the LC cavity passing RD resonance.

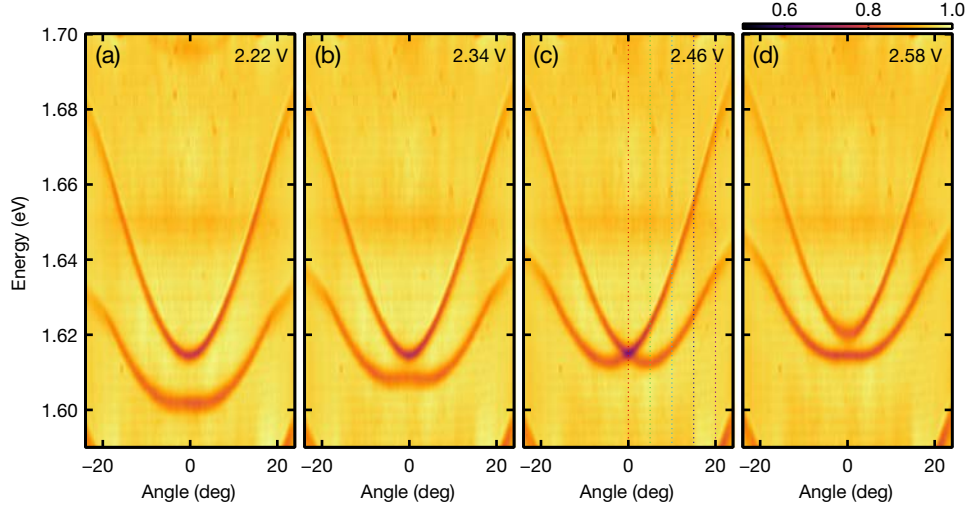


Fig. 2.24 | Tuning around Rashba-Dresselhaus resonance. Experimentally measured angle resolved reflectance maps at different voltages applied to the LC microcavity close to the Rashba-Dresselhaus resonance at 2.46 V.

Fig. 2.24 presents angle resolved reflectance maps measured along y angle of RD term collected at different voltages around RD resonance. In Fig. 2.24a 2.22 V voltage is below the RD resonance, which in Eq. (2.37) is described by a negative δE_{HV} . For increasing voltage the modes approach each other at normal incidence, and become degenerate at 2.46 V in Fig. 2.24c. For higher voltage the separation between the modes at normal incidence increases, this time corresponding to positive values of ΔE_{HV} .

Those reflectance spectra can be also shown as a function of applied voltage at certain observation angle (in y - z plane). Fig. 2.25a at normal incidence corresponding to $k_y = 0$ RD term is equal to zero and two modes cross each other. Simultaneously at nonzero observation angle the modes exhibit anticrossing behavior Fig. 2.25b. As the observation angle increases the minimal energy separation between the modes increases, as shown in Fig. 2.25c-e. The minimum energy splitting between the modes plotted as a function of the k_y wave vector of the reflected light in Fig. 2.25f shows expected linear dependence, with the slope $\alpha = 3.19 \times 10^{-3} \text{ eV}\mu\text{m}$.

Fig. 2.26a,d,g presents calculated with the Berreman method dispersion relation of the cavity modes around $(N + 1, N)$ regime: above the resonance (Fig. 2.26a: $\Delta E_{\text{HV}} > 0$, $U > U_{\text{RD}}$, $\theta > \theta_{\text{RD}}$), at the RD resonance (Fig. 2.26g)

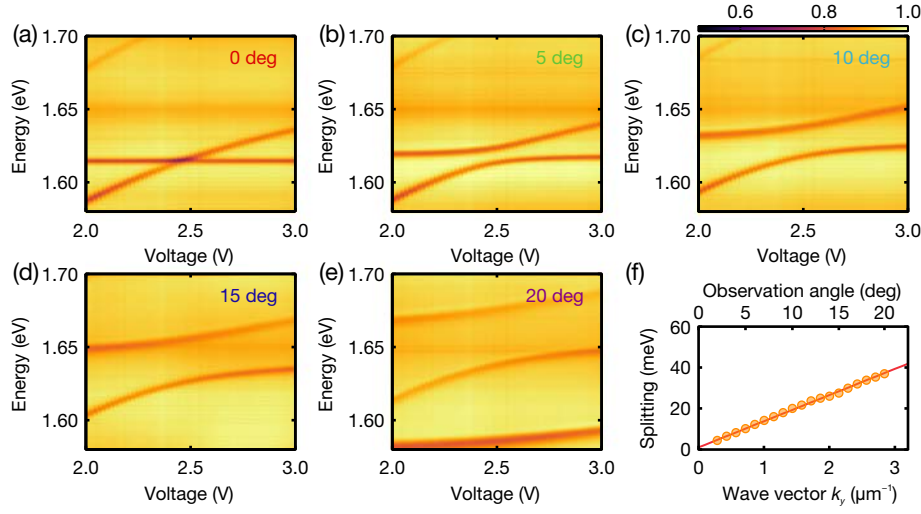


Fig. 2.25 | Rashba-Dresselhaus spin-orbit coupling. Experimental demonstration of Rashba-Dresselhaus SOC between the modes at reflectance maps measured at: (a) normal incidence, (b) 5 deg, (c) 10 deg, (d) 15 deg, (e) 20 deg for increasing voltage applied to the device. (f) Minimal splitting between the cavity modes dependence on the wave vector with linear fit.

and below the resonance (Fig. 2.26g).

The dispersion relation in all of those cases can be easier visualized by plotting the energy of those modes subtracted by their mean value $\langle E \rangle$ (panels Fig. 2.26b,e,h). Corresponding polarization state of the lower energy mode is plotted in Fig. 2.26c,f,i.

Below the RD resonance (Fig. 2.26a–c) at normal incidence the cavity modes are split with positive ΔE_{HV} . As a result lower energy mode exhibit vertical polarization. Along k_y wave vector direction the energy splitting rapidly increases with modes becoming circularly polarized due to the dominant RD SOC term $2\alpha k_y \hat{\sigma}_z$. Depending on the sign of α S_3 component of the polarizations state is negative (σ^-) for positive k_y and $S_3 > 0$ for negative k_y with zero S_3 component along $k_y = 0$.

Along the perpendicular wave vector direction k_x starting from $k = 0$ the splitting between the modes decreases. Here positive value of ΔE_{HV} is compensated by $\delta_x k_x^2$ contribution as $\delta_x < 0$. Especially at $k_x = \pm \sqrt{\frac{\Delta_{\text{HV}}}{2\delta_x}}$ both modes are degenerate. Crossing the degeneracy along k_x the polarization state of lower energy mode flips from vertical to horizontal.

Observed here two degeneracy points constitute another pair of diabolical points (DB). Those two DB are distinctly different than those observed

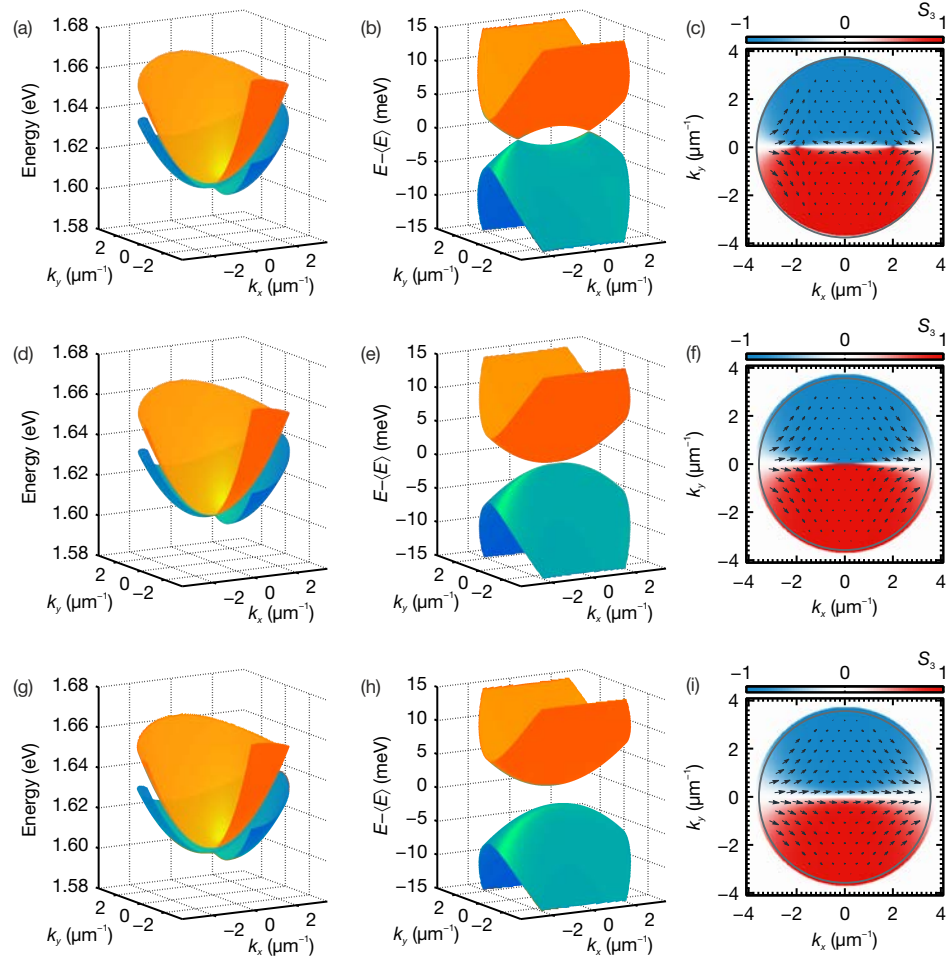


Fig. 2.26 | Cavity modes dispersion in $(N + 1, N)$ regime based on Berreman matrix simulations. Above $(N + 1, N)$ resonance for $\theta = 47.7$ deg: (a) calculated dispersion of the cavity modes. (b) Energy of the modes subtracted by their mean value $\langle E \rangle$. (c) Polarization state of the lower energy mode. $(N + 1, N)$ resonance for $\theta = 47.2$ deg: (d) calculated dispersion of the cavity modes. (e) Energy of the modes subtracted by their mean value $\langle E \rangle$. (f) Polarization state of the lower energy mode. Below $(N + 1, N)$ resonance for $\theta = 46.7$ deg: (a) calculated dispersion of the cavity modes. (b) Energy of the modes subtracted by their mean value $\langle E \rangle$. (c) Polarization state of the lower energy mode.

around (N, N) regime. In (N, N) regime spin state winded up within S_1 – S_2 plane during encircling of either of DP. Here due to circular polarization splitting from RD term encircling one of the DB polarization state rotates

predominantly in S_1 – S_3 plane.

Is in (N, N) regime lifting degeneracy at DB required external magnetic field to split circularly polarized state [39], here lifting such degeneracy at DB is possible by addition of anisotropy in $\hat{\sigma}_y$ direction (S_2) which does not require external magnetic field. Such scheme will be described in detail in Chapter 6.

By lowering voltage/molecules rotation angle/ ΔE_{HV} position of the diabolical points moves towards $k = 0$. Especially at RD resonance they merge to the single degeneracy point at normal incidence (Fig. 2.26d,e). Regarding polarization of the lower cavity mode Fig. 2.26f it is dominated by RD term resulting in circular polarization of the modes with helicity depending on the sign of k_y . When $k_y = 0$ along k_x the splitting of the modes and horizontal polarization of the lower energy mode arises from negative δk_x^2 term. As at $k = 0$ both modes are degenerate, thus no eigenpolarization, there is a sharp jump between σ^+ and σ^- polarization when crossing zero along k_y wave vector direction.

Moving further below RD resonance the dispersion relations of the two closest in energy modes exhibit no degeneracies (Fig. 2.26g,h). Polarization of the lower energy band follows this for RD resonance with homogeneous increase of S_1 contribution due to negative HV detuning, as shown in Fig. 2.26i. At normal incidence polarization of the lower energy mode is horizontal with smooth transition between σ^+ and σ^- through horizontal polarization when following along k_y through normal incidence point.

Overall behavior in the ($N + 1, N$) regime in LC cavities is dominated by a Rashba-Dresselhaus like spin-orbit coupling term $2\alpha k_y \hat{\sigma}_z$. It leads to the dispersion relation consisting of two circularly polarized paraboloids separated in k_y direction. By taking into account smaller in value contributions to linear polarization reveals that for positive HV detunings the dispersion relation exhibit two diabolical points.

Since the initial discovery which we described in [21], we used the photonic Rashba-Dresselhaus spin orbit coupling in LC cavity system to obtain photonic persistent spin helix polarization texture [47] and observation of photonic lasing from inseparable valleys [48]. Rashba-Dresselhaus term is also crucial in engineering nontrivial band geometry in LC cavities, as it will be described in Chapter 6.

Apart from LC cavities Rashba Dresselhaus-like SOC was also recently observed with birefringent, optically active organic crystals incorporated inside an optical cavity [49].

2.2.5 $(N + 2, N)$ regime

In $(N + 2, N)$ regime two modes closest in energy are characterized with mode numbers of the same parity: both even or odd.

Based on the waveguide approximation such dispersion relation of the cavity modes can be described by 2×2 Hamiltonian in circular polarization basis, as we introduced in [50]:

$$\hat{H} = \left(\frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} \right) \hat{\sigma}_0 - (\delta_x k_x^2 - \delta_y k_y^2) \hat{\sigma}_x + \frac{1}{2} \Delta E_{\text{HV}} \hat{\sigma}_x - \delta_{xy} k_x k_y \hat{\sigma}_y. \quad (2.41)$$

Here similarly to $(N + 1, N)$ regime effective masses are anisotropic with different along k_x and k_y wave vector directions and HV detuning controllable by external voltage can be introduced by $\Delta E_{\text{HV}} \hat{\sigma}_x$ term. In this regime RD SOC term disappears, but the symmetry allows for additional $\delta_{xy} k_x k_y \hat{\sigma}_y$ term. Appearance of such splitting will be demonstrated experimentally in Sec. 3.2.3. Combined with also previously present terms splitting HV polarized modes quadratically with wave vector $(\delta_x k_x^2 - \delta_y k_y^2) \hat{\sigma}_x$ they constitutes an extended TE-TM splitting in which all $\delta_x = \delta_y = \delta_{x,y}$.

As previously precise dispersion of the cavity modes in such regime can be obtained by Berreman matrix method, demonstrated in Fig. 2.27.

Extracted from the simulations dispersion relation of the cavity modes above ($\Delta E_{\text{HV}} > 0, U > U_{(N+2,N)}, \theta > \theta_{(N+2,N)}$) is presented in Fig. 2.27a. As previously details of the dispersion relations are easier to observe by plotting energy of the modes subtracted by their mean value $\langle E \rangle$, as in Fig. 2.27b. Polarization state of the lower energy branch is shown in Fig. 2.27c. At normal incidence due to positive ΔE_{HV} the lower energy mode is polarized vertically. As in $(N + 1, N)$ regime following along k_x wave vector direction the splitting decreases as for negative sign of δ_x the $\delta_x k_x^2$ compensates HV detuning. This leads to emergence of two diabolical points where both contributions cancels out each other at $k_x^{\text{DP}} = \pm \sqrt{\frac{\Delta E_{\text{HV}}}{2\delta_x}}$. For higher k_x splitting term goes more negative and the lower energy mode becomes horizontally polarized. At perpendicular wave vector direction k_y starting from $k = 0$ the behavior is qualitatively very similar. with increasing wave vector negative $\delta_y k_y^2$ contribution initially decreases the splitting, up to degeneracy point at $k_y^{\text{DP}} = \pm \sqrt{\frac{\Delta E_{\text{HV}}}{2\delta_y}}$ and flips polarization of the mode to horizontal at higher wave vectors. As for any other wave vector directions contributions before $\hat{\sigma}_x$ and $\hat{\sigma}_y$ cannot simultaneously cancel out each other the dispersion relation exhibit in total four diabolical points: two at each main wave vector directions, clearly visible in Fig. 2.27b. Following polarization state around

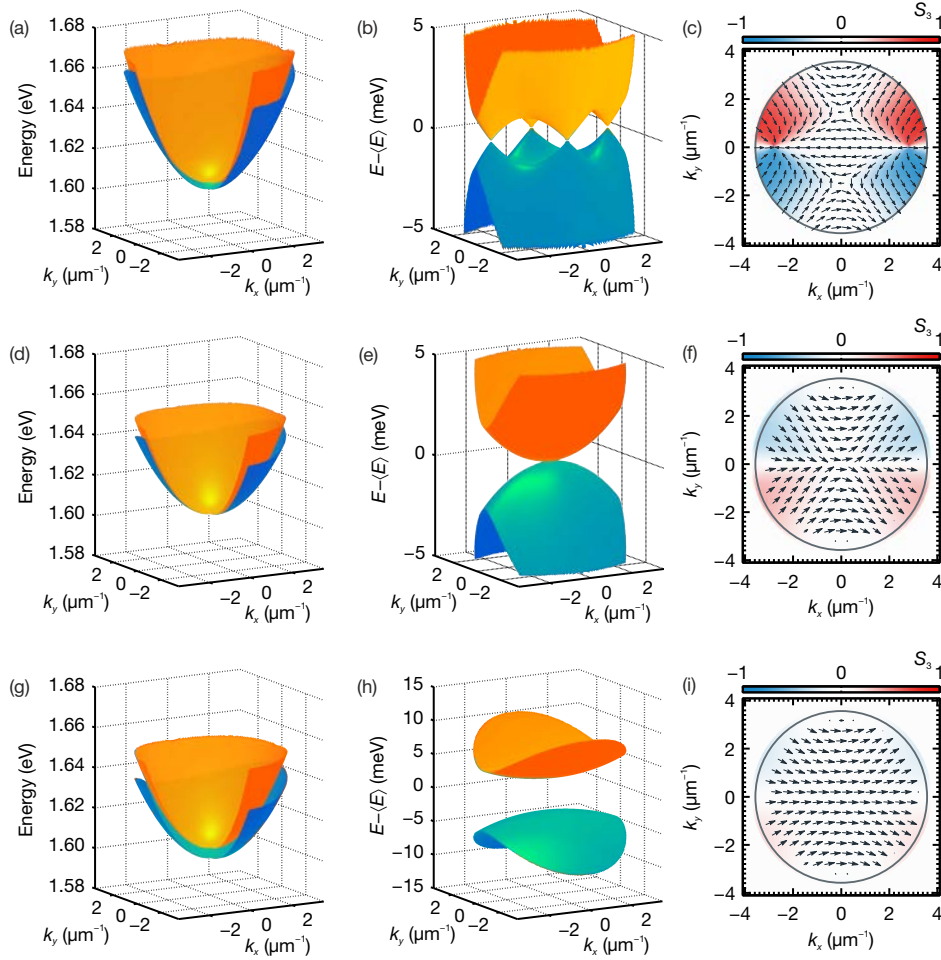


Fig. 2.27 | Cavity modes dispersion in ($N + 2, N$) regime based on Berreman matrix simulations. Above ($N + 2, N$) resonance for $\theta = 26.4$ deg: (a) calculated dispersion of the cavity modes. (b) Energy of the modes subtracted by their mean value $\langle E \rangle$. (c) Polarization state of the lower energy mode. ($N + 2, N$) resonance for $\theta = 25.7$ deg: (d) calculated dispersion of the cavity modes. (e) Energy of the modes subtracted by their mean value $\langle E \rangle$. (f) Polarization state of the lower energy mode. Below ($N + 2, N$) resonance for $\theta = 24.7$ deg: (a) calculated dispersion of the cavity modes. (b) Energy of the modes subtracted by their mean value $\langle E \rangle$. (c) Polarization state of the lower energy mode.

the diabolical points those located at $k_y = 0$ are characterized with +1 positive winding number, whereas those positioned at $k_x = 0$ have negative -1 , thus total topological charge is equal to zero.

It is worth noting that the polarization state obtained by the Berreman matrix simulations predicts nonzero S_3 component, especially nearby the DB at $k_y = 0$, which is not captured by analytical model given by Eq. (2.41), but those contributions are relatively smaller than contributions to S_1 and S_2 .

Decreasing HV detuning the energy splitting at normal incidence decreases and the diabolical points approach each other and merge at single degeneracy point at $k = 0$ when $\Delta_{HV} = 0$ (Fig. 2.27d,e). As total topological charge of four DB were equal to zero at $(N + 2, N)$ resonance winding of the pseudospin around the degeneracy point is equal to zero. The polarization of the lower energy mode calculated by Berreman method (Fig. 2.27f) is predominantly horizontal originating from negative $(\delta_x k_x^2 - \delta_y k_y^2)\hat{\sigma}_x$ contributions with diagonal (antidiagonal) contribution in the first and third (second and fourth) quadrant of wave vector directions from $\delta_{xy}k_x k_y \hat{\sigma}_y$ term.

With further decrease of the HV detuning (Fig. 2.27g-i) there are no longer any degeneracies between the cavity modes and the polarization of the lower energy branch becomes more and more horizontally polarized.

In summary $(N + 2, N)$ regime when two modes of the same parity are close to the resonance despite lack of Rashba-Dresselhaus SOC is distinctly different than what is observed in typical microcavities with relatively small birefringence or XY splitting. At positive HV detuning dispersion of the modes contain four diabolical points with zero total topological charge, instead two degeneracy points with total $+2$ topological charge [39, 40, 36, 37, 38].

As it will be shown in the two following chapters this brings new effects in terms of spatial polarization textures and behavior of exceptional points.

2.3 Optical planar microcavities: summary

In summary, this chapter presented an introduction to the physics of planar microcavities, with a special emphasis on resonators that incorporate an optically anisotropic medium.

At first, we describe two theoretical approaches: transfer matrix method and the Berreman matrix method, which can be used to simulate optical properties of layered structures such as planar microcavities. We used those techniques to illustrate the optical properties of the distributed Bragg reflectors and planar microcavities. Where possible, we compared the results of the calculations with the experimental results. We described an experimental setup allowing for measurement of the dispersion relation of the cavity modes of the resonator in both wave vector directions.

Second, we analyzed dispersion of the cavity modes in a multimode resonator filled with a birefringent, nematic liquid crystal based on the Berreman matrix simulations. Depending on the orientation of the anisotropy direction of the LC layer, we distinguished three different regimes in which distinct modes of the cavity are close to degeneracy. For all of these regimes we simulate polarization of the cavity modes based on the Berreman matrix method and provide effective 2×2 Hamiltonians describing the dispersion relation of the two closest in the energy cavity modes.

Chapter 3

Photonic spin textures in LC microcavities

The results described within this chapter are published in:



- [50] M. Król et al. Observation of second-order meron polarization textures in optical microcavities. *Optica* 8, 255 (2021).

Previous chapter showed various different configurations of spin polarized bands which can be obtained within a cavity filled with a nematic liquid crystal. Within this chapter we will show how the transmission of polarized light through such bands can result in real space polarization textures with nonzero winding, namely merons and antimerons of the first and second order.

The first section (Sec. 3.1) introduces to different pseudospin textures in 2D space with nonzero topological charge: skyrmions, antiskyrmions, merons and antimerons of different order.

The second section (Sec. 3.2) considers transmission of polarized light through LC microcavity based on Berreman matrix simulations. It shows that depending on the incident light and orientation of the LC microcavity the transmitted light polarization texture can consist of meron-antimeron lattice, or form second order meron or second order antimeron. Those two last polarization textures will be demonstrated also experimentally based on polarization-resolved transmission measurement.

3.1 Spin textures

3.1.1 Skyrmion and meron textures in 2D

Skyrmion polarization textures for three dimensional order parameter $\mathbf{S}$ in two dimensional space can be written in polar coordinates (r, φ) as [51]:

$$\mathbf{S} = \begin{pmatrix} \cos \Phi(\varphi) \sin \Theta(r) \\ \sin \Phi(\varphi) \sin \Theta(r) \\ p \cos \Theta(r) \end{pmatrix}, \quad (3.1)$$

where

$$\Phi(\varphi) = v\varphi + \gamma. \quad (3.2)$$

Skyrmion texture written in this form has three parameters. The first one is the vorticity v , which has an interpretation of a winding number. Vorticity can take integer values (except 0). For a skyrmion the vorticity value is equal to 1, and the negative values leads to antiskyrmions (keep in mind, that some authors differentiate between skyrmions and antiskyrmions based on topological number Q , not vorticity e.g. [52]). Vorticity with modulus higher than one are characteristic for higher order (anti)skyrmions.

Exemplary skyrmion textures with vorticities $v = \pm 1, \pm 2$ and phases $\gamma = 0, -\pi/2$ with positive polarity are shown in Fig. 3.1.

Additional phase factor γ is known as skyrmion helicity. For the vorticity

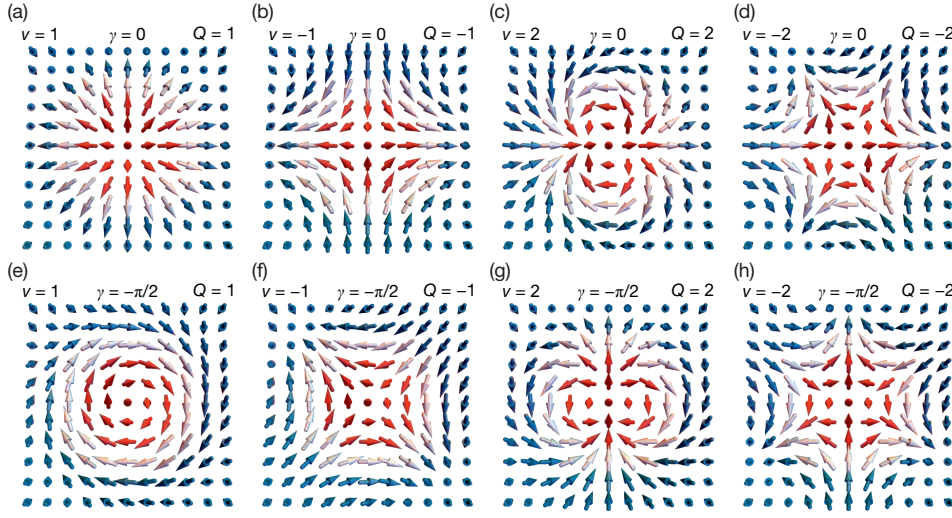


Fig. 3.1 | Skyrmion textures. Example skyrmions with positive polarity and various winding number v and helicities γ with their topological charges Q .

$v = 1$ the helicity $\gamma = 0, \pi$ leads to so called Néel skyrmion. Moving radially from the skyrmion core the order parameter rotates in the plane given by radial direction and z axis, like in Néel domain wall known in magnetism. If helicity γ is equal to $\pi/2$ or $3\pi/2$ moving radially from the core the order parameter rotates around radial direction as in Bloch domain wall leading to the so-called Bloch skyrmion. Helicities of different values are also possible leading to an intermediate state between Néel and Bloch skyrmions.

For higher order skyrmions there is no distinction between Bloch and Néel skyrmions, as such textures contain both: Néel and Bloch walls. Helicity in this case controls the orientation of the skyrmion/antiskyrmion texture.

The last parameter is the polarity p . For skyrmions out of plane spin component $[S_3 = p \cos \Theta(r)]$ texture changes to antiferromagnetic configuration between the core and infinity. Positive polarity means that core has positive out of plane direction.

Topological invariant of skyrmion textures called skyrmion number or topological charge is given by [53]:

$$Q = \int \rho_{\text{sk}} dx dy = \frac{1}{4\pi} \int \mathbf{S} \cdot \left(\frac{\partial \mathbf{S}}{\partial x} \times \frac{\partial \mathbf{S}}{\partial y} \right) dx dy, \quad (3.3)$$

where ρ_{sk} is referred to as topological charge density.

For skyrmion texture given by (3.1) the topological charge simplifies to

product of vorticity and polarity [54]:

$$Q = v \cdot p. \quad (3.4)$$

As the idea of skyrmions originate from higher dimensional theory of elementary particles [55], textures of 3D order parameter in 2D space [(2 + 1)] investigated in this work are also referred to as 'baby skyrmions' [56].

Another distinct group of textures of three dimensional order parameter in two dimensional space are merons. Contrary to antiferromagnetic configuration between core and infinity for skyrmions, in merons the order parameter away from the core lays in-plane. Merons are also called as half-skyrmions or Mermin-Ho vortices.

Merons textures can be described by (3.1) in which in S_3 component $\cos \Theta(r)$ varies between 1 and 0 instead of between ± 1 . In result, the topological charge of the meron is half of the corresponding skyrmion (thus is named as half-skyrmion).

3.1.2 Realizations of skyrmion/merons

Most popular example of skyrmion/meron are magnetization textures observed in magnetic materials. Multiple review articles have been devoted to magnetic skyrmions [57, 58, 59, 54], some concentrating on specific experimental methods like neutron scattering [60].

Magnetic skyrmions are promising candidates for data storage and processing. They are robust and small which allow for high data density, can be created by spin polarized currents [61] or optically [62, 63], moved by current [64] and read-out.

There have been theoretical proposals for construction of logic gates based on magnetic skyrmions [65]. Operations have been performed by movement of the skyrmions through specifically designed channels thinner than size of a single skyrmion. Due to boundary effects skyrmions have lost their robustness thus could merge or annihilate leading to AND or OR logic gates.

Stability of magnetic skyrmions have been predicted theoretically [66, 67] and first experimental evidence of skyrmion lattice has been made in MnSi [68]. Néel-type skyrmions [69], antiskyrmions [70] have been also experimentally observed.

In magnetic materials textures with higher than one topological charge typically have a form of a biskyrmion: coupled pair of two skyrmions [71]. Recently the observation of second order antimeron has been reported [72].

In magnetic materials isolated merons have been typically not stable. Merons have been present only in a form of lattice [52] or in pairs when magnetization was in-plane in constrained geometries [73, 74] or in pairs [75].

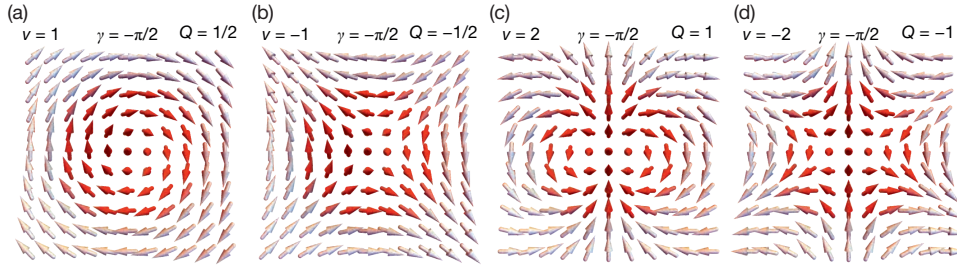


Fig. 3.2 | Meron textures. Spin textures of (a) meron, (b) antimeron and (c) second order meron and (d) second order antimeron with positive polarity and helicity $\gamma = -\pi/2$.

In photonic systems the skyrmions have been observed in evanescent field of plasmonic structures [76, 77] and have been theoretically investigated in chiral materials [78]. Skyrmion polarization textures with size significantly smaller than the wavelength have appeared also in strongly focused beam with orbital angular momentum [79].

Of special interests are merons in reciprocal space [80], as they are the evidence of a nontrivial band geometry leading to nonzero Berry curvature [39].

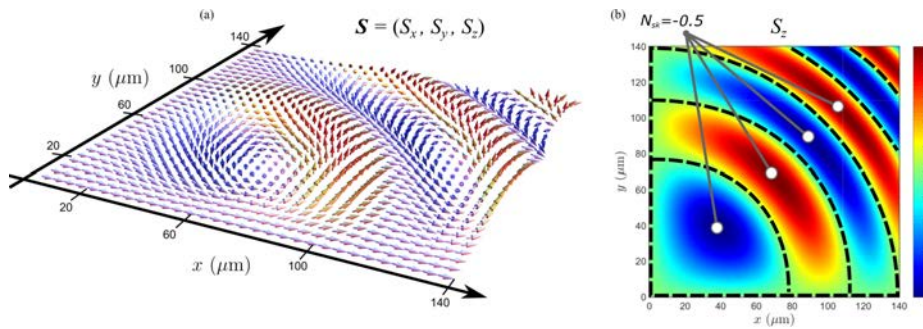


Fig. 3.3 | Meron polarization texture of exciton-polariton condensate. Theoretical spatial polarization pattern described by (a) Stokes vector and (b) only $S_3 = S_z$ component for microcavity exciton-polariton condensate. Figure from Ref. [81]

Skyrmions have been also investigated in optical microcavities. Donati et al. [82] investigated the evolution of the polarization pattern after reso-

nant excitation with beams with nontrivial topology. Skyrmion patterns in incident light have been created using liquid crystal q -plates [83].

Theoretical investigations of resonantly driven microcavity in strong coupling regime have led to a spatial skyrmion lattice polarization pattern [84] due to TE-TM splitting what has resulted in optical spin Hall effect.

Cilibrizzi et al. [81] observed analogous spatial spin texture for nonresonantly created exciton polariton condensate in GaAlAs microcavity (shown in Fig. 3.3).

Those two results gave rise to the question if the same results can be observed in cavities filled with liquid crystals without strong coupling with excitonic component, following the observation of optical spin Hall effect in uncoupled cavities [29, 85]. Additional question was how observed polarization textures will be affected by tunability allowed by liquid crystals?

3.2 Meron polarization textures in liquid crystal microcavity

3.2.1 Polarization textures in (N, N) regime

Meron lattice polarization textures observed in semiconductor microcavities results from TE-TM splitting which leads to optical spin Hall effect (OSHE). OSHE has been predicted [24] for optical microcavities in strong coupling regime as a result of TE-TM splitting. OSHE can be described as an effective magnetic field dependent on the wave vector direction:

$$\mathbf{B}_{\text{eff}} = \frac{\Omega}{k^2} \begin{pmatrix} k_x^2 - k_y^2 \\ 2k_x k_y \\ 0 \end{pmatrix}, \quad (3.5)$$

where Ω is a constant proportional to polariton longitudinal-transverse splitting. In the original model [24, 30] this splitting originates from exciton longitudinal-transverse splitting.

Effects of TE-TM splitting on cavity photons with parabolic dispersion relation can be written using effective Hamiltonian in circular polarization basis:

$$\hat{H}_{\text{TE-TM}} = \begin{pmatrix} E_0 + \frac{\hbar^2 \mathbf{k}^2}{2m} & \beta \mathbf{k}^2 e^{-2i\varphi} \\ \beta \mathbf{k}^2 e^{2i\varphi} & E_0 + \frac{\hbar^2 \mathbf{k}^2}{2m} \end{pmatrix} = \begin{pmatrix} E_0 + \frac{\hbar^2 \mathbf{k}^2}{2m} & \beta(k_x - ik_y)^2 \\ \beta(k_x + ik_y)^2 & E_0 + \frac{\hbar^2 \mathbf{k}^2}{2m} \end{pmatrix} \quad (3.6)$$

where parameter β gives strength of TE-TM splitting and $\varphi = \arctan k_x/k_y$.

OSHE has been observed by multiple groups in strongly coupled micro-

cavities [30, 86, 87]. Later experiments have predicted OSHE also in cavities without strong coupling to excitonic resonance [29]. In a previous work, we demonstrated OSHE in a microcavity filled with liquid crystal [85].

OSHE Hamiltonian given by magnetic field in Eq. (3.5) is equivalent to eigenvalue problem for cavity filled with liquid crystal at (N,N) regime (when the anisotropy axis is oriented along the growth axis z) based on the waveguide approach developed by prof. W. Bardyszewski [21]:

$$\left(\frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} \right) \mathbf{P} + \frac{\hbar^2}{4m'_0} \left(\frac{\varepsilon_{xx} - \varepsilon_{zz}}{\varepsilon_{zz}\varepsilon_{xx}} \right) \left[(k_x^2 - k_y^2)\hat{\sigma}_x + 2k_x k_y \hat{\sigma}_y \right] \mathbf{P} = E\mathbf{P}. \quad (3.7)$$

Prediction of this model is in agreement with Berreman matrix simulations. Fig. 3.4a shows simulated cavity mode dispersion from a dielectric cavity filled with liquid crystal with $\Delta n = 0.41$ and the anisotropy axis oriented along z axis. In this configuration the effective refractive index for linearly polarized light along both x (X , green) and y (Y , purple) axes is the same, which leads to the degeneracy of the modes at normal incidence. For light incident at nonzero angle this degeneracy is lifted and modes are TE and TM linearly polarized. Polarization of the modes for different wave vector directions is shown in Fig. 3.4b. Yellow arrows marks orientation of effective magnetic field given by Stokes vector $\mathbf{S} = (S_1, S_2, S_3)$ plotted in the figure for inner cavity mode at energy marked with dashed vertical line in Fig. 3.4a. Polarization of the mode follows the direction perpendicular to the incidence plane, which is equivalent to TE polarization. Polarization of the outer mode (not shown in the figure) is perpendicular to the inner one.

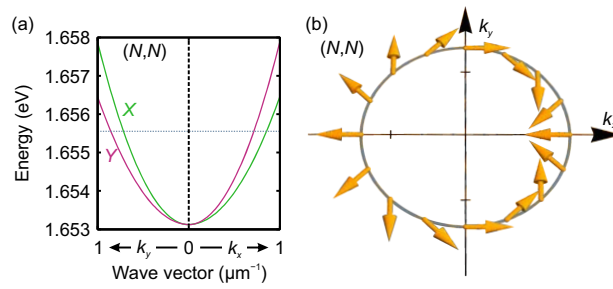


Fig. 3.4 | Liquid crystal microcavity dispersion in (N,N) regime. (a) Dispersion relation along two perpendicular wave vector directions. Modes polarized horizontally (vertically) are plotted in green (violet). (b) Cavity mode polarization direction at the energy marked with the gray dashed line in (a) with inner cavity mode. Obtained by Berreman matrix model.

For microcavities filled with liquid crystal – according to Eq. (3.7) – the role of longitudinal-transverse splitting in semiconductor microcavities in strong coupling regime is played by optical anisotropy. Precisely, the splitting is proportional to the difference between dielectric tensor elements along x and y direction $\varepsilon_{xx} - \varepsilon_{zz}$. In (N, N) resonance this difference is equal to the difference between ordinary and extraordinary dielectric tensor elements:

$$\varepsilon_{xx} - \varepsilon_{zz} = \varepsilon_o - \varepsilon_e.$$

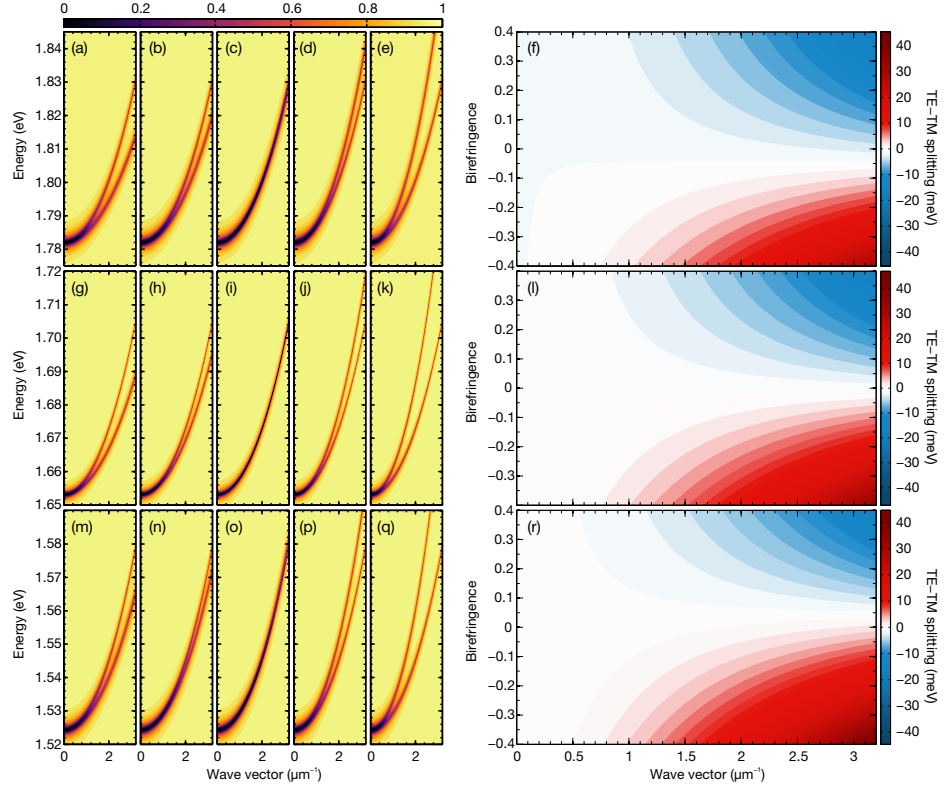


Fig. 3.5 | Angle-resolved reflectance for subsequent cavity modes for variant birefringence in multiple-mode cavity. (a–e) mode with $N = 9$, energy above center of the stop band for birefringence 0.4, 0.2, 0.0, -0.2 and -0.4 . (g–k) mode with $N = 10$ and energy equal exactly to the center of the stop band. (m–q) mode with $N = 11$ and energy below the center of the stop band.

Fig. 3.5g–k shows simulated with Berreman matrix model reflectance for a cavity filled with liquid crystals of increasing birefringence $\Delta n = \sqrt{\varepsilon_e} - \sqrt{\varepsilon_o}$. DBRs forming the cavity are centered at $\lambda_0 = 750 \text{ nm}$. To keep the resonance energy constant at the same LC thickness ordinary refractive index of the cavity is equal to $n_o = 1.539$ with cavity length $10 \frac{\lambda}{2n_o}$ and only

extraordinary refractive index is changed. In Fig. 3.5(g) for negative birefringence of -0.4 with increasing wave vector the cavity mode splits with TM-polarized one with higher energy. As cavity birefringence increases to -0.2 (Fig. 3.5h) the energy of TE-polarized mode at a given, nonzero, wave vector increases and the splitting decreases. When $n_e = n_o$ and the cavity is optically isotropic the TE-TM splitting disappears (Fig. 3.5i). With further increase of the birefringence (Fig. 3.5j–k) splitting reappears. This time the energy of TE-polarized mode remains constant, but the energy of TM-polarized one increases faster with wave vector for higher values of n_e . Dependence of the TE-TM splitting on birefringence of the cavity layer and incident light wave vector is summarized in Fig. 3.5l.

Overall simulated values of TE-TM splitting for high birefringence at the order of 40 meV are significantly higher than those observed in semiconductor microcavities [88, 23, 29, 32]. This is expected as those structures are assumed to be optically isotropic. Berreman matrix simulations can be used to demonstrate, that in addition to the optical anisotropy TE-TM splitting can be also affected by the design of the resonator.

Fig. 3.5a–e and Fig. 3.5m–q shows the same simulations of reflectance spectra for increasing values of birefringence of the cavity layer, but calculated at the energy ranges corresponding to cavity modes with mode numbers $N = 9$ and $N = 11$. General behavior is the same as for the central cavity mode, with TE-TM splitting strongly decreasing with birefringence. Decreasing birefringence eventually leads to the change of sign of TE-TM splitting, but for various modes vanishing of the splitting occurs at different, nonzero birefringence value. This can be observed in Fig. 3.5f,r, especially for high values of the wave vector. For higher energy mode in Fig. 3.5f splitting vanishes for $\Delta n \approx 0.05$ whereas for lower energy mode (Fig. 3.5r) zero splitting corresponds to $\Delta n \approx -0.05$. Those effects are summarized in Fig. 3.6 presenting strong dependence of the TE-TM splitting strength parameter β dependence on LC birefringence.

TE-TM splitting appearing for cavity modes away from the center of the stopband for DBRs is observable in Berreman matrix simulations also for fully optically isotropic cavities. The magnitude of the splitting increases with the energy difference between mode energy and DBR center as long as the quality factor of the mode starts to significantly decrease when the cavity mode approaches a boundary of the photonic stopband.

Another mechanism for increasing of TE-TM splitting, even in optically isotropic cavities, is to introduce interfaces with high contrast of refractive indices. TE-TM splitting has already been reported to be significantly higher for open cavities design which is primarily caused by interfaces with high

refractive index contrast between air and semiconductor [33].

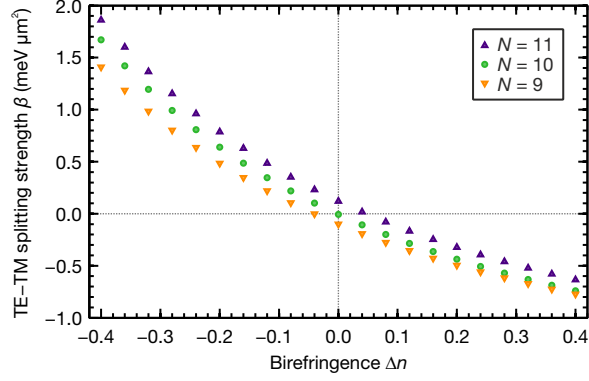


Fig. 3.6 | TE-TM splitting strength. Simulated value of TE-TM splitting parameter β for different birefringence.

3.2.2 Polarization textures in real space in (N, N) regime

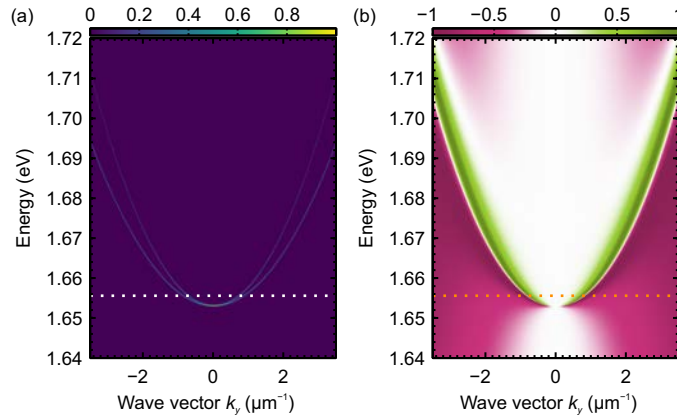


Fig. 3.7 | Transmittance spectra simulated by Berreman method. (a) Angle-resolved transmittance spectra. (b) Transmitted light S_1 parameter.

Thanks to the relation between in-plane photon wave vector and incidence/transmittance angle, the Berreman method allows to directly simulate transmittance properties of LC microcavities in momentum space. Fig. 3.7a presents transmittance spectra of the cavity with $\Delta n = 0.41$ and the anisotropy axis oriented in z direction. Simulations were performed for 8 pairs of $\text{TiO}_2/\text{SiO}_2$ DBR layers in order to achieve spectrally narrow cavity modes, which results in visibly separated TE and TM polarized modes.

Polarization of the modes is shown in Fig. 3.7b which presents S_1 parameter of transmitted light. Presented data is a cross section for wave vectors along y direction thus for positive birefringence higher energy mode is TE polarized resulting in positive sign of the first Stokes parameter.

(It's important to note that Stokes parameters are normalized in respect to intensity, thus can be nonzero even when photon energies and wave vectors does not correspond to a cavity mode.)

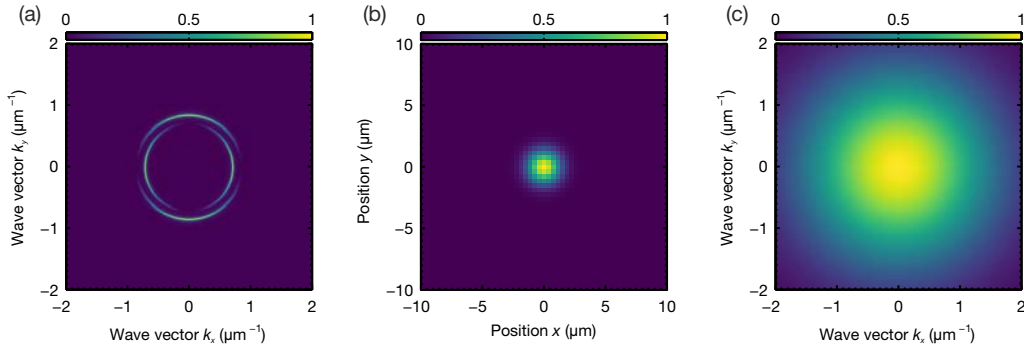


Fig. 3.8 | Simulations in real space. (a) Transmittance through LC microcavity in reciprocal space of vertically polarized light. (b) Gaussian beam spot in real space. (c) Normalized Fourier transform of Gaussian profile from (b).

To investigate polarization textures in real space we considered transmission of monochromatic light through LC microcavities. Fig. 3.8a presents transmission intensity in momentum space of horizontally polarized light at energy marked by dashed line in Fig. 3.7. This cross section consists of two concentric circles with intensity dependent on the wave vector direction, as modes are TE-TM polarized and incident light has the horizontal polarization. For $k_y = 0$ transmittance is high for inner mode, but decreases to zero at perpendicular wave vector direction where $k_x = 0$. the polarization of the outer mode is opposite which leads to the maximum transmittance for $k_x = 0$, and the intensity vanishes at $k_y = 0$.

Transition between real and momentum space can be realized by Fourier transform. We considered tightly focused beams, which are required to cover broad range of LC microcavity dispersion in reciprocal space, as dimensions in momentum and real space are inversely proportional. Example Gaussian beam spot with full width at half maximum (FWHM) of $2.14 \mu\text{m}$ is shown in Fig. 3.8b. Fourier transform of Gaussian spatial beam profile is also Gaussian. Numerically calculated Fourier transform in Fig. 3.8c corresponds to FWHM of $2.59 \mu\text{m}^{-1}$ in momentum space. Thus such tightly

focused beam spot can cover wave vector range of dispersion relation of LC microcavity at the investigated energy.

To simulate the electric field transmitted in real space, at first the Berreman method was used to obtain electric field in x (E_x) and y directions (E_y) at a given wave vector for a given incident light polarization. Incident light amplitude for a given wave vector was modulated by a Gaussian envelope, such as shown in Fig. 3.8c, which carries information of desirable spot size in simulations. Obtained transmitted electric fields E_x and E_y in momentum space can be then transformed to real space by numerical 2 dimensional fast Fourier transform.

To investigate spatial polarization textures calculated electric field can be used to determine the stokes parameters by:

$$S_1 = \frac{|E_x|^2 - |E_y|^2}{|E_x|^2 + |E_y|^2}, \quad (3.8)$$

$$S_2 = \frac{E_x E_y^* + E_y E_x^*}{|E_x|^2 + |E_y|^2}, \quad (3.9)$$

$$S_3 = \frac{i(E_x E_y^* - E_y E_x^*)}{|E_x|^2 + |E_y|^2}. \quad (3.10)$$

Results of spatial polarization state described by Stokes vector $\mathbf{S} = (S_1, S_2, S_3)$ of transmitted light through LC microcavity are shown in Fig. 3.9. Obtained polarization texture is close to the meron-antimeron lattice observed in exciton-polariton condensates in semiconductor microcavity Fig. 3.3 from Ref. [81].

Polarization texture consists of four quadrants separated by directions parallel and perpendicular to incidence polarization plane. Radially from the incidence spot, each quadrant consists of lobes with opposite values of S_3 and whirling S_1 and S_2 . Each neighboring lobe exhibit opposite sign of S_3 parameter. Each lobe contains meron or antimeron, what can be shown by calculation of the topological charge Q . Based on equation (3.3) numerical calculations of topological charge is approximately equal to ± 0.5 in each region separated by gray line in Fig. 3.9, which matches with topological charge of meron/antimeron of $\pm 1/2$ (precise values of Q are given in the figure).

For a given quadrant the topological charge of all merons/antimerons is the same. For the first and third quadrant ($x, y > 0$, $x, y < 0$) lobe closest to the center (within radius of $23 \mu\text{m}$) contains a Bloch meron with negative polarity ($S_3 < 0$) which results in topological charge of -0.50 . Lobe

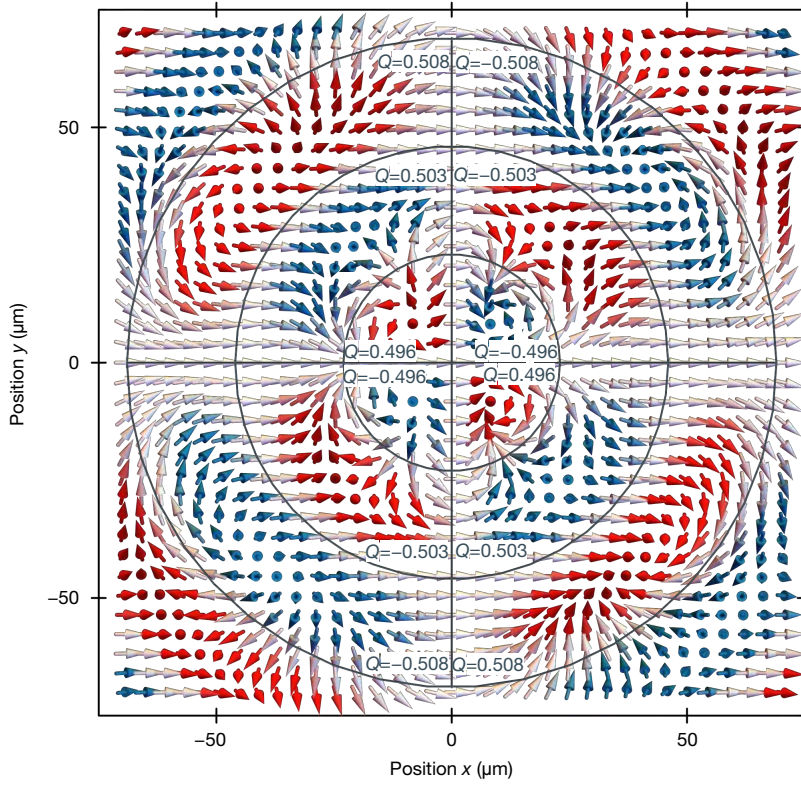


Fig. 3.9 | Spatial polarization texture for light transmitted through LC microcavity. Spatial polarization state given by Stokes vector S resulting in meron–antimeron lattice. Region of each single meron/antimeron is separated by solid gray line with calculated numerically topological charge. Incident light is linearly polarized along x direction (horizontal).

further from the center ($23\ \mu\text{m} < r < 46\ \mu\text{m}$) with positive polarity is also characterized with the negative topological charge, thanks to its -1 winding number. Further from the center the patterns repeat and meron with negative polarity is followed by antimeron with positive polarity.

For the second and fourth quadrant the polarity is reversed leading to positive sign of topological charge. Lobe closest to the center contains meron with positive polarity and $Q = 0.50$. Lobe further from the center contains antimeron with negative polarity.

Fig. 3.10 presents how observed polarization texture changes with the polarization of incident light. Fig. 3.10 concentrates on the area close to the center containing four merons for horizontally-polarized incident beam. For increasing degree of circular polarization of incident beam (Fig. 3.10b–c)

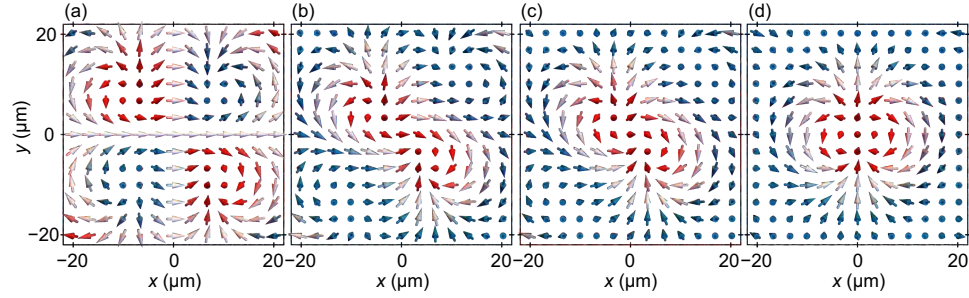


Fig. 3.10 | Formation of second order meron. Spatial polarization state for light transmitted through LC microcavity for varying polarization of incidence light from (a) linear to (d) σ^+ circularly polarized.

pattern evolves and two merons with positive polarity approaches to the incidence spot. For fully circularly polarized excitation, shown in Fig. 3.10d (and Fig. 3.12 in wider range) they merge into a single second order meron.

As each of the meron carries topological charge of $1/2$ second order meron is characterized by two times higher topological charge of 1. This expectation is confirmed by the numerical calculation of the topological charge. At the distance of $11.5 \mu\text{m}$ from the center the S_3 parameter of transmitted light reaches zero. Within this radius from the center—marked with gray circle in Fig. 3.12—numerically calculated topological charge is equal to $Q = 1.00$.

Considering distances further from the center S_3 parameter decreases further and reaches -1 . Such anti parallel orientation to the core is characteristic to skyrmions. Within this area, marked by yellow circle in Fig. 3.12, topological charge is equal to 1.98 what is consistent with skyrmion number of 2, the second order skyrmion.

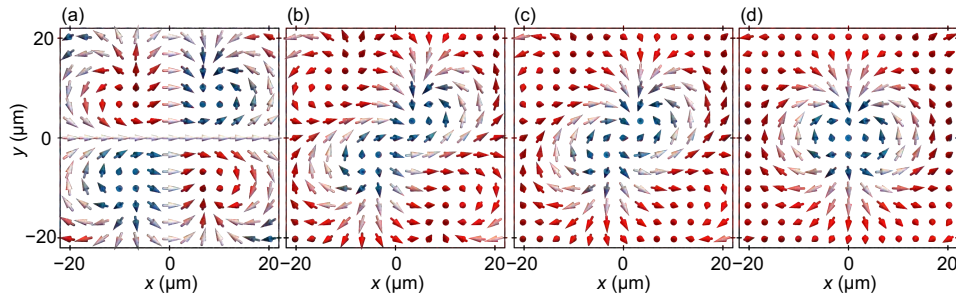


Fig. 3.11 | Formation of second order meron with negative polarity. Spatial polarization state for for light transmitted through LC microcavity for varying polarization of incident light from (a) linear to (d) σ^- circularly polarized.

For changing incident light polarization from linear to circular with the opposite chirality (σ^-) two merons with negative polarity merge at the excitation spot forming second order meron with negative polarity, as is presented in Fig. 3.11.

Overall, the Berreman matrix simulations predict that after the change of the polarization of excitation from linear to circular meron/antimeron lattice evolves into a second order skyrmion at the center.

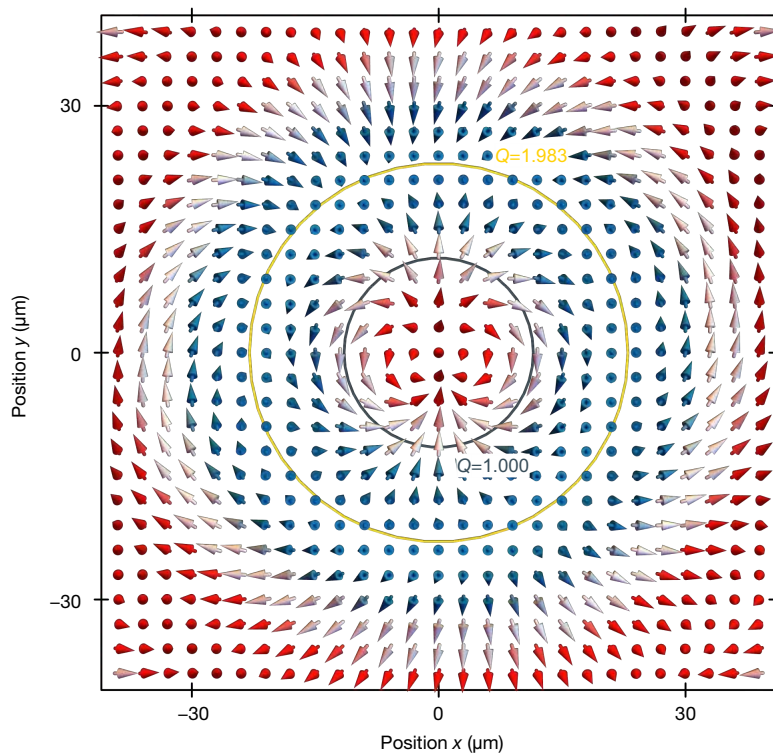


Fig. 3.12 | Spatial polarization texture for σ^+ polarized light transmitted through LC microcavity. Spatial polarization state given by Stokes vector $\mathbf{S}$ resulting in the formation of second order skyrmion. Area within radius of $11.5 \mu\text{m}$ (gray circle) contains second order meron with topological charge of $Q = 1.00$. Wider area within radius of $23 \mu\text{m}$ (marked with yellow circle) contains second order skyrmion with topological charge of $Q = 1.98$.

Observation of second order skyrmions or merons were not reported in semiconducting microcavities. A similar radially varying S_3 parameter was modeled for circularly-polarized excitation of exciton-polariton condensates, but this regime has not been described in more details [89].

To experimentally verify the prediction of the model we performed transmission measurements on a microcavity filled with liquid crystal previously used in Ref. [85]. The sample consists of 2 DBRs designed for 700 nm wavelength with 6 pairs of $\text{SiO}_2/\text{TiO}_2$. Cavity with thickness of around $1.9 \mu\text{m}$ was filled with liquid crystal with $n_o = 1.504$ and $n_e = 1.801$ in HT configuration (extraordinary direction perpendicular to cavity plane), corresponding to (N,N) degeneracy regime without external electric field.

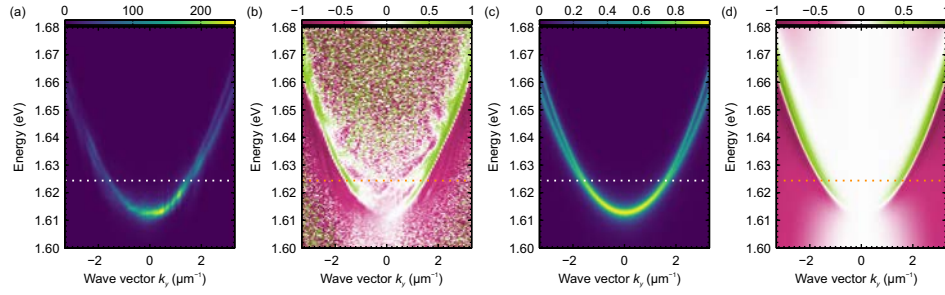


Fig. 3.13 | Liquid crystal microcavity dispersion in (N,N) regime.

(a) Angle-resolved transmittance spectra shown as dispersion relation along k_y wave vector. (b) S_1 parameter of transmitted light. (c) Transmittance and (d) S_1 Stokes parameters simulated by Berreman method.

Angle-resolved transmission of the sample is shown in Fig. 3.13a. Spectrum was obtained using broadband light from halogen lamp. To achieve small excitation spot and wide range of incidence angles light beam was focused on the sample with high numerical aperture microscope objective ($NA = 0.55$). Transmitted light was collected with another microscope objective with the same numerical aperture. Dispersion relation in Fig. 3.13a shows, as expected, a parabolic dispersion relation with splitting increasing with the wave vector. Split modes are linearly polarized what can be seen in Fig. 3.13b, which presents S_1 parameter of transmitted light.

At normal incidence linewidth of the mode based on fitting Lorentzian peak profile is equal to 3.6 meV, which with resonance at 1.613 eV leads to quality factor of the resonator of 453.

Experimental results are in perfect match with the Berreman matrix simulations. Fig. 3.13c,d presents calculated transmission spectrum and S_1 parameter of transmitted light. To obtain the same resonance energy as in the experiment, the thickness of the LC layer in simulations was set to 1855.2 nm. Simulations with 6 pairs of dielectric layers resulted in calculated quality factor of 846, which is significantly higher than experimental value. To better match linewidths in simulations Fig. 3.13c,d shows results in

which simulated structure consists of DBRs made with 5 pairs of $\text{SiO}_2/\text{TiO}_2$ with Q -factor of 442.

Based on the quality factor the lifetime of the cavity photon can be estimated to approx. 0.2 ps. As a result of this limited lifetime, cavity photon in-plane propagation is significantly limited to distances around 10 μm . For example, in simulated meron lattice for linearly polarized excitation the cores of individual merons were far from the excitation spot. Therefore, we concentrated in that case on observation of second order meron, as it is centered at the excitation spot.

Observation of polarization textures in real space was based on the same broadband halogen lamp as in Fig. 3.13a,b. The incident light was circularly polarized by a set of linear polarizer and a quarter-wave plate. Access to cross section at a chosen photon energy was achieved by a tomography measurement.

Light transmitted through the sample was imaged on the entrance slit of a monochromator equipped with CCD camera providing images resolved in photon energy and position in y direction. To obtain the resolution in x direction, the image was scanned through the spectrometer slit by motorized movement of the imaging lens. Each scan was performed with polarization optics set to detect light intensity in polarizations: horizontal, vertical, diagonal, antidiagonal, σ^+ and σ^- to calculate Stokes parameters.

Measured spatial polarization state of σ^+ polarized light transmitted through LC cavity at energy marked by dashed line in Fig. 3.13 is shown in Fig. 3.14a. S_3 parameter is presented in colorscale and in plane Stokes vector $\mathbf{S}_{\parallel} = (S_1, S_2)$ is plotted by arrows. Transmitted light preserves incident circular polarization, especially close to the incidence spot with additional nonzero degree of linear polarization. Orientation of this linear polarization described by in plane Stokes vector $\mathbf{S}_{\parallel}$ wraps around the center in similar way to second order meron. For comparison, the polarization texture of second order meron described by Eq. (3.1) is shown in Fig. 3.14a. Good agreement can be easier observed for S_1 and S_2 parameters plotted separately by a colorscale in Fig. 3.14b–c for model texture and in Fig. 3.14e–f for experiment. In both results S_1 is positive in the first and the third quadrant with negative values in remaining second and fourth quadrants. Similar pattern but rotated by 45 degrees counter-clockwise can be observed in S_2 component, with positive values along vertical direction, and negative along x direction.

Experimental results can be also directly compared with exact simulations by Berreman method for the same parameters as in Fig. 3.13. This results are plotted in Fig. 3.14g–i. Simulations also recreate with a good agree-

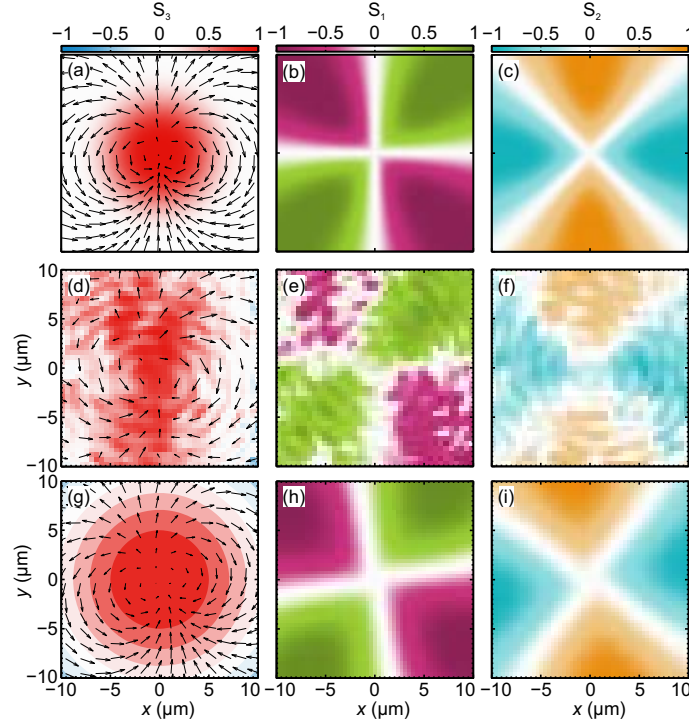


Fig. 3.14 | Second order meron polarization texture in LC microcavity. (a–c) Second order meron polarization texture. (a) $\mathbf{S}_{\parallel} = (S_1, S_2)$ marked with arrows with S_3 mapped by a colorscale; (b) S_1 (c) S_2 parameters mapped by colorscale. (d–f) Experimental spatial polarization texture of transmitted light: (a) $\mathbf{S}_{\parallel} = (S_1, S_2)$ marked with arrows with S_3 mapped by a colorscale; (e) S_1 (f) S_2 parameters mapped by colorscale. (g–h) Simulated by Berreman method spatial polarization texture of transmitted light: (g) $\mathbf{S}_{\parallel} = (S_1, S_2)$ marked with arrows with S_3 mapped by a colorscale; (h) S_1 (i) S_2 parameters mapped by colorscale.

ement the experiment and the model texture of second order meron, but with slightly rotated polarization pattern.

This rotation comes from an overlap of the perpendicularly polarized modes. In Fig. 3.7 shown previously, where the calculations were performed for 8 DBR pairs leading to high quality factor of the cavity, both modes were clearly separated. Contrary to experiment for which energy splitting at considered wave vector is significantly smaller than the spectral linewidths of the modes.

In simulations different overlaps between the modes can be obtained by the same parameters that alters TE-TM splitting: birefringence with extra-

ordinary optical axis in z direction and energy position of the mode within photonic stopband.

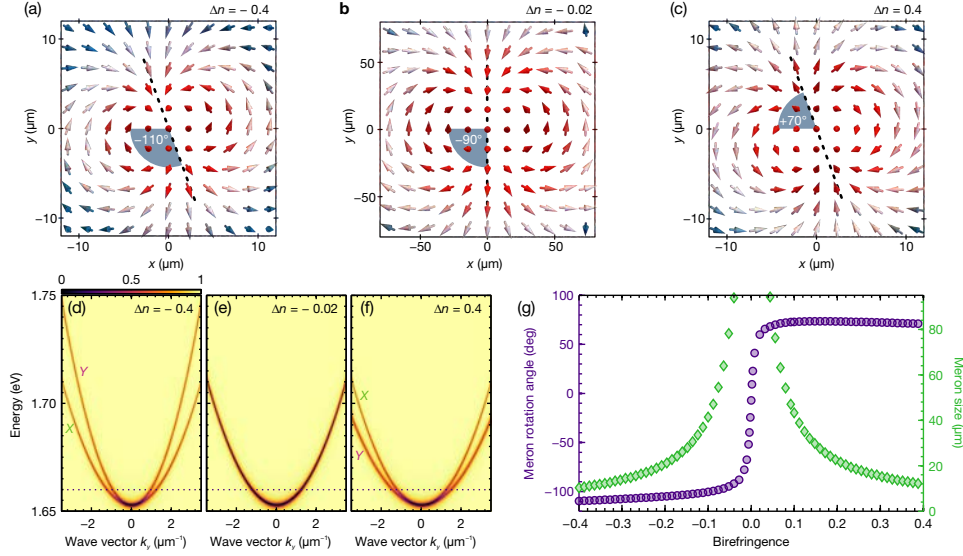


Fig. 3.15 | Rotation of second order polarization texture with varying birefringence. Spatial polarization textures obtained by Berreman method for LC birefringence of (a) $\Delta n = -0.4$, (b) -0.02 , (c) $+0.4$ at the energy marked with dashed line at corresponding angle-resolved reflectance spectra in (d-f). (g) Meron orientation angle and size for varying birefringence.

Fig. 3.15a-c shows evolution of second order meron texture for increasing birefringence with corresponding angle-resolved reflectance spectra in Fig. 3.15d-f. Transmitted light polarization for each considered birefringence Δn between -0.4 and $+0.4$ results in second order meron texture but with varying orientation and size which are plotted in Fig. 3.15. Meron orientation angle changes from -110° for negative birefringence to $+70^\circ$ for positive Δn . The dependence is steep for birefringence close to zero as TE-TM splitting decreases and modes spectrally overlap (Fig. 3.15e), as for slightly negative $\Delta n = -0.02$ rotation angle is close to -90° . At the same time the birefringence and the overlap between the modes affects spatial size of resulting polarization textures. Merons are the smallest (below $10 \mu\text{m}$) for high absolute values of cavity birefringence. For small optical anisotropy the size of the meron strongly increases even above $100 \mu\text{m}$, where simulated light intensity is too low to numerically determine the polarization state in performed simulations.

Another previously mentioned parameter affecting the TE-TM splitting

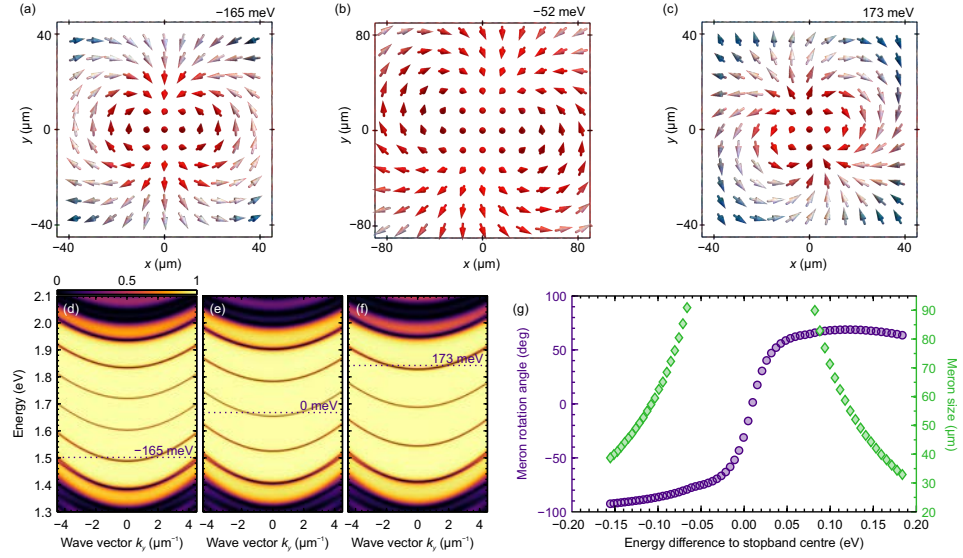


Fig. 3.16 | Rotation of second order polarization texture for varying energy position within photonic stopband. Spatial polarization textures obtained by Berreman method at cavity mode energy detuning of (a) -165 meV, (b) -52 meV, (c) 173 meV from central energy of photonic stopband at 1.653 eV. (d–f) Corresponding angle-resolved reflectance spectra. (g) Meron orientation angle and size for the detuning of the cavity mode.

is the energy position of the given mode within photonic stopband region. Fig. 3.16 summarizes simulations in which energy of the mode was changed above 0.3 eV by varying the length of optically isotropic cavity between 2137 nm and 2787 nm. Fig. 3.16a–c shows transmitted light polarization texture at three different cavity mode detunings, each realizing second order meron pattern. Energy of incident light in each case was set 10 meV above given resonance energy at normal incidence, which is marked at corresponding reflectance spectra in Fig. 3.16d–f. Overall dependence of both meron orientation and size (Fig. 3.16g) follows the same behavior as in the case of birefringence.

Apart from rotation and scale the overlap between the modes affect transmitted polarization pattern in one more way. Due to relatively low quality factor of the cavity, in plane propagation is limited and experimental determination of polarization state of transmitted light was possible in approx. ± 10 μm range. This allowed for experimental observation of only a second order meron core of a second order skyrmion shown in Fig. 3.12. Apart from this experimental limitations, the Berreman matrix simulations show that

generation of generate second order skyrmion textures is possible only in high- Q cavities and the low- Q are limited to second order meron textures. Fig. 3.17a shows simulated dependence of S_3 parameter of transmitted light on the distance from the incidence spot for cavities made of different number of dielectric layer pairs in DBRs. For high- Q structure made of 8 DBR pairs the degree of circular polarization oscillates in a full range between $+1$ and -1 . This ability to reach the pseudospin orientation antiparallel to the core allows to reach topological charge of $+2$ of a second order skyrmion, as shown in Fig. 3.17b. As the quality factor decreases, the radial oscillations of S_3 parameters dampens with distance. With 5 DBR pairs antiparallel configuration is almost possible, but for 4 DBR pairs the minimal S_3 value reaches only -0.66 . This results in decreasing maximal value of the topological charge, in which value of 2 of the second order skyrmion is not reached, but the observation of meron requiring topological charge of 1 is still possible.

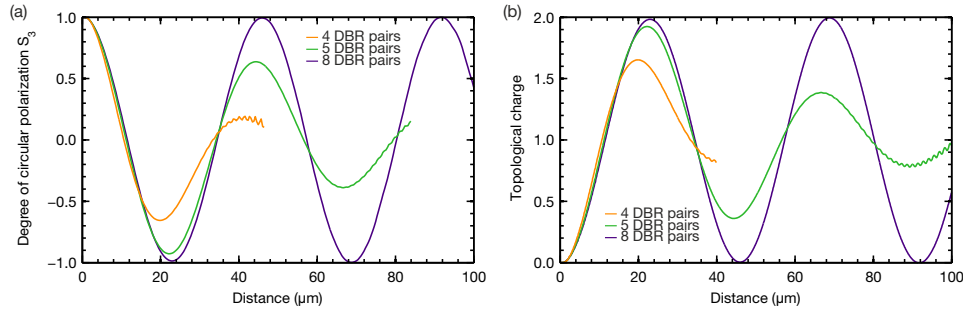


Fig. 3.17 | Second order meron polarization texture for cavities of different Q -factor. (a) S_3 parameter of transmitted light dependence on distance from incidence spot calculated for cavities made of different number of dielectric pairs. (b) Topological charge density integrated within disk of increasing radius.

In summary, in (N, N) regime the cavity modes are TE/TM polarized and can be described by in-plane effective magnetic field with winding number in momentum space equal to 2. Transmission of linearly polarized light results in the formation of meron/antimeron spatial polarization pattern at the surface of the cavity. For circularly polarized pump beam the transmitted polarization forms second order meron, what was verified experimentally.

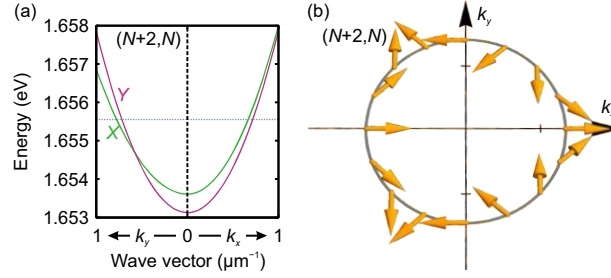


Fig. 3.18 | Liquid crystal microcavity dispersion close to $(N+2, N)$ regime.

(a) Dispersion relation along two perpendicular wave vector directions. Modes polarized horizontally (vertically) are plotted in green (violet). (b) Cavity mode polarization direction at the energy marked with gray dashed line in (a) with inner cavity mode. Obtained by Berreman matrix model.

3.2.3 Polarization textures in $(N+2, N)$ regime

Described in the previous section results in (N, N) regime showed that for circularly polarized incident light, the transmitted light formed spatial polarization pattern with the same winding number (2) as induced by an effective magnetic field in momentum space. Following this observation and using tunability of the liquid crystal inside the cavity we find a regime in which the winding number of the effective magnetic field is equal to -2 . Following Berreman matrix simulations for dispersion relation as in Fig. 3.4 by rotation of LC anisotropy around y axis introduces energy splitting for linearly polarized modes along x and y direction. At specific angle of 24.77° , shown in Fig. 3.18a, at constant energy the cross section along k_x and k_y wave vector modes polarization can be reversed compared to initial (N, N) regime. Thanks to effective mass difference for X -polarized mode along k_x and k_y at small, positive detuning around $(N+2, N)$ resonance the inner mode is X -polarized along k_x and Y -polarized at k_y wave vector directions. Additionally, simulations for all wave vector directions presented in Fig. 3.18b shows that in this configuration the effective magnetic field wraps two times anticlockwise resulting in winding number of -2 , reverse to (N, N) regime with winding number of 2. As can be observed in Fig. 3.18b the smooth rotation of effective magnetic field orientation with wave vector direction is the evidence of a coupling between X and Y polarized modes of the same parity with difference of mode number of 2.

This coupling can be described by an effective eigenvalue problem derived in linear polarization basis, as we introduced in [50], following the

approach developed by prof. W. Bardyszewski:

$$\left(\frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} \right) \mathbf{P} - \left[(\delta_x k_x^2 - \delta_y k_y^2 - \Delta E) \hat{\sigma}_z - \delta_{xy} k_x k_y \hat{\sigma}_x \right] \mathbf{P} = E \mathbf{P}. \quad (3.11)$$

At $k_x = 0$ and $k_y = 0$ corresponding Hamiltonian is directly diagonal resulting in X - Y polarized modes. Coupling of this modes by nondiagonal term $k_x k_y \hat{\sigma}_x$ mixes those initially X - Y modes at diagonal/antidiagonal wave vector directions.

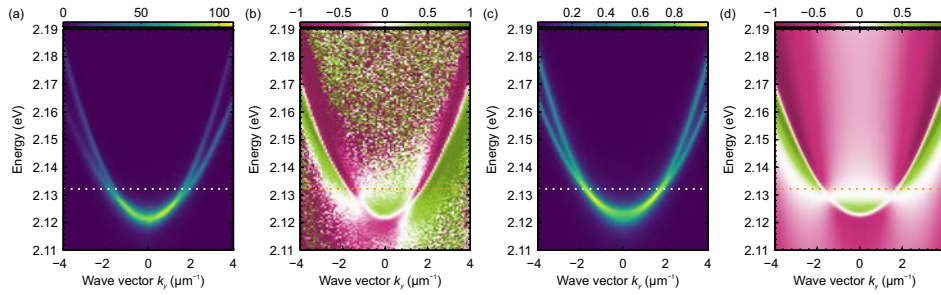


Fig. 3.19 | Liquid crystal microcavity dispersion close to $(N + 2, N)$ regime.

(a) Angle-resolved transmittance spectra shown as dispersion relation along k_y wave vector with applied square waveform with the frequency 1 kHz and the amplitude of 1.425 V. (b) S_1 parameter of transmitted light. (c) Transmittance and (d) S_1 Stokes parameters simulated by Berreman method.

Appearance of this term can be accessed also in the experiment. Fig. 3.19a presents angle-resolved transmittance spectrum of LC cavity in $(N + 2, N)$ coupling regime. Investigated cavity is based on two DBRs centered at 580 nm with 5 $\text{SiO}_2/\text{TiO}_2$ pairs. Approx. $2 \mu\text{m}$ thick cavity is filled with nematic liquid crystal with $n_e = 1.949$ and $n_o = 1.539$. To tune the cavity modes to $(N + 2, N)$ coupling regime square waveform with frequency of 1 kHz and 1.425 V amplitude is applied to transparent ITO electrodes. Voltage was set to obtain situation as in Fig. 3.18 with slightly positive detuning of X -polarized mode. This can be easier to observe in Fig. 3.19b showing S_1 parameter of transmitted light. At normal incidence, equivalent to $k_y = 0$, lower energy mode is polarized vertically ($S_1 < 0$) and higher energy mode horizontally ($S_1 > 0$). Due to the difference in effective masses of the modes they cross each other at around $1.5 \mu\text{m}^{-1}$.

Observed transmittance spectra can be well reproduced by Berreman method. Fig. 3.18c–d contains transmittance and S_1 of transmitted light for structure simulated with LC layer thickness of 1902 nm with molecules ro-

tation angle $\theta = 26.27$ deg. As previously, to obtain a better agreement with spectral linewidths, the simulations were performed with lower number of dielectric layers in DBRs, 4 in the model.

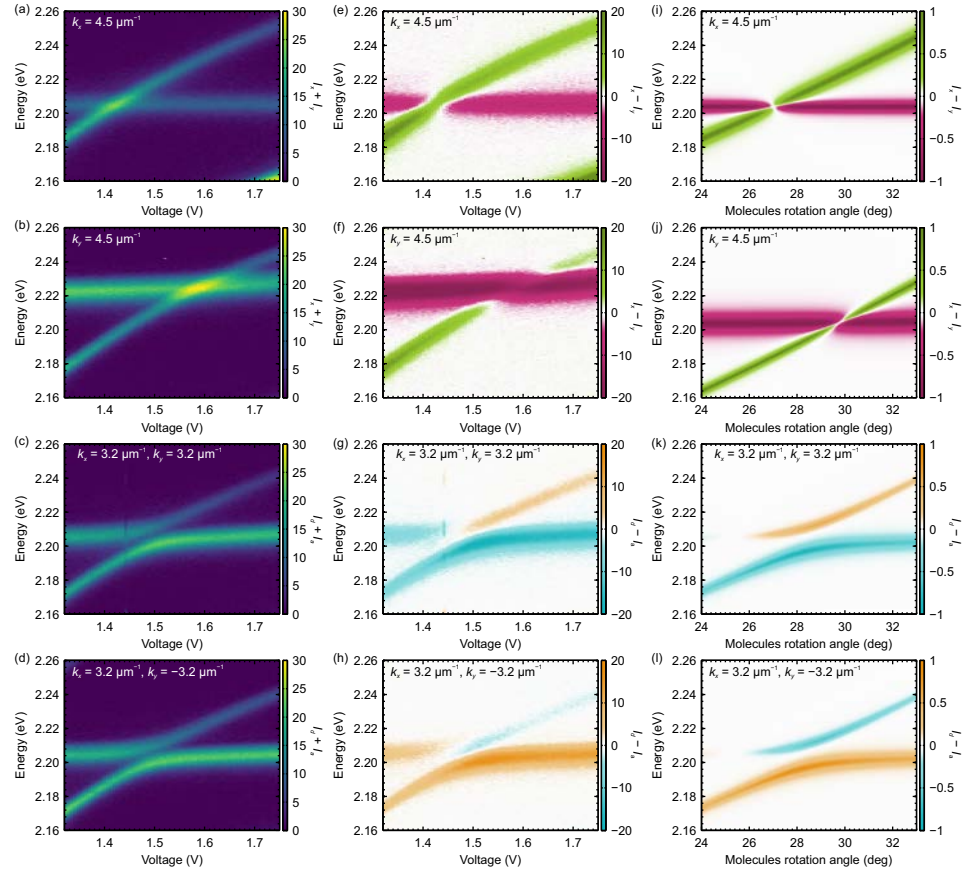


Fig. 3.20 | Voltage tuning of LC microcavity in $(N + 2, N)$ regime at high wave vector at different directions. Total transmission intensity at (a) $k_x = 4.5 \mu\text{m}^{-1}$, (b) $k_y = 4.5 \mu\text{m}^{-1}$, (c) $k_d = 4.5 \mu\text{m}^{-1}$ and (d) $k_a = 4.5 \mu\text{m}^{-1}$. (e-f) Difference between transmission intensities of X- and Y-polarized light corresponding to (a,b). (g-h) Difference between transmission intensities of diagonally and anti-diagonally polarized light corresponding to (c,d). (i-l) Corresponding simulated difference between transmittance in relevant polarizations with rotation of the director of LC molecules.

Presented spectra show transmission with wave vectors of light along y direction. To directly observe the coupling between the modes additional experiment was performed in which modes were tuned by varying voltage applied to ITO electrodes to go across $(N + 2, N)$ resonance. To probe dif-

ferent wave vector direction, the whole sample was rotated around z axis (with corresponding switching of the detection of polarizations). Fig. 3.20 shows polarization resolved transmittance spectra at high wave vectors of transmitted light at different orientations.

For wave vectors limited to $k_x = 0$ or $k_y = 0$ directions the Hamiltonian (3.11) is diagonal which results in modes polarized along this main axes. Additional rotation of LC molecules obtained by varying voltage applied to ITO electrodes is described by $\Delta E \hat{\sigma}_z$. Experimental transmittance spectra for this conditions are shown in Fig. 3.20a,b. With increasing voltage the energy position of one of the modes increases and eventually crosses the energy of the other mode which is independent on the applied electric bias. Polarization of the modes can be traced in Fig. 3.20e,f presenting difference of intensities detected in horizontal (I_x) and vertical polarizations (I_y). As expected, the mode responding to voltage is polarized in the same plane as LC molecules director ($I_x - I_y > 0$), while stationary mode exhibit perpendicular linear polarization.

For diagonal and antidiagonal wave vector directions (Fig. 3.20c,d) the evidence of coupling term $k_x k_y \hat{\sigma}_x$ appears as anticrossing behavior between the modes. Even at this specifically chosen high wave vector value the splitting is comparable with spectral linewidths, but two modes are clearly separated. Additionally, the polarization of the modes changes to diagonal basis, as can be observed in nonzero difference of transmitted light intensity in diagonal (I_d) and antidiagonal (I_a) polarizations presented in Fig. 3.20c,d.

All of these experimental transmission results are well reproduced by Berreman method simulations presented in Fig. 3.20i-l presented as a function of molecules rotation angle θ , showing crossing of the modes when the incident light wave vector has only k_x or k_y component in Fig. 3.20i,j, in contrast to the anticrossing behavior for diagonal and antidiagonal wave vector direction in Fig. 3.20k,l.

As we discussed all of the components in the Hamiltonian Eq. (3.11), we can proceed to the analysis of the polarization textures appearing in the real space.

Fig. 3.21a presents simulated spatial polarization texture for linearly polarized light transmitted through LC microcavity at the energy marked by dashed line in Fig. 3.18. As for (N, N) regime the polarization pattern can be separated into four quadrants, each consisting of lobes in radial direction. However, the topology of individual polarization vortices is different than previously. In the first and third quadrant ($x, y > 0$, $x, y < 0$) the lobe closest to the center is formed by antimeron, contrary to the meron in (N, N)

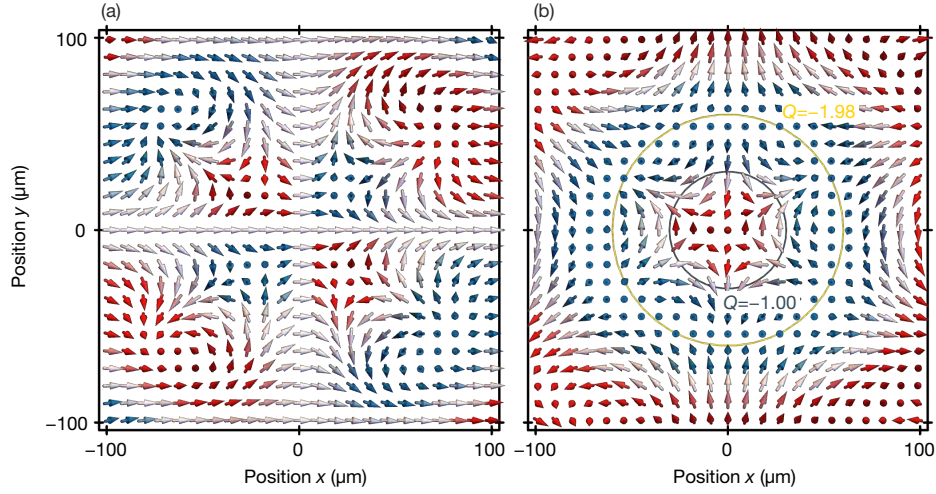


Fig. 3.21 | Formation of second order antimeron. Spatial polarization state for for light transmitted through LC microcavity close to $(N + 2, N)$ regime for polarization of incidence light from (a) linear and (b) circular σ^+ .

regime. Negative polarity of this antimerons result in positive topological charge. The same topological charge is carried with further lobe in the radial direction formed by meron with positive polarity.

In the remaining second and fourth quadrants ($\pm x \cdot \mp y > 0$) the polarity of the individual lobes is opposite to the first quadrant what leads to the negative topological charge with antimeron closest to the center and meron at higher distance.

Effectively, compared to the (N, N) regime in the polarization pattern observed in $(N + 2, N)$ regime the topological charge sign is reversed as antimerons appear in the place of merons and vice versa.

Following simulations for different incident light polarizations with increasing value of S_3 parameter of excitation beam two antimerons with positive polarity approach to the excitation spot. At σ^+ incident light polarization presented in Fig. 3.21b they merge forming second order antimeron. Integrating topological charge density within radius of 30.4μ from the center results in topological charge of -1.00 as expected from second order antimeron. Similarly, as for the (N, N) case for high- Q cavities S_3 oscillates in full range of ± 1 . At the radius of 60.1μ numerically calculated topological charge reaches local minimum of -1.975 , close to -2 expected from second order anitiskyrmion.

To observe second order antimeron polarization pattern in transmitted light we performed polarization resolved tomography measurement analo-

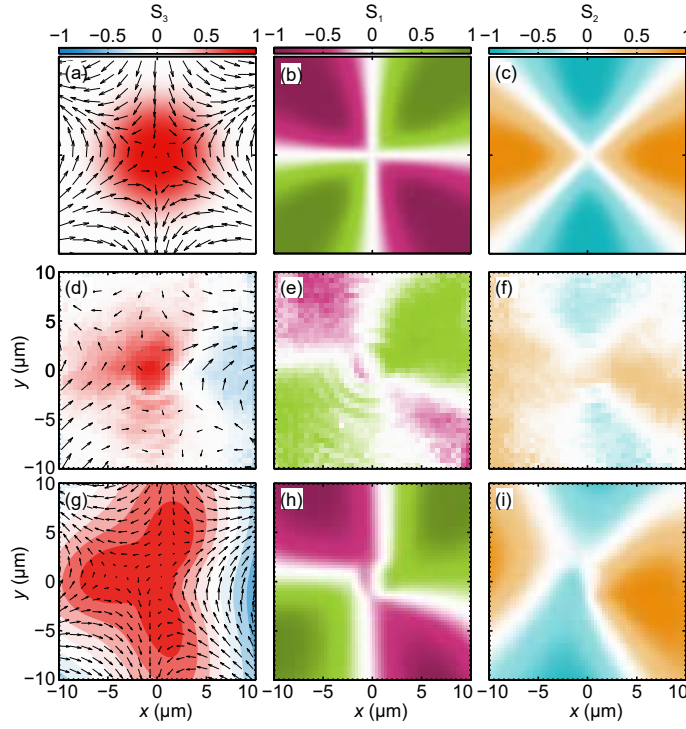


Fig. 3.22 | Second order antimeron polarization texture in LC microcavity in $(N + 2, N)$ regime. (a–c) Second order antimeron polarization texture. (a) $\mathbf{S}_{\parallel} = (S_1, S_2)$ marked with arrows with S_3 mapped by a colorscale; (b) S_1 (c) S_2 parameters mapped by colorscale. (d–f) Experimental spatial polarization texture of transmitted light: (a) $\mathbf{S}_{\parallel} = (S_1, S_2)$ marked with arrows with S_3 mapped by a colorscale; (e) S_1 (f) S_2 parameters mapped by colorscale. (g–h) Simulated by Berreman method spatial polarization texture of transmitted light: (g) $\mathbf{S}_{\parallel} = (S_1, S_2)$ marked with arrows with S_3 mapped by a colorscale; (h) S_1 (i) S_2 parameters mapped by colorscale.

gous to experiment for (N, N) regime.

Fig. 3.22d–f presents spatial polarization state separated into Stokes parameters observed at energy cross section corresponding to dashed lines in Fig. 3.19. Experimental results show the same polarization structure as second order antimeron pattern plotted in Fig. 3.22a–c. S_1 parameter is positive in the first and third quadrant with opposite sign in second and fourth quadrant, the same as in (N, N) regime. Changes can be observed in S_2 parameter where polarization is opposite to previously observed with positive S_1 values along $y = 0$ line and negative at $x = 0$ direction.

Observed polarization pattern matches well with Berreman matrix simulations for parameters chosen to recreate angle-resolved transmittance spectra in [Fig. 3.19](#).

3.3 Polarization textures in LC cavities: summary

In summary, the incorporation of optically anisotropic material such as nematic liquid crystal into planar optical cavity leads to multiple-polarization dependent phenomena.

When the anisotropy axis of LC is oriented in the growth direction, such that there is no optical anisotropy in the plane, it strongly affects strength of TE-TM splitting of the cavity modes observed at nonzero incidence angles. Especially it allows to change the sign of the TE-TM splitting when using materials with positive or negative birefringence. Regardless of the birefringence sign, the presence of LC significantly enhances the TE-TM splitting strength.

It has been previously reported in the literature, the presence of TE-TM splitting in an optical cavity combined with linearly polarized pump beam leads to polarization texture in the real space composed of meron-antimeron lattice [\[81\]](#).

In this work, we presented theoretically that the same effect can be observed in the cavity filled with liquid crystal. Experimental demonstration of this effect turned out to be challenging due to limited quality factor of the investigated samples, what resulted in limited in-plane propagation length, insufficient to show the meron-antimeron lattice. More importantly, when the same effect is analyzed for circularly polarized beam the polarization texture changes to a second order meron for realistic cavities. As its core is located at the incident light spot the presence of this texture was confirmed also in the experiment.

For significantly reduced linewidths of the cavity mode the simulations predict that it is possible to observe second order skyrmion polarization texture.

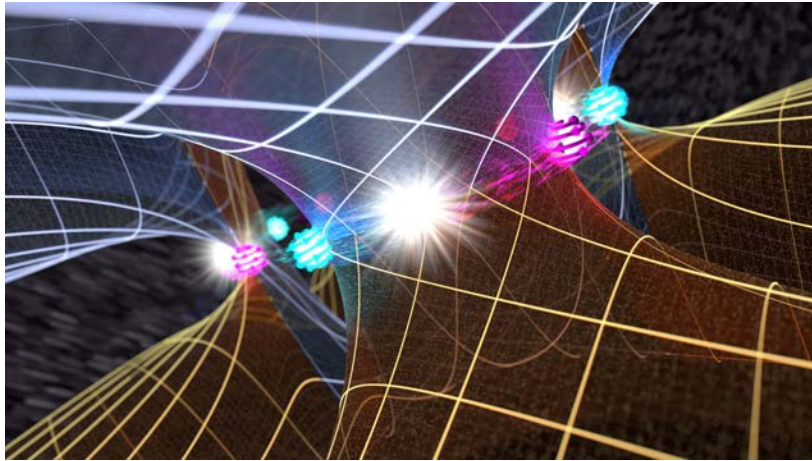
Even more importantly, the tunability of the optical anisotropy direction within the LC layer allows to arrange the cavity modes in such a fashion, that the effective magnetic field at a given incident light wavelength can be inverted from $+2$ characteristic for typical TE-TM splitting to -2 . In such configuration theoretical simulations show reversal of the topological charge at the spatial polarization texture for polarized incident light in comparison with the typical microcavities. It allowed for the observation of second

order antimeron polarization texture in both theory and experiment.

Chapter 4

Annihilation of exceptional points in LC microcavity

Results described in a manuscript currently under review:



- [90] M. Król, I. Septembre et al., Annihilation of exceptional points from different Dirac valleys in a 2D photonic system. [arXiv:2112.06621](https://arxiv.org/abs/2112.06621) (2021).

In the previous chapter the manipulation of the spin polarized excitation beam was used to obtain polarization textures in the real space.

An overlooked feature of those band was the existence of spectral degeneracies when two band cross in the momentum space. Such intersection points are of particular importance especially in the systems exhibiting polarization dependent gain or loss described by non-Hermitian terms. An optical cavity with inherent losses originating from the limited lifetime of the cavity photons is a natural candidate for such studies.

In this chapter the first section (Sec. 4.1) is devoted to a brief introduction into non-Hermitian physics and its most important feature comprising of spectral degeneracies referred to as exceptional points. In the second section the behavior of exceptional points in planar microcavities will be introduced starting from a realistic parameters for a liquid crystal microcavity in (N, N) regime.

The main result of this chapter will be presented in the last section (Sec. 4.3). Starting from the analytical model for liquid crystal microcavity in $(N + 2, N)$ regime it will be shown that in such condition it is possible to observe the annihilation of the exceptional points, occurring when those spectral singularities disappear upon meeting in the reciprocal space. Those theoretical predictions will be followed by an experimental demonstration of the photonic band structure before and after the annihilation.

4.1 A brief overview of non-Hermitian physics

Recent interests in non-Hermitian physics started with the seminal work by C. Bender and S. Boettcher who noticed that for a wide class of Hamiltonians conserving parity-time ($\mathcal{PT}$) symmetry despite clear non-Hermiticity the spectrum is purely real and positive [91]. This brought widespread attention to the systems trying to balance gain and losses.

Of special interests in the non-Hermitian physics are so-called exceptional points (EPs) which are the singularities in the parameter space of Hamiltonians at which at least two eigenvalues (real and imaginary parts) coalesce and are degenerate. Moreover around the EP the vector space is skewed that at EP supports only one eigenvector [92]. This is in a striking difference in comparison to the Hermitian systems in which at spectral degeneracies, referred to as diabolical points, the eigenvectors remain linearly independent [27]. Starting from a diabolical point the introduction of a non-Hermitian perturbation can spawn a pair of exceptional points [93], which are connected with a Fermi arc [94].

Concepts based on $\mathcal{PT}$ symmetry and unique properties of non-Hermitian systems have been applied in diverse fields of physics like: acoustics [95], electronics [96], microwave resonators [97], mechanical systems [98], cold atoms [99], and metamaterials [100], but probably the most fruitful area is optics [101, 102].

One of the most promising applications of the non-Hermitian physics is enhanced sensing in the vicinity of EP [103, 104]. In the Hermitian systems according to Weyl's perturbation theorem the change of the eigenvalue is smaller or equal to the highest eigenvalue ϵ of any Hermitian perturbation [92]. However at the exceptional point the shift of the spectrum scales as $|\epsilon|^{\frac{1}{n}}$, where n is the order of the EP, which for small perturbations $|\epsilon| < 1$ is smaller than the perturbation itself [92]. Such behavior has been observed experimentally for second [103] and third order EPs [104]. Although such approach gives hopes for a new generation of sensor based on EPs, practical applications point to other inherent limitations, [105], which are described in more details in specific review articles [106].

Maintaining $\mathcal{PT}$ symmetry requires precise balancing of losses and gain within the system. This can lead to such counter-intuitive observations like suppression of lasing under increasing pumping power [107] or revival of lasing upon increasing loss [108]. In more practical applications lasing in a ring resonator with a specific single mode can be strongly favored even in materials with broadband gain spectral range by coupling with additional lossy ring [109]. Another scheme allowing for the selection of a given lasing mode has been based on a precise design of loss and gain regions [110].

In Hermitian systems adiabatic encircling of a diabolical points leads to accumulation of π Berry phase [26, 27], as has been demonstrated experimentally in microwave cavities [111].

In the non-Hermitian case due to self-intersecting Riemann sheet geometry of the complex energy around the EPs single encircling of a second order EP leads to switching between the modes. Full π geometric Berry phase is accumulated only when the same EP is encircled twice, as has been experimentally demonstrated in a microwave resonator [97] as well as an optical cavity [112] or exciton-polariton billiard [113].

Encircling EP in a dynamical system results in the final state dependent only on the encircling direction, regardless of the initial state [114, 115, 116]. In practice this allows for realization of unidirectional transport, where the final transmitted state is dependent on the direction of the propagation within the system. Within the same framework conceptually simple structure consisting of a stack of successive absorbing and non-absorbing layers can be used to design and produce a unidirectional reflectors, where reflection

and transmission varies strongly depending on the direction of the incident light [117]. It was proposed that a similar scheme can be used to obtain an omni-polarizer, a system allowing to produce any polarization state independently on the input [118].

Presented here extremely concise overview of the most intriguing phenomena arising in non-Hermitian system does not aim to exhaust wide and rapidly expanding area. More precise information can be found in recent review papers concentrating on general $\mathcal{PT}$ symmetry [119], exceptional points in optics [102, 120], or topology of non-Hermitian systems [121].

Before proceeding into specifics of exceptional points in liquid crystal microcavities it is instructive to consider effects of inclusion of non-Hermitian contributions into a Hermitian system containing diabolical points. For the analysis of an exemplary 2D Hamiltonian in circular polarization basis:

$$\hat{H}(\mathbf{k}) = k_y \hat{\sigma}_x - k_x \hat{\sigma}_y. \quad (4.1)$$

Eigenvalues of this model Hamiltonian are given by:

$$E_{1,2}(\mathbf{k}) = \mp \sqrt{k_x^2 + k_y^2}, \quad (4.2)$$

which are plotted in Fig. 4.1a. The resulting dispersion relation consists of a Dirac cone with a single diabolical point at $\mathbf{k} = 0$, where both energies are equal to each other (plotted also as energy difference between the branches in Fig. 4.1d). Pseudospin texture of the lower energy branch solution of Hamiltonian (4.1) plotted in Fig. 4.1g shows +1 winding while encircling the Dirac point at $\mathbf{k} = 0$.

Here Hamiltonian with the form of (4.1) is used as both dispersion relation and the pseudospin texture resembles the diabolical point appearing in typical planar microcavities with small XY splitting described in Sec. 2.2.3 and plotted in Fig. 2.22e,f.

Hermicity of Eq. (4.1) assures that the eigenvalues are purely real with vanishing imaginary part. Gain/losses distinguishing horizontal and vertical pseudospin states can be included into the Hamiltonian by an additional non-Hermitian term $i\xi \hat{\sigma}_x$ leading to:

$$\begin{aligned} \hat{H}(\mathbf{k}) &= k_y \hat{\sigma}_x - k_x \hat{\sigma}_y + i\xi \hat{\sigma}_x = \\ &= \begin{pmatrix} 0 & k_y + ik_x + i\xi \\ k_y - ik_x + i\xi & 0 \end{pmatrix}. \end{aligned} \quad (4.3)$$

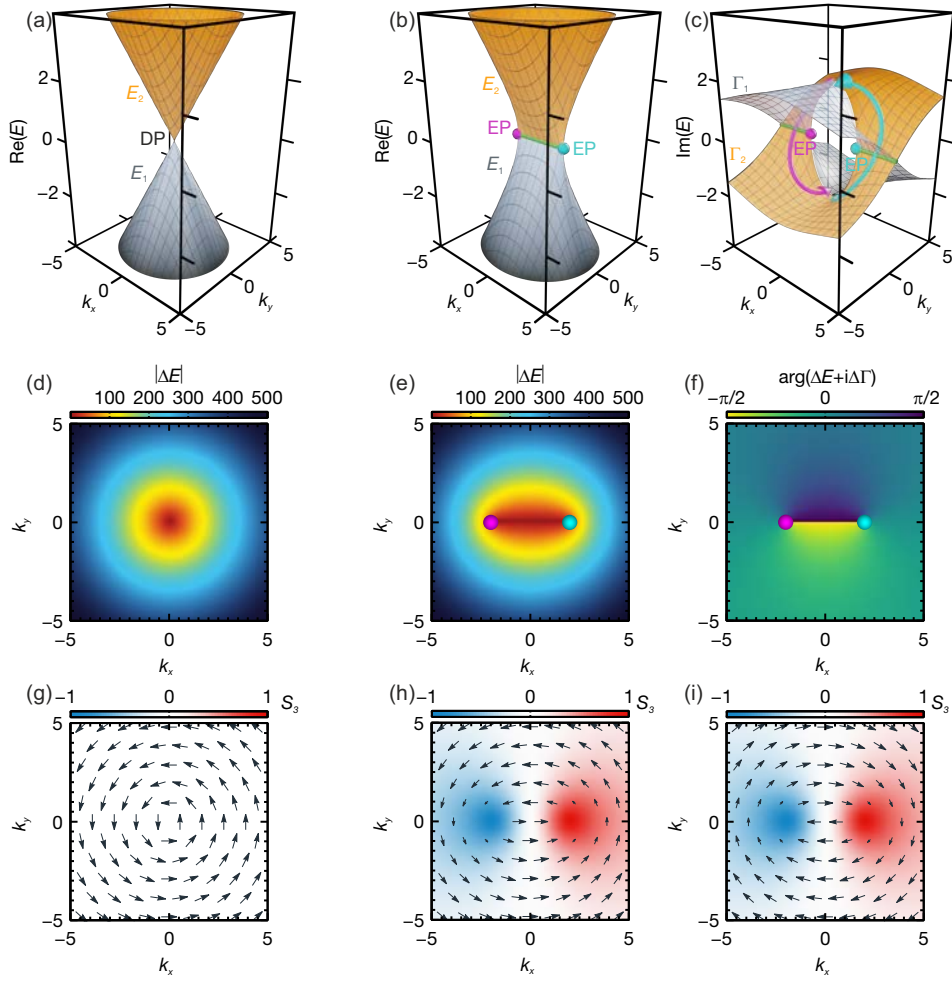


Fig. 4.1 | Spawning of exceptional points (EPs) from a diabolical point (DP) by additional non-Hermiticity. (a) Dispersion relation of a Dirac cone exhibiting single DP, with (d) energy difference between the bands and (g) pseudospin orientation at the lower energy branch. (b) Real spectrum of the Hamiltonian with additional non-Hermiticity exhibiting two EPs connected with a real Fermi arc (green line) and (e) corresponding energy difference between the bands. (c) Imaginary part of the spectrum of the non-Hermitian Hamiltonian with (f) corresponding argument of the complex energy. Pseudospin orientation at (h) lower and (i) upper energy branch.

Eigenvalues of Eq. (4.3) are given by:

$$E_{1,2} = \mp \sqrt{k_x^2 - (\xi - ik_y^2)^2}. \quad (4.4)$$

For general values of the wave vector they are complex with nonzero real and imaginary part. Real part of the spectrum $\text{Re}(E_{1,2})$ and imaginary part $\text{Im}(E_{1,2})$ for $\xi = 2$ are plotted in Fig. 4.1b,c, correspondingly. Starting from the real spectrum the single degeneracy point in the Hermitian limit expands into a degeneracy line laying within $|k_x| < \xi$, which can be observed also in Fig. 4.1e, which presents the difference in real parts of the energies. Both ends of this degeneracy line—also referred to as real Fermi arc, which is marked in Fig. 4.1b by a green line—contain an exceptional points (EPs), where both real and imaginary part of the eigenenergies of Hamiltonian (4.3) are equal at the same time.

Imaginary part of the eigenenergies are plotted in Fig. 4.1c. Imaginary parts of the eigenenergies (Γ_i) are degenerate at EPs and for $|k_x| > \xi$, which forms an imaginary Fermi arc. Imaginary part of the spectrum consisting of surfaces corresponding to lower real energy branch (gray) and upper real energy branch (orange) form a Riemannian surface, winding around both EPs. Especially winding around each of the two EPs is opposite, as marked with cyan and magenta arrows in Fig. 4.1c. Encircling clockwise EP located at $k_x = \xi$ following surface corresponding to upper real energy branch (marked with orange color), we follow upwards in imaginary part of the energy along the cyan arrow. Moving with the same direction around the second EP at $k_x = -\xi$ the imaginary part of the energy decreases following the magenta arrow. Winding around the EP can be also visualized on a 2D image by plotting an argument of the complex energy $\arg(\Delta E + i\Delta\Gamma)$. It is plotted in Fig. 4.1f, which clearly shows vortex-like phase profile around both of the EPs with opposite winding between them. It is worth noting that Fig. 4.1f does not emphasize the location of the imaginary Fermi arc, which follows $\arg(\Delta E + i\Delta\Gamma) = 0$ condition.

Another important property of the exceptional points that not only energies are degenerate (both real and imaginary part) but also the eigenvectors coalesce. At the position of the EP where $|k_x| = \xi$ the Hamiltonian reads:

$$\hat{H}(k_x = \xi, k_y = 0) = \begin{pmatrix} 0 & 2i\xi \\ 0 & 0 \end{pmatrix} \quad (4.5)$$

$$\hat{H}(k_x = -\xi, k_y = 0) = \begin{pmatrix} 0 & 0 \\ -2i\xi & 0 \end{pmatrix} \quad (4.6)$$

For (4.5) the only eigenvector is equal to $(1,0)^T$ corresponding to σ^+ polarization, whereas for (4.6) the eigenvector $(0,1)^T$ is orthogonal, corresponding to σ^- polarization of the state. Pseudospin texture for all wave vectors

is plotted in Fig. 4.1h for lower real energy mode and in Fig. 4.1i for higher real energy band. Far away from $k = 0$ the pseudospin texture tends to the Hermitian limit (Fig. 4.1g), with the orthogonal spin state at the upper energy branch (Fig. 4.1i). The situation changes approaching the EPs positions, where eigenstates of both energy branches is the same and circularly polarized with opposite helicities at different EPs. Additionally the spin state plotted in Fig. 4.1h,i is discontinuous while crossing the real Fermi arc along k_y direction. Such behavior is allowed as moving in this direction corresponds to switching between lower and upper real energy branches, which are degenerate along the discontinuity line, but both states differ in imaginary part of the energy (compare with encircling single EP in Fig. 4.1c).

Following such exemplary model brings intuitions that by introducing non-Hermitian terms to the Hamiltonian leads to eigenenergies with nonzero real and imaginary parts. Special singularities in spectra of such Hamiltonians are so-called exceptional points where real and imaginary parts of the eigenenergies are degenerate and corresponding eigenvectors coalesce. In a specific case of polarization-dependent losses between linearly polarized spin states leads introduced into a system exhibiting a Dirac (diabolical) point leads to spawning of two exceptional points with opposite windings.

Here we followed a single model which will be close to a realistic microcavity system, as it will be shown shortly. Starting from the same Hermitian limit addition of losses differentiating diagonal and antidiagonal pseudospin orientation ($i\xi\hat{\sigma}_y$) leads to very similar behavior, but with the EPs positioned at $k_x = 0$ line. Losses can also be introduced by term multiplied by the third Pauli matrix ($i\xi\hat{\sigma}_z$), which in this configuration leads to spawning of an so called exceptional ring, in which over a whole closed loop both real and imaginary parts are degenerate [100, 121].

4.2 Exceptional points in (N, N) regime

Following description presented in Sec. 2.2.3 close to the (N, N) resonance, when the LC director is oriented closely to the normal direction of the cavity plane, the dispersion relation can be described by a Hamiltonian:

$$\begin{aligned} \hat{H}_{\text{TE-TM}} = & \left(E_0 + \beta_x k_x^2 + \beta_y k_y^2 \right) \hat{\sigma}_0 + \\ & + \beta(k_x^2 - k_y^2) \hat{\sigma}_x + 2\beta k_x k_y \hat{\sigma}_y + \frac{1}{2} \Delta E_{\text{HV}} \hat{\sigma}_x + \\ & + i\xi \hat{\sigma}_x, \end{aligned} \quad (4.7)$$

which includes TE-TM splitting proportional to the parameter β and HV splitting ΔE_{HV} (also referred to as XY splitting) originating from tilt of the LC director from the z axis. Parameters $\beta_{x/y} = \frac{\hbar^2}{2m_{x,y}}$ are different notation for effective mass contribution.

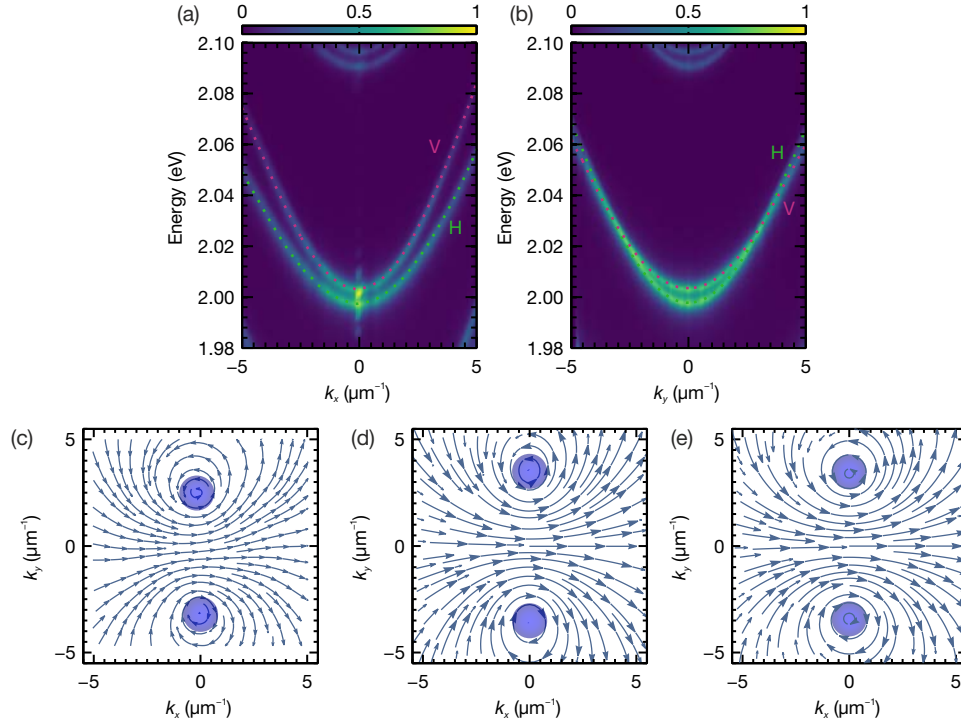


Fig. 4.2 | Polarization-resolved Fourier space tomography in (N,N) regime. Dispersion relations along (a) k_x and (b) k_y wave vector direction obtained from transmission measurement. Pseudospin state along lower energy branch from (c) experiment, (d) Berreman matrix simulations (e) fitting of the to the experiment.

Fig. 4.2a,b presents cross sections through polarization-resolved Fourier space tomography measured in transmission configuration. The measurement was performed at a comparably thick LC cavity (approx. $3.9 \mu\text{m}$ LC width) at a high voltage of 11 V, which corresponded to the operation close to the (N,N) regime degeneracy for this ample. The green (purple) dotted lines show fitting of the polarization-resolved measurements in horizontal (vertical) incident light polarization. Along the k_x wave vector direction (Fig. 4.2a) H and V polarized energy bands are separated in energy and do not cross. In contrast, in the perpendicular k_y wave vector direction in Fig. 4.2b the H-polarized lower energy mode at normal incidence cross with the other mode due to lower effective mass. As discussed in Sec. 2.2.3, those

two crossing points along k_y wave vector direction act as a diabolical points with characteristic Dirac cone dispersion relation in their vicinity in the momentum space.

Winding of the pseudospin texture around those diabolical points can be observed in Fig. 4.2c, which shows experimental polarization state of the lower energy mode extracted from the tomography measurement. At normal incidence the lower energy mode is horizontally polarized, and keeps its polarization when following k_x wave vector direction as in Fig. 4.2a. At perpendicular wave vector direction the polarization changes to vertical at high wave vectors as H and V modes crosses each other, like in Fig. 4.2b. Combining with positive (negative) S_2 contributions at diagonal (antidiagonal) wave vector directions around the diabolical crossing points (marked with blue circles) the pseudospin shows +1 winding.

The same pseudospin texture of the lower energy branch can be recreated by Berreman model. Fig. 4.2d shows results of calculations in which in a cavity with total thickness of the LC of 3865 nm (with two isotropic interface layers, each 240 nm thick) in which molecules rotation angle $\theta = 83$ deg. Clearly two singularities with +1 winding of the pseudospin around them can be observed along the $k_x = 0$ line.

Using cross sections along k_x and k_y all parameters of the Hermitian part of the model given by Eq. (4.7) can be extracted by fitting the dispersion relation of the observed modes. Based on the experimental results presented in Fig. 4.2a,b the fitting parameters are equal to $E_0 = 2.002$ eV, $\Delta E_{HV} = -7.6$ meV, $m_x = 2.8$ meV μm^2 , $m_y = 2.5$ meV μm^2 , $\beta = -0.33$ meV μm^2 .

Parameters of the model can be then further used to investigate system exhibiting additional non-Hermitian H-V polarization dependent gain/losses provided by the $i\xi\hat{\sigma}_x$ term. Calculations performed for $\xi = 5$ meV are presented in Fig. 4.3a,b.

The difference of real energy part of eigenenergies plotted in Fig. 4.3a shows two degeneracy arcs originating one from each diabolical point. At the ends of this real Fermi arcs the argument of the complex energy difference, presented in Fig. 4.3b, shows vortex-like winding of the phase as expected from the exceptional points.

Thus similarly to the model simplified Hamiltonian (4.1) exhibiting single diabolical point in the Hermitian limit, in microcavity system exhibiting two diabolical point each becomes an origin of two exceptional points with introduction on losses dependent of H/V polarized states. Contrary to simplified model presented earlier in the LC microcavity in (N,N) do not move excursively in k_x , as the real Fermi arcs exhibit dislocation from the diabolical point origin in both k_x and k_y components. Precise momentum posi-

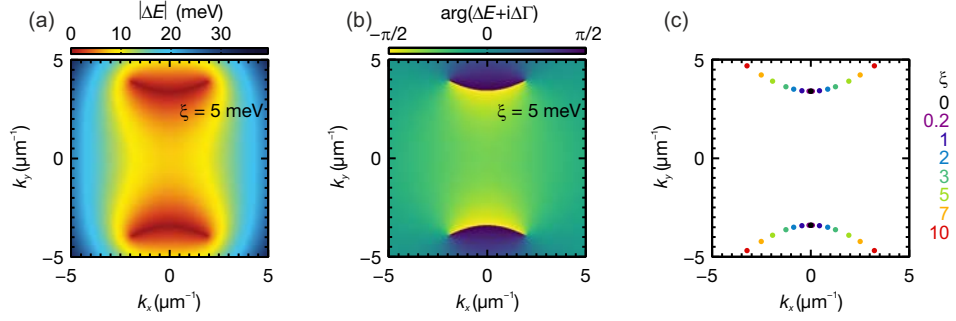


Fig. 4.3 | Exceptional points in (N,N) regime. (a) Difference of real parts of the eigenenergies of the modes at $\xi = 5$ meV. (b) Corresponding argument of the complex energy. (c) Position of the EP (colored dots) for increasing non-Hermitian contribution ξ .

tion of the exceptional points can be calculated based on Eq. (4.7) by setting conditions requiring equality in both real and imaginary parts of both eigenenergies. Analytical positions are given by:

$$k_x^{\text{EP}} = \pm \frac{1}{2} \sqrt{\frac{-\Delta E_{\text{HV}}}{\beta} + \sqrt{\frac{\Delta E_{\text{HV}}^2 + 4\xi^2}{\beta^2}}} \quad (4.8)$$

$$k_y^{\text{EP}} = \pm \frac{1}{2} \sqrt{\frac{\Delta E_{\text{HV}}}{\beta} + \sqrt{\frac{\Delta E_{\text{HV}}^2 + 4\xi^2}{\beta^2}}}. \quad (4.9)$$

Based on this solutions the positions of the EPs for increasing non-Hermitian contribution ξ are plotted in Fig. 4.3c. For low non-Hermiticity (up to $\xi = 1$ meV) spawned EPs move in the momentum space mostly in k_x direction, but for higher polarization-dependent losses their separate also in k_y direction. In particular in the limit of high non-Hermiticity contribution $\xi \rightarrow \infty$ asymptotically tend to diagonal/antidiagonal wave vector directions as:

$$\frac{k_x^{\text{EP}}(\xi \rightarrow \infty)}{k_y^{\text{EP}}(\xi \rightarrow \infty)} = \pm 1. \quad (4.10)$$

Here possibility of observation of EPs in LC cavities is shown only theoretically based on the analytical model, which shows good agreement with the experiment in the Hermitian limit. As described here (N,N) regime is closest to the typical planar microcavities the appearance of exceptional points in this regime was studied by different groups.

In the literature emergence of exceptional points in planar microcavi-

ties was initially observed by S. Richter et al. in numerical simulations of a planar microcavity incorporating birefringent ZnO material [41]. This theoretical predictions were later on followed by the same group with an experimental demonstration described in Ref. [42]. Later on exceptional points were also demonstrated experimentally in a planar microcavity containing a birefringent perovskite crystal [43]. Berry curvature and quantum metric, especially their divergence at the EPs, investigated in a planar microcavity containing strongly anisotropic organic crystal [122].

4.3 Exceptional points in $(N + 2, N)$ regime

The ability to tune a LC cavity into a different regimes of degeneracy between the modes brings a great opportunity for realization of different behavior of the exceptional points.

In particular, as described in Sec 2.2.5 around $(N + 2, N)$ resonance the dispersion relation of the cavity modes can exhibit four diabolical points. Such regime is demonstrated in Fig. 4.4.

It presents Fourier space tomography results performed on the same sample as in Fig. 4.2, but at lower voltage applied to the LC cell of 1.39 V, which corresponds to $(N + 2, N)$ regime. Fig. 4.4a,b presents cross section along k_x and k_y wave vector directions through polarization-resolved tomography in transmission configuration. Here in both of the main wave vector directions the cavity modes of perpendicular linear polarization crosses. Along the k_y direction the crossing of the modes is evident, as the effective mass difference between the modes is higher. It is less visible at k_x wave vector direction but the both modes also crosses each other, as can be easier to trace by following dotted lines, which present fittings to the H/V polarization resolved data. In total the dispersion relation contains 4 diabolical points where energies of the two nearest cavity modes are degenerate.

Corresponding to this experimental results pseudospin texture of the lower energy mode is shown in Fig. 4.4c, with the position of the diabolical points marked with circles with different color depending on winding of the pseudospin texture around them. Both diabolical points oriented along $k_y = 0$ exhibit positive +1 winding of the pseudospin. This is contrary to the remaining two diabolical points located at $k_x = 0$ wave vector direction which exhibit negative -1 winding of the pseudospin. As the band structure contains two positive and two negative poles the total winding of the texture is equal to zero.

The same pseudospin texture is a result of Berreman matrix simulations,

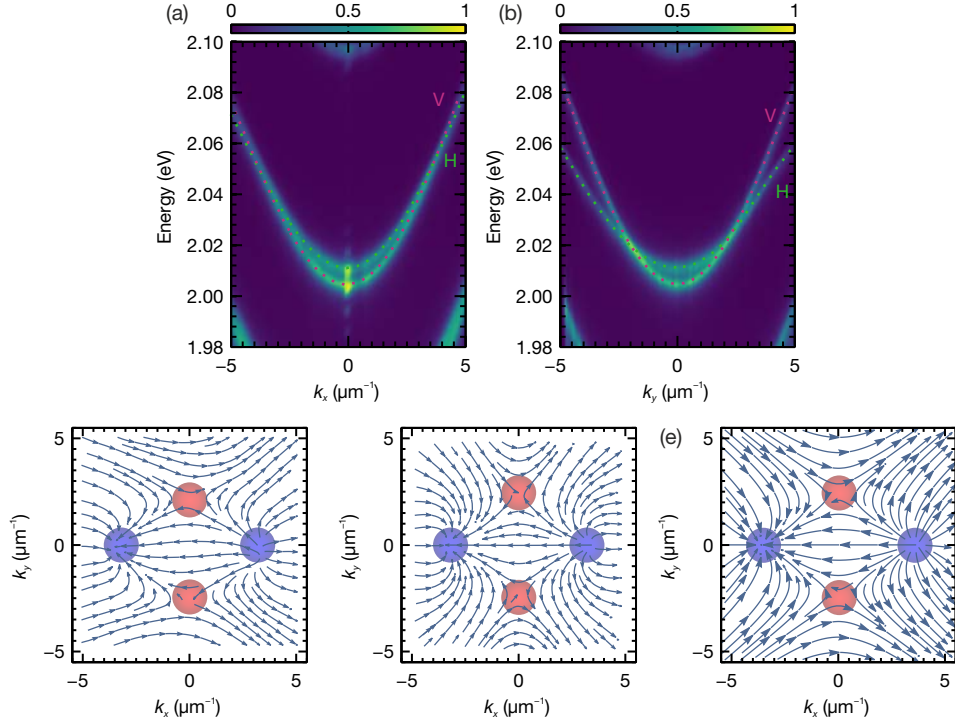


Fig. 4.4 | Polarization-resolved Fourier space tomography in $(N + 2, N)$ regime. Dispersion relations along (a) k_x and (b) k_y wave vector direction obtained from transmission measurement. Pseudospin state along lower energy branch from (c) experiment, (d) Berreman matrix simulations (e) fitting of the to the experiment.

presented in Fig. 4.4d. The simulations were performed for the same structure as the one in Fig. 4.2d, with the only difference in the LC molecules rotation angle $\theta = 44.4$ deg to recreate proper energy difference between the modes in $(N + 2, N)$ regime. As in the experiment simulated texture exhibit four singularities with alternating ± 1 windings. As in the experiment the diabolical points located at $k_x = 0$ line are located closer to the momentum space origin than the ones located at $k_y = 0$ line.

Earlier described Hamiltonian providing description of the cavity modes close to the $(N + 2, N)$ resonance is given by:

$$\begin{aligned} \hat{H} = & \left(E_0 + \beta_x k_x^2 + \beta_y k_y^2 \right) \hat{\sigma}_0 + \\ & + (\delta_x k_x^2 + \delta_y k_y^2) \hat{\sigma}_x + \frac{1}{2} \Delta E_{\text{HV}} \hat{\sigma}_x + \delta_{xy} k_x k_y \hat{\sigma}_y. \end{aligned} \quad (4.11)$$

Eigenvalues of this Hamiltonian can be fitted to the experimentally obtained

dispersion relation with parameters: $E_0 = 2.010$ eV, $\Delta E_{\text{HV}} = 6.1$ meV, $\beta_x = 3.0$ meV μm^2 , $\beta_y = 2.5$ meV μm^2 , $\delta_x = -0.23$ meV μm^2 , $\delta_y = -0.58$ meV μm^2 , $\delta_{xy} = -0.60$ meV μm^2 .

For this given parameters the dispersion relation calculated by Eq. (4.11) is plotted in Fig. 4.5a. It consists of 2 paraboloid-like energy branches which touches in only 4 positions marked by colored spheres denoted as DP (diabolical points). Their position within the dispersion relation is easier to trace by plotting energy of the photonic branches subtracted by their mean value $\langle E \rangle$, as is shown in Fig. 4.5b.

Starting from this Hermitian limit we can investigate theoretically the influence of non-Hermitian contribution $i\xi\hat{\sigma}_x$ providing H/V polarization dependent losses/gain. In this case the total Hamiltonian of the system is given by:

$$\begin{aligned}\hat{H} = & \left(E_0 + \beta_x k_x^2 + \beta_y k_y^2\right) \hat{\sigma}_0 + \\ & + (\delta_x k_x^2 + \delta_y k_y^2) \hat{\sigma}_x + \frac{1}{2} \Delta E_{\text{HV}} \hat{\sigma}_x + \delta_{xy} k_x k_y \hat{\sigma}_y + \\ & + i\xi \hat{\sigma}_x.\end{aligned}\quad (4.12)$$

Starting from relatively low non-Hermitian contribution $\xi = 1$ meV, the real and imaginary energy spectrum are presented in Fig. 4.5c,d. The corresponding real part of the energies difference and complex energy difference argument is plotted in Fig. 4.5e,f, correspondingly. In total, the spectrum contains 8 exceptional points, two originating from each of the 4 diabolical points in the Hermitian limit. Each two EPs spawned from the same diabolical point are connected by a real Fermi arc of vanishing difference of the real parts of the energy. On the imaginary spectrum (Fig. 4.5d) each two closest in the momentum space EPs originating from different diabolical points are connected by imaginary Fermi arcs (green lines in Fig. 4.5d). The complex energy phase plotted in Fig. 4.5f shows that each of the EPs exhibit opposite winding to their closest neighbor in momentum space originating from both: the same or different diabolical point.

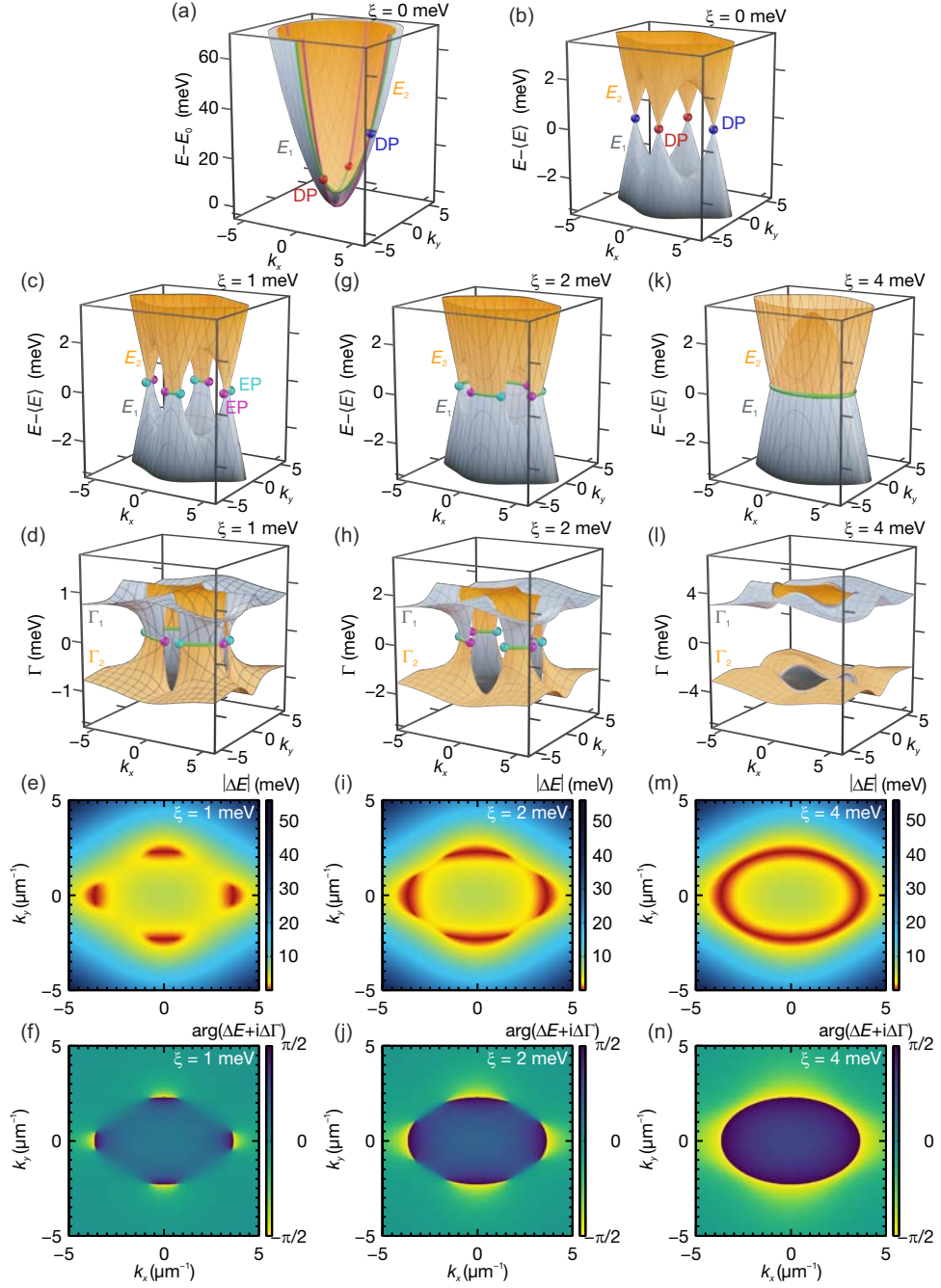


Fig. 4.5 | $(N+2, N)$ regime with non-Hermiticity. Dispersion relation in Hermitian limit (a) energy of the cavity bands (b) $E - \langle E \rangle$. Low non-Hermiticity $\xi = 1$ meV: (c) real and (d) imaginary spectrum; (e) real energy difference $|\Delta E|$ and (f) complex energy phase $\arg(\Delta E + i\Delta\Gamma)$. Intermediate non-Hermiticity $\xi = 2$ meV: (g) real and (h) imaginary spectrum; (i) real energy difference and (j) complex energy phase. High non-Hermiticity $\xi = 4$ meV: (k) real and (l) imaginary spectrum; (m) real energy difference and (n) complex energy phase.

Based on the Eq. (4.12) and the requirement of vanishing both real and imaginary parts of energy difference between the modes positions of all of the EPs can be given by:

$$k_x^{\text{EP}} = \pm \frac{1}{2} \sqrt{\frac{-\Delta E_{\text{HV}} \delta_{xy}^2 \pm \sqrt{\Delta E_{\text{HV}}^2 \delta_{xy}^4 - 16 \delta_x \delta_y \delta_{xy}^2 \xi^2}}{\delta_x \delta_{xy}^2}} \quad (4.13)$$

$$k_y^{\text{EP}} = \pm \frac{1}{2} \sqrt{\frac{-\Delta E_{\text{HV}} \delta_{xy}^2 \pm \sqrt{\Delta E_{\text{HV}}^2 \delta_{xy}^4 - 16 \delta_x \delta_y \delta_{xy}^2 \xi^2}}{\delta_y \delta_{xy}^2}}. \quad (4.14)$$

With increasing non-Hermitian contribution to $\xi = 2 \text{ meV}$ presented in Fig. 4.5g-j, the momentum position of the EPs changes. Real Fermi arcs expands, while imaginary Fermi arcs shrink as the EPs issued from different approach each other in the momentum space.

Especially at sufficiently high non-Hermitian contribution the EPs can meet each other in the momentum space and annihilate. Fig. 4.5k-n presents situation calculated for $\xi = 4 \text{ meV}$. Within the real spectrum Fig. 4.5k,m both modes exhibit a degeneracy ring. At the same time in the imaginary part of the spectrum shown in Fig. 4.5l for all of the wave vectors the imaginary parts of the eigenenergies are clearly separated from each other. As a result the spectrum no longer contains any exceptional points where both real and imaginary parts of the eigenenergy are equal to each other. This theoretical prediction shows that upon increasing non-Hermiticity exceptional points originating from different valleys in momentum space corresponding to different diabolical points can annihilate leaving no singularities in the dispersion relation.

Annihilation of the EPs occurs when the inner square root in Eq. (4.13) is equal to zero, which leads to condition:

$$\xi^2 = \frac{\Delta E_{\text{HV}}^2 \delta_{xy}^2}{16 \delta_x \delta_y}. \quad (4.15)$$

As within this regime in order to observe 4 diabolical points in the Hermitian limit $\Delta E_{\text{HV}} > 0$ and all $\delta_i < 0$:

$$\xi = \frac{\Delta E_{\text{HV}} \delta_{xy}}{4 \sqrt{\delta_x \delta_y}}. \quad (4.16)$$

This equation can be rewritten to the form:

$$\frac{\xi}{\Delta E_{\text{HV}}} = \frac{\delta_{xy}}{4\sqrt{\delta_x \delta_y}}, \quad (4.17)$$

which shows that the condition required for annihilation of the EPs can be equivalently obtained by increasing non-Hermiticity ξ or decrease of the real part provided by HV splitting ΔE_{HV} . The latter approach is especially useful as HV detuning is directly controllable by external voltage applied to the LC cavity. Such behavior indicate possibility for observation of the annihilation of exceptional points issued from different Dirac points within a single system with simple change of macroscopic parameter as amplitude of externally applied voltage.

Presented here predictions of behavior of EPs, especially their annihilation, in $(N + 2, N)$ regime is strikingly different than described earlier for (N, N) regime in which upon increasing the non-Hermitian contribution they depart from each other moving into infinity in the reciprocal space. The most obvious difference between those regime which is visible when comparing spectra of Hamiltonians describing those systems [Fig. 4.3](#) and [Fig. 4.5](#) is the number of diabolical points in the Hermitian regime. Though more fundamental feature allowing or not for the annihilation of EPs is total topological charge of the bands. In the (N, N) regime both diabolical points carry the same topological charge of $+\frac{1}{2}$ ([Fig. 4.2c](#)), thus the spectral singularities cannot be removed from the system by non-Hermitian contribution without changing the total topological charge. In contrast in the $(N + 2, N)$ regime from total of 4 diabolical points there are two pairs exhibiting the opposite topological charge of $\pm\frac{1}{2}$. As a result the total topological charge vanishes and spectral singularities can be removed without any change of the global topological invariant.

It is also important to note, that after the annihilation the LC cavity system does not realize so-called exceptional ring, which requires that along a whole of a closed loop both real and imaginary parts of the spectrum being degenerate [100]. Comparing with [Fig. 4.5k,l](#) where the real part of the spectrum is degenerate along a ring, but both solutions are distinguishable by their imaginary part of the spectrum, where values for each of the modes are clearly different.

Such possibility was proposed proposed by I. Septembre, D. Solnyshkov and G. Malpuech. To analyze the effect they started with with the Hermitian

Hamiltonian:

$$\begin{aligned}\hat{H}_{\mathbf{k}}^{\text{real}} &= \begin{pmatrix} E_0 + \beta_x k_x^2 + \beta_y k_y^2 & \frac{1}{2}\Delta E_{\text{HV}} - \beta' \mathbf{k}^2 - \beta(k_x - ik_y)^2 \\ \frac{1}{2}\Delta E_{\text{HV}} - \beta' \mathbf{k}^2 - \beta(k_x + ik_y)^2 & E_0 + \beta_x k_x^2 + \beta_y k_y^2 \end{pmatrix} = \\ &= (E_0 + \beta_x k_x^2 + \beta_y k_y^2) \hat{\sigma}_0 + \\ &+ \left(\frac{1}{2}\Delta E_{\text{HV}} - \beta k_x^2 + \beta k_y^2 - \beta' \mathbf{k}^2 \right) \hat{\sigma}_x - 2\beta k_x k_y \hat{\sigma}_y\end{aligned}\quad (4.18)$$

Within this Hamiltonian E_0 provides sets energy of the modes at normal incidence and terms $\beta_{x/y} k_{x/y}^2$ provide parabolic dispersion relation on the cavity photon wave vector with x, y anisotropic effective masses described by parameters $\beta_{x/y} = \frac{\hbar^2}{2m_{x/y}}$. Apart from this spin-independent terms tunability of H/V polarized modes controlled in experiment by external voltage applied to the sample is described with HV splitting parameter ΔE_{HV} . Additionally the cavity exhibit TE-TM splitting described by $\beta(k_x \mp ik_y)^2$ term. The last contribution $\beta' \mathbf{k}^2 \hat{\sigma}_x$ provides difference in effective masses of horizontally (m_{H}) and vertically (m_{V}) polarized modes equal to $\beta' = \hbar^2(m_{\text{H}} - m_{\text{V}})/4m_{\text{H}}m_{\text{V}}$.

The model leads to 4 diabolical points from which two are located at $k_y = 0$ line at

$$k_{0x} = \pm \sqrt{\frac{\Delta E_{\text{HV}}}{2(\beta' + \beta)}}, \quad (4.19)$$

and the remaining two diabolical points are located at $k_x = 0$ line at

$$k_{0y} = \pm \sqrt{\frac{\Delta E_{\text{HV}}}{2(\beta' - \beta)}}. \quad (4.20)$$

Losses can be introduced into Hamiltonian (4.18) by additional terms leading to non-Hermitian Hamiltonian:

$$\begin{aligned}\hat{H}_{\mathbf{k}}^{\text{com}} &= (E_0 + \beta_x k_x^2 + \beta_y k_y^2) \hat{\sigma}_0 + \\ &+ \left(\frac{1}{2}\Delta E_{\text{HV}} - \beta k_x^2 + \beta k_y^2 - \beta' \mathbf{k}^2 \right) \hat{\sigma}_x - 2\beta k_x k_y \hat{\sigma}_y + \\ &+ i\Gamma_0 \hat{\sigma}_0 + i\delta\Gamma \hat{\sigma}_x,\end{aligned}\quad (4.21)$$

where $\Gamma_0 = (\Gamma_{\text{H}} + \Gamma_{\text{V}})/2$ term describes the spin-independent mean decay rate and $\delta\Gamma = (\Gamma_{\text{H}} - \Gamma_{\text{V}})/2$ defines a constant imaginary effective field discerning losses for horizontally and vertically polarized modes.

Within this model momentum position can be written as:

$$k_y^{\text{EP}} = \pm \frac{k_{0y}}{\sqrt{2}} \sqrt{1 \pm \sqrt{1 - \frac{\delta\Gamma^2}{\beta^2 k_{0x}^2 k_{0y}^2}}}, \quad (4.22)$$

$$k_x^{\text{EP}} = \pm \frac{k_{0x}}{\sqrt{2}} \sqrt{1 \pm \sqrt{1 - \frac{\delta\Gamma^2}{\beta^2 k_{0x}^2 k_{0y}^2}}}, \quad (4.23)$$

where k_{0x}, k_{0y} are positions of the diabolical points in the Hermitian limit given by Eqs (4.3–4.20).

By introducing a new parameter describing the degree of non-Hermiticity:

$$\chi = 2 \frac{\delta\Gamma}{\Delta E_{\text{HV}}} \frac{\sqrt{\beta'^2 - \beta^2}}{\beta}, \quad (4.24)$$

condition for annihilation of the exceptional points is $\chi > 1$. Condition this can be equivalently met by increasing strength of the non-Hermitian contribution $\delta\Gamma$, or easier to perform experimentally by decreasing HV detuning ΔE_{HV} .

To test predictions of the theoretical model, we collected polarization resolved tomographies performed at two different voltages applied to the LC cavity. The experiments were performed on a cavity consisting of two $\text{SiO}_2/\text{TiO}_2$ DBRs each containing 6 Bragg pairs with maximum reflectance at 550 nm, filled with liquid crystal of high birefringence $\Delta n = 0.41$. Additionally the LC mixture contained 0.2% of pyromethene 580 organic dye. Based on the energy separation between the consecutive cavity modes, the total thickness of the LC layer was approx. 1.8 μm .

Obtaining the range of annihilation of EPs required operation at low values of HV splittings. The first dataset was obtained at 1.77 V applied to the LC cavity. Cross sections along two main wave vector directions are plotted in Fig. 4.6a–e. Based on the cross sections at normal incidence, plotted in Fig. 4.6e, HV splitting in this case was equal to 3.0 meV. The second measurement was performed at lower 1.72 V with resulting splitting between the modes at normal incidence equal to 1.2 meV (Fig. 4.6f–j).

The experimental data was analyzed by I. Septembre. By fitting solutions of the Eq. (4.18) to those dispersion relations one can access present in the model: the effective masses, TE-TM splitting strength β and HV effective mass difference β' , which are equal to $m_x = (1.34 \pm 0.07) \times 10^{-5} m_0$, $m_y = (1.08 \pm 0.06) \times 10^{-5} m_0$, $\beta = 0.080 \pm 0.03 \text{ meV}\mu\text{m}^2$, $\beta' = 0.47 \pm 0.03 \text{ meV}\mu\text{m}^2$.

Difference of the energies between the modes for higher HV splitting for

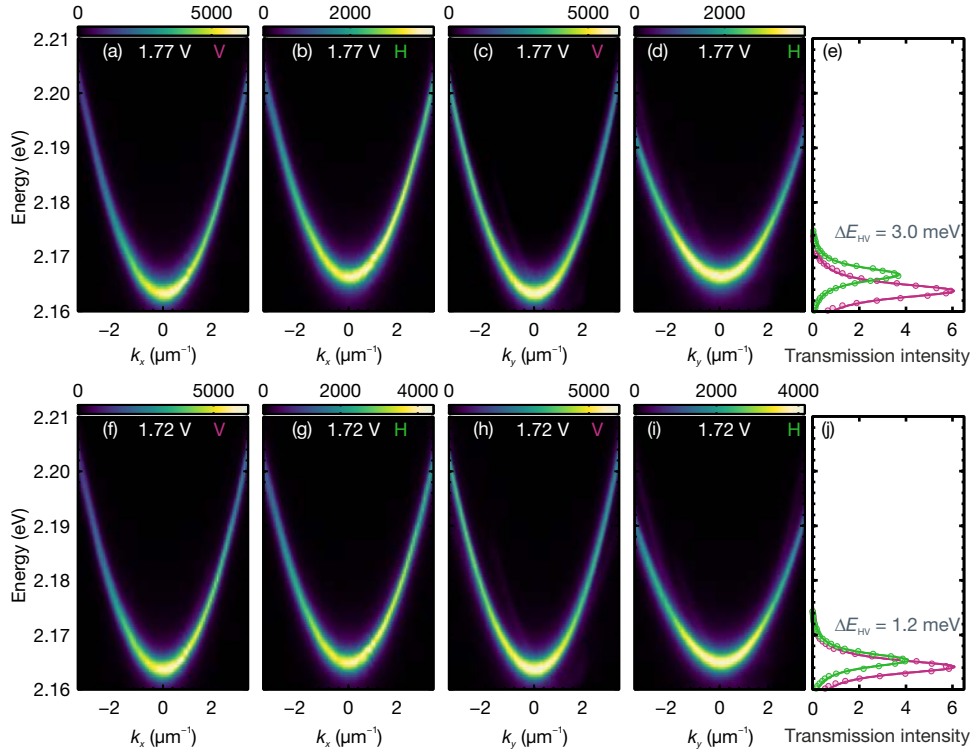


Fig. 4.6 | k -space tomography in $(N + 2, N)$ regime in transmission configuration. (a)–(d) Cross sections through H/V polarization resolved tomographies measured at 1.77 V applied to the cavity. (e) Cross section through normal incidence fitted with Lorentzian profiles providing $\Delta E_{HV} = 3.0$ meV detuning. (f)–(i) Cross sections through H/V polarization resolved tomographies measured at 1.72 V applied to the cavity. (e) Cross section through normal incidence fitted with Lorentzian profiles providing $\Delta E_{HV} = 1.2$ meV detuning.

all wave vector directions is plotted in Fig. 4.7a. In this image wave vector coordinates are normalized to the momentum position of the initial diabolical points $k_{0x/y}$. The energy difference spectra contains 4 degeneracy arcs located at the unit ring in reduced wave vector coordinates around the cross sections with the main axes. Following the unit ring at diagonal/antidiagonal wave vector directions the energy difference visibly increases, and the modes are no longer degenerate, as expected from the regime with non-Hermiticity degree χ below 1. The positions of the 8 EPs located at the edges of the degeneracy lines are marked with white x. The results are in a perfect agreement with the calculated real part of the energy difference,

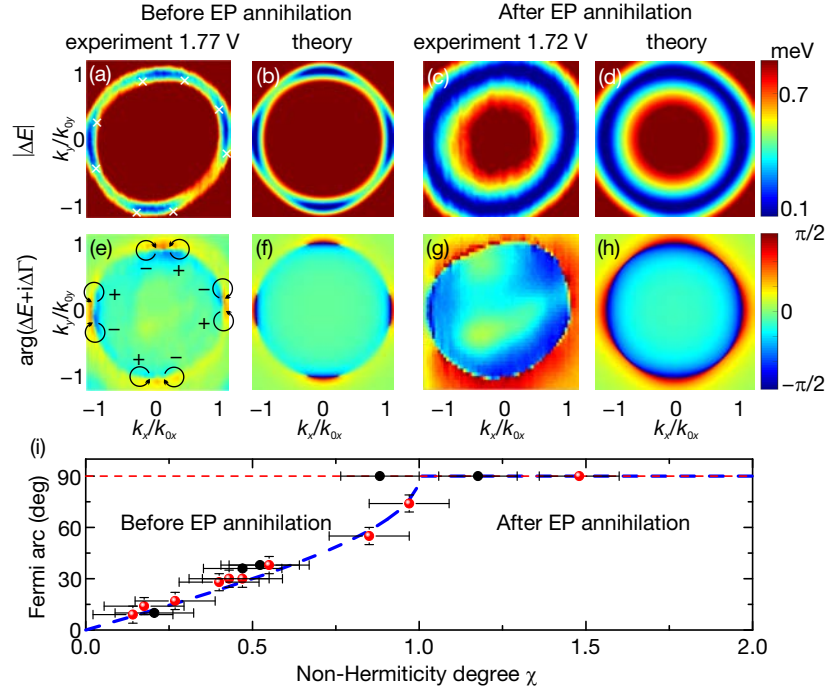


Fig. 4.7 | Observation of EPs and their annihilation. Real part of the energy difference in (a) experiment at 1.77 V ($\Delta E_{\text{HV}} = 3$ meV) and (c) at 1.72 V ($\Delta E_{\text{HV}} = 1.2$ meV) with corresponding theoretical results in (b) and (d) correspondingly. The color bar is saturated above 0.9 meV. The argument of the complex energy difference in (e) experiment at 1.77 V and (g) at 1.72 V with corresponding theoretical results in (f) and (h) correspondingly. (i) Length of the real Fermi arcs as a function of non-Hermiticity degree χ . Black (red) points were obtained for the sample with approx. 1.8 μm (3.2 μm) thickness of the LC layer. Figure and analysis prepared by I. Septembre.

presented in Fig. 4.7b. Theoretical map was obtained for parameters fitted from the $k_{x/y}$ cross sections and $2\delta\Gamma = 0.24 \pm 0.05$ meV.

The energy difference between the modes extracted from the experiment at lower voltage shows a degeneracy ring for the whole ring at unitary reduced wave vector, as shown in Fig. 4.7c. Such behavior is expected from the regime with annihilated EPs for non-Hermiticity degree $\chi > 1$. Once again the experimental results are consistent with calculations in which the only change was the value of the HV splitting.

The access to the imaginary part of the spectrum was possible by analysis of the linewidths of the observed cavity modes. In order to distinguish

homogeneous line broadening described by non-Hermitian terms in Hamiltonian Eq. (4.21) from the nonhomogeneous broadening originating from effects like the disorder, the experimental results were fitted with the Voigt profile. As a convolution of Lorentzian and Gaussian peak functions it allows to distinguish both contributions, with only Lorentzian linewidth taken into account in further analysis.

Fig. 4.7e plots the argument of the complex energy difference extracted from the experiment with higher HV detuning. It shows winding (marked with the arrows) of the phase around position of each of the 8 exceptional points traced at the real part of the spectrum. Consistent with the theoretical prediction each of the EPs exhibit opposite winding (marked by the direction of the arrows and $+/-$ sign) to its nearest neighbor originating from the same, or different momentum valley. Exactly the same behavior with more pronounced phase difference is visible in the corresponding theoretical plot in Fig. 4.7f.

For the regime with lower HV splitting complex energy phase in both experiment and the theory plotted in Fig. 4.7g,h contain no positions with winding of the phase characteristic for the presence of the exceptional points. In both consistent between each other results the phase undergoes transition between $\pm\pi/2$ when crossing the degeneracy line at the real part of the spectrum following in radial direction in wave vector.

In order to provide additional experimental confirmation for observation of annihilation of the exceptional points additional measurements were performed on another sample, with significantly thicker LC layer of approx. $3.2\ \mu\text{m}$. The results are summarized in Fig. 4.7i which shows extracted from the experiment length of a real Fermi arcs as a function of non-Hermiticity degree χ . For χ below 1 exceptional points can be observed in the momentum space. They are connected with the real Fermi arcs with lengths increasing with the non-Hermiticity degree. At $\chi = 1$ length of the Fermi arcs increases to 90° they connect to single degeneracy ring as EPs issued from different valleys in momentum space annihilate. Exactly the same behavior was observed on two independent samples: thinner marked with black points and thicker marked with the red dots.

4.4 Exceptional points in LC microcavities: summary

In summary by changing voltage applied to the LC cavity for constant polarization dependent losses we can precisely control the non-Hermiticity degree by adjustment of the real part contribution. Such possibility is extreme-

ly fruitful especially in the regime close to degeneracy between $(N + 2, N)$ cavity modes. In this regime in the Hermitian limit the dispersion relation contains 4 distinct valleys in momentum space each with diabolical point.

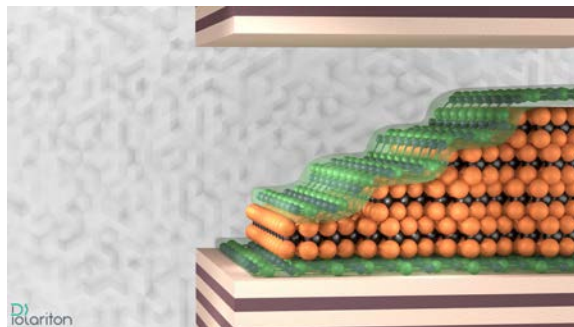
With additional polarization H/V dependent losses each of the four valley gives rise to two exceptional points. With increasing degree of non-Hermiticity the exceptional points with opposite topological charges issued from different valleys approach each other in the momentum space. Most importantly at some critical value of the non-Hermiticity they meet in the momentum space and annihilate. After the annihilation any position in the reciprocal space exhibit degeneracy in both real and imaginary part of the spectrum.

It is important to note, that as possibility of annihilation of the exceptional points is known [121]. It is however typically thought of as the opposite process to creation of two exceptional point from a diabolical point with addition of non-Hermiticity. To our best knowledge this is the first demonstration of annihilation of exceptional points originating from different valleys in momentum space.

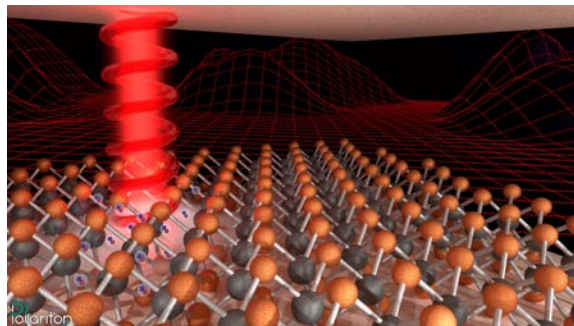
Chapter 5

Valley effects in transition metal dichalcogenide exciton-polaritons

The results described within this chapter are published in the following publications:



- [123] M. Król et al. Exciton-polaritons in multilayer WSe₂ in a planar microcavity. *2D Mater.* **7**, 015006 (2020).



- [124] M. Król et al. Valley polarization of exciton-polaritons in monolayer WSe₂ in a tunable microcavity. *Nanoscale* **11**, 9574 (2019).

In the previous chapter spin (polarization) effects arose as a direct result of optical anisotropy of the photonic system in microcavities filled with birefringent liquid crystal.

Within this chapter a completely different approach will be employed, where in fully isotropic planar microcavities observed spin effects will be provided through admixture of spin-polarized excitons in strong light-matter coupling regime.

The investigated system consists of a tunable open cavity made of two distributed Bragg reflectors containing thin layers of tungsten diselenide (WSe_2), which is a member of broader group of semiconducting transition metal dichalcogenides.

The first section (Sec. 5.1) will be devoted to optical properties of thin flakes of transition metal dichalcogenides, with special emphasis on excitons in the monolayer limit.

The second section (Sec. 5.2) concentrates on the design of the optical resonator allowing for observation of strong coupling regime with exciton in WSe_2 flake. Initial concepts will be optimized using the theoretical calculations based on the transfer matrix method. At the end, the experimental realization of the best design will be compared with the theoretical predictions.

The third section (Sec. 5.3) will present the optical properties of thin WSe_2 flakes based on photoluminescence and reflectance measurements. Based on the obtained reflectance contrast spectra the properties of the excitonic transitions in WSe_2 will be extracted.

The fourth section (Sec. 5.4) will demonstrate strong coupling regime in the fully assembled resonator with WSe_2 flakes. It will be shown, that the strong coupling can be obtained not only in the direct bandgap monolayers, but also in thicker flakes. Experimental results will be compared with theoretical simulations based on the properties of the excitonic transitions extracted from prior measurements of the bare flakes.

Finally, the fifth section (Sec. 5.5) will concentrate on the valley physics of the monolayer WSe_2 exciton-polaritons showing transfer of the incident angular momentum of the pump laser into the cavity polariton states.

5.1 Transition metal dichalcogenides (TMDCs)

Extraordinary properties of single layer graphite in the form of graphene [125, 126] brought attention as well as development of experimental techniques for obtaining, exploiting and characterization of thin flakes of other

thin forms of various layered materials.

Especially desirable was a semiconducting 2D material to complement semimetallic graphene and insulating hexagonal boron nitride. One specific group of such materials fulfilling such requirements were transition metal dichalcogenides (TMDCs).

General chemical formula for TMDCs is MX_2 , where M is the transition metal and X is the chalcogen. Semiconducting properties exhibit materials in which M stands for: molybdenum (Mo) or tungsten (W) and X for: sulfur (S), selenium (Se) or tellurium (Te).

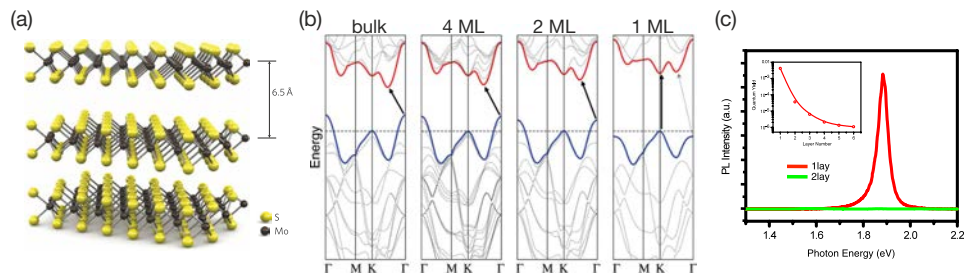


Fig. 5.1 | Properties of MoS₂. (a) A sketch of the crystallographic structure of TMDCs after Ref. [127], (b) Band structure of MoS₂ calculated for the bulk form as well as for 4, 2 and 1 monolayer-thick flakes. Image taken from Ref. [128], (c) Increase of the photoluminescence intensity between bilayer and monolayer MoS₂ reported in Ref. [129].

Similarly to graphite TMDCs are built from a stack of planar layers bound between each other by the weak van der Waals forces and within each layer the atoms are bound with strong covalent bonds, schematically depicted in Fig. 5.1a. What differentiates TMDCs from the graphene is the composition of the monolayer which consists of all metal ions lying within single plane sandwiched from the top and the bottom with the chalcogen ions.

In the bulk form semiconducting TMDCs exhibit indirect band-gap [130], with lowest energy transition between the Γ point in the valance band and Λ point in the conduction band (laying in between Γ and K points), as presented in the band structure in Fig. 5.1a. This property is maintained upon thinning the material up to the two monolayer limit (2 ML), but is simultaneously accompanied with the decrease of the energy of the valence band maximum and increase of the energy at the Λ point. Between the bilayer and the monolayer form of TMDCs there is a crossover and for the single layer flake the material becomes a direct bandgap semiconductor with lowest

energy transition located at the $\pm K$ points in the Brillouin zone. As expected from a direct bandgap semiconductor the light emission from TMDCs monolayers is much more efficient, what has been reported in the first reports for single layer TMDCs (MoS_2) [129, 128].

From the beginning of the studies, the semiconducting TMDCs with insulating hBN and conducting graphene have been playing a crucial role in the concept of "2D Lego" [131]. Nowadays the idea is expanded by a plethora of different materials like 2D magnetic materials [132] and additional degrees of freedom like twist angles leading to moiré lattices [133, 134].

From the top view TMDC structure resembles a hexagonal lattice of graphene but with nonequivalent A and B sites. As a result TMDC structure lacks the inversion symmetry, which leads to opening energy gap in Dirac cones of the graphene-like dispersion relation and emergence of nonequivalent K and $-K$ valleys with opposite Berry curvature [135].

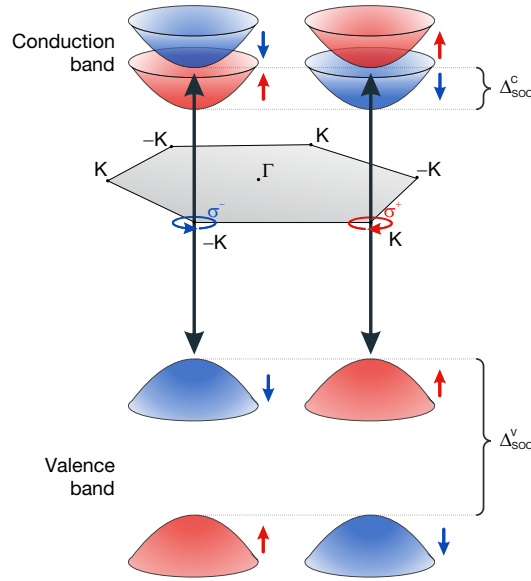


Fig. 5.2 | Band structure in TMDCs. Schematic representation of the band structure in TMDCs monolayers around the $\pm K$ points in the Brillouin zone.

The sketch of the band structure of TMDCs concentrating on the most important for the optical transitions $\pm K$ points is presented in Fig. 5.2. Due to the heavy ions of the transition metal both conduction and valence bands are spin-split due to strong spin orbit coupling at the range of several hundred meV in the valence band ($\Delta_{\text{SOC}}^{\text{v}}$) and of a few to tens of meV in the conduction bands ($\Delta_{\text{SOC}}^{\text{c}}$) [136]. Without the external magnetic field both

$\pm K$ valleys are connected by time reversal symmetry, which results in the opposite spin configuration in the opposite valleys.

The optical selection rules allows for circularly polarized light to couple only to the transition at K or $-K$ valley [137]. As a result within the K ($-K$) valley σ^+ (σ^-) polarized photon can promote an electron to the conduction band leaving in the valence band a hole. This two oppositely charged quasi-particles are subjected to strong Coulomb interactions between each other. As a result, an electron with a hole can form an analogous to the hydrogen atom bound state, referred to as neutral exciton.

In TMDCs the electron with the hole are highly bound with the exciton binding energy at the order of 0.2 eV [138, 139, 140], thus excitonic transitions dominate over optical properties of the TMDCs from cryogenic to room temperature.

As another consequence of high exciton binding energy in TMDCs monolayers, the exciton Bohr radius—defining mean distance between the electron and the hole—is comparably small, on the order of a few nanometers [136]. This gives an intermediate value between two main models of an exciton. The first type, so called Wannier-Mott exciton, is characterized by a Bohr radius expanding over multiple lattice constants. It is applicable to typical semiconductors like GaAs. On the other side of the spectrum is so called Frenkel exciton where both the electron and the hole are strongly spatially localized within a single site, and such behavior is more specific to e.g. organic molecules.

For the atomically-thin monolayers Coulomb interaction between charge carriers is highly sensitive to the screening provided by the local dielectric environment [141]. This fact strongly affects also the excited exciton states, which energies deviates from the simple hydrogen-like model for 2 dimensions [138, 139].

According to the band structure depicted in Fig. 5.2 an electron or a hole in order to change the valley has to undergo a spin flip or energetically unfavorable process to higher energy state, which significantly inhibits intervalley scattering. This effect, combined with the spin valley locking allows for optical generation of the imbalance in carrier concentration between the valleys by circularly polarized light. Additionally, this imbalance can be observed in the polarization of the emitted light: phenomenon referred to as valley polarization. It has been shown, that even for nonresonant excitation, the chirality of the incident light is partially preserved in the emitted light [142, 143]. Moreover, the valley coherence in which excitation with a linearly polarized light can create a coherent superposition of the exciton in both $\pm K$ valleys which is in part maintained in the emitted light has been de-

monstrated, for the first time in WSe₂ in Ref. [144]. Valley coherence can be robust enough to allow for rotation of the linear polarization plane direction when the flake is subjected to external magnetic field in the Faraday configuration [145].

Opportunity to optically address and readout of the valley index in TMDCs brought great hopes for valleytronics [146], which aims to employ valley degree of freedom for data processing, manipulation and storage [147].

Coming back to the band structure in Fig. 5.2, depending on the material the spin splitting at the $\pm K$ point in the valence band $\Delta_{\text{SOC}}^{\text{V}}$ varies between molybdenum based TMDCs from approx 0.2 eV to 0.4 eV in tungsten based TMDCs. As a result of this splitting two optical transitions are possible: lower energy one marked by black arrows in Fig. 5.2 referred to as A exciton, which will be investigated within this study. The transitions between the bands of the opposite spin-split valence/conduction bands leads to higher energy so-called B exciton.

Smaller value spin splitting $\Delta_{\text{SOC}}^{\text{C}}$ is also present in the conduction band. Its sign depends on the material distinguishing dark or bright materials. For "dark" TMDCs, as in Fig. 5.2, the optical transition from the lowest energy ground state is spin forbidden. The discussions about the sign of the conduction band splitting are based on experiments in the function of temperature, magnetic field in Voigt configuration.

An exciton with additional charge carriers can form more complex multi-particle bound states, which were observed and identified in the TMDC monolayers. An exciton bound with another single charge carrier forms so called trion [148]. A bound four-particle state arranged from two electrons and two holes forms a biexciton [149], which with an additional charge carrier forms a five-particle state of charged biexciton [150].

Some authors suggest that such multi-particle excitonic complexes in TMDCs instead of a bound sets of the electron and holes should be treated rather in the polaron picture in which excitons interact with a Fermi sea of charge carriers [151].

For more complex structures with stacking of two different TMDCs, a type-II band alignment between them results in the lowest energy spatially indirect exciton state, where the electron and the hole are located in the other flakes [152]. Similarly in thicker TMDCs flakes such interlayer exciton states can be identified [153].

In specially designed heterostructures even such excitonic complexes as doubly charged excitons (quatermions) [154] can be observed.

Main means to obtain monolayer TMDC flakes are: mechanical exfo-

liation [155], chemical vapor deposition [156] or molecular beam epitaxy [157, 158, 159].

For scientific purposes the first widely adopted method was mechanical exfoliation from the bulk crystal. Based on earlier experiences with graphene it allowed for preparation of the thin flakes with the typical lateral size of tens of micrometers. The main disadvantage of this approach is randomness of the method. Location, size and thickness of the obtained flakes is completely irregular, which requires time for characterization and selection of the most promising fragments with typically very low yield. Nevertheless, combined with methods of deterministic transfers, like with viscoelastic stamps [160], it allows for preparation of high quality structures appropriate for the scientific investigations. There are demonstrations of a functional fully automatized setups which use mechanical transfer to construct quite complex heterostructures [161].

On the other hand the use of growth methods such as CVD and MBE brings hope for preparation of a wafer-size flakes [162] which maintain very high optical properties [158].

At the initial stage of research, the investigated TMDCs monolayers have been deposited on a silicon substrates with top oxidized layer of silicon dioxide SiO_2 . Depending on the thickness of the SiO_2 layer the interference effects can significantly enhance the visibility of the thin, especially monolayer flakes [163]. Huge technological advantage in optical quality of the prepared monolayers brought encapsulation of the flakes in hexagonal boron nitride (hBN) [164]. As another member of the van der Waals family, the hBN provides an atomically flat surface and isolates from localized charges which has resulted in the significant reduction of the emission and absorption linewidths, up to narrow emission lines close to the homogeneous broadening limit. With the encapsulation by choosing specific thicknesses of hBN layers to some degree the exciton radiative rate can be controlled by the Purcell effect [165].

Thickness of the layers even close to the monolayer limit can be estimated by the contrast on the microscopic images. More confident determination of the thickness of TMDC flakes can be performed based on AFM profile, comparison of Raman spectra [166] or reflectance contrast measurements. To distinguish monolayers from thicker flakes, an easy method is to compare intensity of the photoluminescence, also at room temperature.

In TMDCs even low strain can induce significant shift in the bandgap energy [167], which can explain shifts in the absolute energy of the excitonic resonances between different samples and positions within the same sample.

Presented here the overview of the optical properties of thin flakes of transition metal dichalcogenides is only an excerpt from more broad reviews on this topic [136, 168, 169]. Especially, the specifics of the valley polarization, can be found in Ref. [170, 147, 171].

5.2 Cavity design and realization

Potential of transition metal dichalcogenides for polaritonic applications owing to their high exciton binding energy has been noticed shortly after the discovery of the unique optical properties of their monolayer form, which followed with broad research performed in various types of planar microcavities.

The first observation of strong coupling regime has been demonstrated in a solid dielectric cavity containing MoS₂ monolayer, shortly followed by a realization in an open cavity [172] and at room temperature [173].

Preservation of the chirality of the incident light in polariton systems has been reported by multiple groups [174, 175, 176, 177, 178]. Valley coherence with rotation of the linear polarization plane by magnetic field was later reported in [179], and up to the room temperature in [180]. In polariton systems valley polarization is maintained up to higher temperatures than for the bare flake [174] and can be enhanced close to trion resonance [176].

In planar microcavities valley physics of TMDCs monolayers couples with TE-TM splitting [24, 30] leading to the Valley Hall effect [181, 182].

More elaborate gated heterostructures have been also investigated within optical cavities and allowed for control over the charge carrier density [151], control over the coupling strength [183] and has allowed for achieving room temperature polariton electroluminescence [184].

Multiple excitonic resonances within different materials can couple simultaneously with a single cavity mode forming hybrid polaritons [185] as was also shown in WS₂-organic dye system [186] as well as in MoSe₂-GaAs [187], which in the former case thanks to the admixture of GaAs excitons in the polariton state allowed for the first observation of polariton lasing.

Bosonic condensation of the TMDC-based polaritons was for the first time reported in [188]. In that work the influence of the GaAs quantum well in a hybrid structure was significantly reduced in comparison with earlier demonstration of polariton lasing in analogous system [187].

Recent reports points on characteristic features for the condensation process, especially coherent emission, even at room temperature [189, 190]. Other nonlinear effects in WS₂ monolayer has been a recent demonstration

of a parametric amplifier [191].

Considered in this work planar microcavity is not the only possible type of an optical resonator. Lasing was observed with TMDC flakes in whispering gallery mode cavity [192, 193] as well as with design based on photonic crystals [194] or grating cavity (on indirect excitons within heterobilayers) [195].

In the linear regime helical propagation was observed in a system composed of TMDC monolayer strongly coupled with topological photonic crystal [196, 197].

5.2.1 Planar microcavities with TMDC thin flakes

In the literature two main approaches of realization of planar microcavities incorporating thin TMDCs flakes have been reported. In both of this methods the first step consisted of depositing a thin TMDC flake on top of a mirror. Monolayer TMDCs flakes can be CVD grown [198] or mechanically exfoliated [199]. In the first "regrowth" approach the full cavity structure is finished by another growth process of the second mirror forming the cavity.

The second method is based on so-called open cavity approach. In this configuration the cavity is formed by approaching the second mirror to the TMDC flake deposited on top of the first mirror, and with a free space in between forming an optical cavity [200, 201, 172].

In both configurations mirrors constituting the cavity can be made either from the multilayer DBR structure made of dielectric [198] or semiconductor [202] layers, or the role of the mirror can be played by a thin metallic layer [173]. In specific realizations of the open cavity the second mirror can be grown directly on top of an optical fiber [151].

There exists also a third approach in which a DBR structure is exfoliated from the substrate and deposited directly on top of the TMD flake [182].

Typically for the regrowing approach an additional layer deposited on top of the TMDC flake is required in order to protect the flake against high temperatures during sputtering deposition of dielectric DBRs [199].

In our approach we used an open cavity, schematically depicted in Fig. 5.3. The cavity was formed by two independent dielectric DBRs separated by a free space. Exfoliated TMDC flake was deposited directly on top of one of the mirrors. To allow tunability of the cavity, one of the mirror was placed on a piezoelectric stage.

The main advantage of this approach is ease of the tunability of the optical resonator. In typical solid resonator the cavity mode energy position is set by distance between the DBRs, so this distance has to be precisely con-

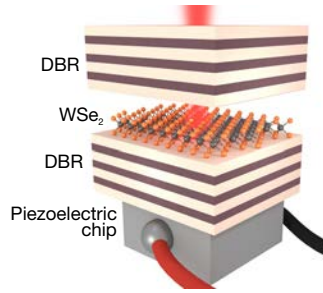


Fig. 5.3 | Scheme of a tunable microcavity incorporating a thin TMDC flake.

trolled during the growth, which could require multiple attempts before mastering the procedure. In the proposed approach the energy of the cavity mode can be freely changed by tuning the distance between the mirrors forming the cavity, which can be performed by external voltage applied to a piezoelectric element even during the experiment.

Another significant advantage of the open cavity approach is the limited number of required growth processes of the dielectric DBRs. Single growth simultaneously performed on multiple glass substrates can be used to obtain multiple parts of the DBRs, which can be used for construction of multiple cavities. This was a significant advantage due to a limited access to a facility allowing for deposition of dielectric layers at that time.

On the other hand, as the coupling strength decreases with the cavity length, the experimental challenge was: how close to each other the two macroscopic DBRs can be approached? It will be shown later that, achieved distances were small enough for the observation of the strong coupling regime for a single cavity mode in the cavity.

5.2.2 Theoretical design of dielectric cavity for WSe₂ thin flakes

Design of dielectric structure allowing for the observation of the strong coupling regime between excitons in WSe₂ and cavity photons was based on transfer matrix method.

Emission from neutral exciton in WSe₂ at liquid helium temperature has been reported at 711 nm [203], 708.5 nm [204] for flakes deposited on silicon substrate with oxidized surface and at: 719 nm [164], 719.4 nm [205]. Additionally thickness of the monolayer flake at the order of 65 pm [206] is significantly smaller than wavelength of the light resonant with the excitonic transition.

Design of the cavity will be discussed assuming for simplicity $\lambda_0 =$

720 nm.

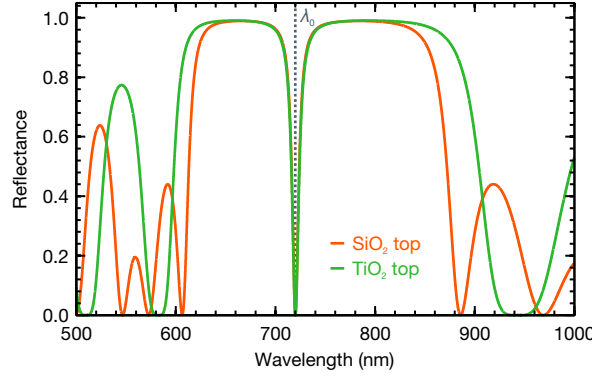


Fig. 5.4 | Reflectance of the open dielectric cavities. Simulated reflectance spectrum from open cavities with top DBR layers made of SiO₂ and TiO₂. Spacing between the mirrors forming cavity is equal to $\lambda_0/2$.

Fig. 5.4 presents simulated reflectance spectra of the open cavity made of two DBRs made of SiO₂ and TiO₂ layers with optical widths of $\lambda_0/2$. In order to use dielectric stacks obtained within only one growth process both mirrors are assumed to be identical. This allows for two possibilities: with the top layer of the DBR made of higher refractive index material (TiO₂ top), or lower refractive index (SiO₂ top). In both of this configurations distance between the mirrors forming the cavity is set to $\lambda_0/2$ to obtain the cavity mode resonant at the λ_0 wavelength. In both of this configurations reflectance spectra exhibit desired reflectance spectra minimum at λ_0 corresponding to the photonic cavity mode within the photonic stopband region of high reflectance coefficient.

Despite the desired reflectance properties of both of these cavity configurations only one of them can be used to couple the photonic field of the cavity with the exciton on TMDC flake deposited on top of the mirror, what can be explained based on Fig. 5.5, which presents mean electric field distribution across the cavity in both of this configurations.

In configuration with higher refractive index material (TiO₂) on top presented in Fig. 5.5a electric field is strongly localized within the cavity, but its maximum lies exactly at the center of the gap region between the mirrors. At the interfaces between air/vacuum and the top TiO₂ layer, where the TMDC flake can be deposited, the electric field has a node. As a result coupling between excitons in TMDC flake and the photonic mode would be minimal.

The contrary is true for the realization with SiO₂ layer as the top of the DBR structure. When two such DBRs are used to form the cavity, simulated

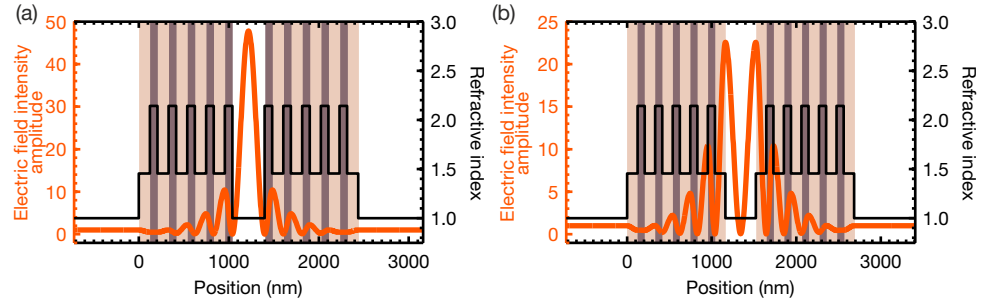


Fig. 5.5 | Electric field in open dielectric cavities. Distribution of electric field intensity through cavities made of two DBRs centred at $\lambda_0 = 720$ nm simulated by transfer matrix method. (a) Top DBR layer made of material with higher refractive index: TiO₂. (b) Top DBR layer made of material with lower refractive index: SiO₂. Spacing between the mirrors forming cavity is equal to $\lambda_0/2$.

electric field distribution exhibit antinodes at the DBR interfaces where thin TMDC flake can be deposited, allowing for efficient coupling between the cavity modes and the excitons in the flake.

Simulations shown in Fig. 5.4 and 5.5 assumed small width of the cavity with the spacing between the mirror equal to $\lambda_0/2 = 360$ nm. In realistic approach achieving such distance can be challenging. For example after the exfoliation process transfer of the thin monolayer flakes leaves significantly thicker material at the order of micrometers, as will be shown in 5.3.1.

Fig. 5.6 presents simulated reflectance spectra for dielectric cavities with varying thickness of the gap between the DBRs. Depending on the distance between the mirrors within the cavity photonic stopband region approx. 600–900 nm multiple spectrally narrow cavity modes appear. Wavelength of those cavity modes is tunable by adjusting the spacing between the DBRs. Especially when the cavity width is equal to multiple of $\lambda_0/2$ one of the cavity modes is in resonance with λ_0 wavelength, which can be also shown at cross sections presented in Fig. 5.6b. For the $\lambda_0/2$ spacing only one cavity mode at desired wavelength of λ_0 can be observed. For wider spacing such as $4\lambda_0 = 2.88 \mu\text{m}$ apart from the central cavity mode at λ_0 within the photonic stopband region two additional cavity modes at 640 nm and 820 nm can be observed. For even greater distance between the mirrors like $8\lambda_0 = 5.76 \mu\text{m}$ spacing between the modes decreases that 5 consecutive cavity modes are visible within the stopband range but one of them is still resonant with the λ_0 .

Corresponding electric field distributions calculated for such wider cavities are shown in Fig. 5.7. For both widths of 4 and $8\lambda_0$ antinodes of the

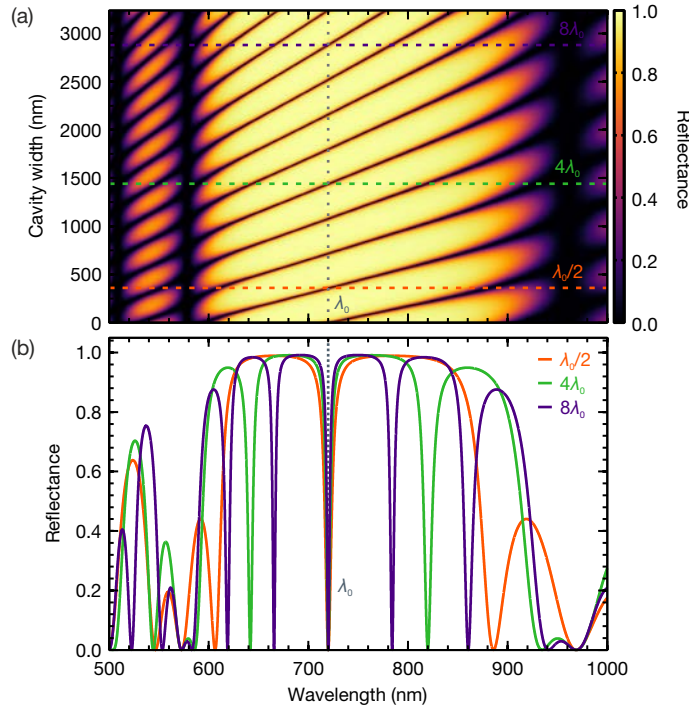


Fig. 5.6 | Reflectance of the open dielectric cavities. (a) Simulated reflectance spectrum from the open cavities with top DBR layers made of SiO₂ and increasing distances between the DBRs. (b) Cross sections through (a) for gap thickness of: $\lambda_0/2$, $4\lambda_0$ and $8\lambda_0$ for $\lambda_0 = 720$ nm.

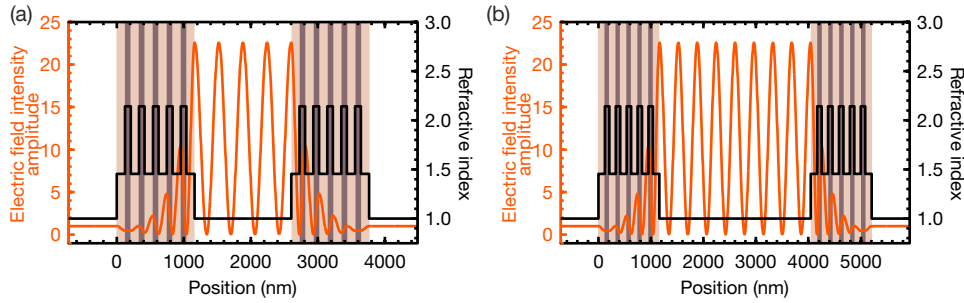


Fig. 5.7 | Electric field in open dielectric cavities. Distribution of electric field intensity through cavities made of two DBRs centered at $\lambda_0 = 720$ nm simulated by transfer matrix method. Top DBR layer made of material with lower refractive index: SiO₂. Spacing between the mirrors forming cavity is equal to (a) $4\lambda_0$ and (b) $8\lambda_0$.

electric field distribution are located at the final position of the thin WSe₂ flake at the interface between the DBRs and the cavity. Such observation is

important as it shows that for DBR structure finished by lower refractive index material for any spacing between the mirrors providing a resonance condition at λ_0 . Additionally for all of the mentioned simulations with the $\frac{1}{2}$, 4 and $8\lambda_0$ cavity widths the electric field amplitude at the position of the flake is the same, slightly exceeding 20 times the incident light amplitude.

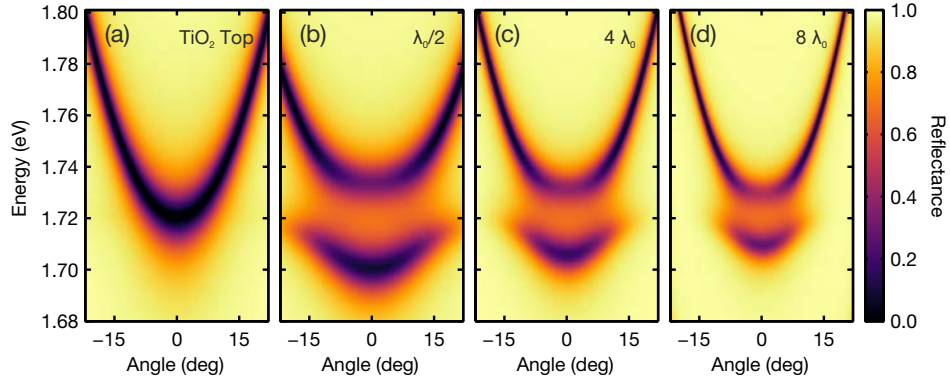


Fig. 5.8 | Simulated cavities incorporating WSe₂ flake. Angle resolved reflectance spectra for (a) cavity with TiO₂ top layer. Cavities with SiO₂ top layer and the increasing spacing between the DBRs equal to (b) $\lambda_0/2$, (c) $4\lambda_0$ and (d) $8\lambda_0$.

According to the simulations the amplitude of the electric field does not change with the distance between the mirrors. Nevertheless the coupling strength decreases with the thickness of the cavity, as was demonstrated in the literature e.g. in [201]. Such effect can be also observed in simulations presented in Fig. 5.8 which include WSe₂ monolayer (the exact approach will be described later in Sec. 5.3).

For cavity made of DBRs with top layer made of higher refractive index material (TiO₂) simulated angle-resolved reflectance spectra shown in Fig. 5.8a consists of single photonic cavity mode displaying no effects of the monolayer flake, as it is positioned at the node of the electric field of the cavity resulting in low coupling efficiency.

This is in contrary to the situation when the flake lays in the antinode of the electric field, as for the cavity made of DBRs with top SiO₂ layer. In this case simulated reflectance spectra in Fig. 5.8b shows characteristic exciton-polariton dispersion relation with two anticrossings: lower and upper polariton modes, resulting from coupling between exciton in the WSe₂ flake and the cavity mode. Simulations in Fig. 5.8b were performed for low spacing between the mirrors forming the cavity equal to $\lambda_0/2$. As this distance increases to $4\lambda_0$ (Fig. 5.8c) and $8\lambda_0$ (Fig. 5.8d) the exciton-photon co-

upling strength given by the minimum energy splitting between lower and upper polariton modes clearly decreases. This decrease is plotted directly in Fig. 5.9, presenting coupling strength dependence on the distance between the mirrors. The highest value of 36 meV is observed in the configuration of the cavity with no additional distance between the DBRs and the WSe₂ flake. Coupling strength significantly decreases with the distance, but even at 5 μm spacing (corresponding to 30 $\lambda_0/2$) energy splitting of the upper and lower polariton branches is above 14 meV.

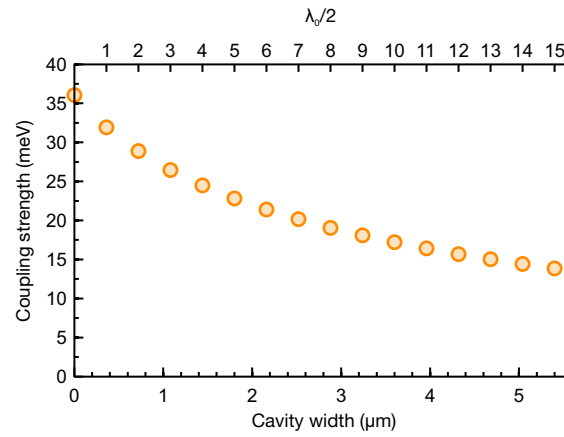


Fig. 5.9 | Coupling strength dependence on the cavity width. Simulated dependence of the exciton–photon coupling strength with the distance between the DBR mirrors forming the cavity.

To summarize, based on the simulations the cavity allowing for the observation of the strong coupling regime between excitons in WSe₂ monolayer and cavity photons can be constructed of two identical dielectric DBRs. In order to maximize photonic field at the position of the flake the top DBR layer must be made of lower refractive index material. In such configuration resonant energy of the cavity is dependent on the spacing between the mirrors. Cavity mode is resonant with the desired λ_0 wavelength for inter-mirror distance equal to any integer multiplication of $\lambda_0/2$. Nevertheless the expected exciton–photon coupling strength decreases with the cavity width, what requires low distance between the mirrors.

Cavity design for hBN-encapsulated flakes

Described above design of the cavity assumes that the TMDC flake is deposited on top of the dielectric DBR, especially on top of SiO₂ layer. As described in 5.1 flakes with the highest optical quality and spectrally narrow lines are obtained by encapsulation in hexagonal boron nitride (hBN) [164].

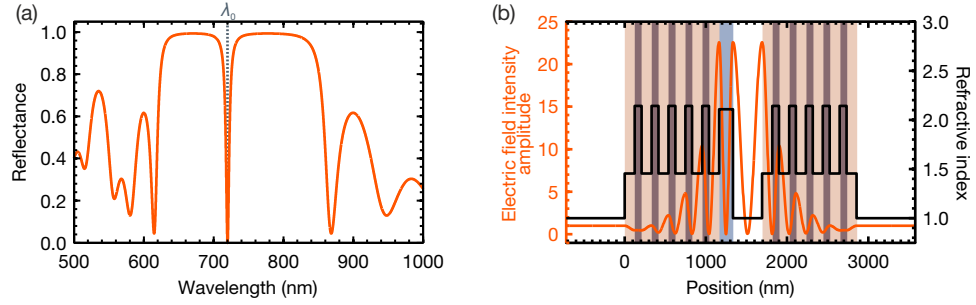


Fig. 5.10 | Dielectric cavity with hBN. Cavity formed from dielectric DBRs. One of the DBRs ends with $\lambda_0/2n_{\text{hBN}}$ thick hBN layer. Simulated (a) reflectance spectrum, (b) distribution of electric field at $\lambda_0 = 720$ nm.

Fig. 5.10 presents simulations showing that additional hBN layer can be incorporated within previously considered microcavity design in which hBN flake is deposited on top of one of the DBRs. When the thickness of this additional hBN layer is equal to $\lambda_0/2n_{\text{hBN}}$ corresponding reflectance spectra presented in Fig. 5.10a clearly shows the cavity mode at the desired wavelength of λ_0 . Additionally, the electric field distribution within such cavity, presented in Fig. 5.10b, shows high value of the electric field at the interface between hBN and air, at the position where WSe₂ flake can be deposited.

Such configuration corresponds to TMDC flake deposited on top of the hBN. To obtain fully encapsulated flake additional hBN layer has to be deposited on top. Fig. 5.11 presents simulations performed for the thicknesses of the covering hBN of 10 nm and 20 nm.

Fig. 5.11a shows simulated amplitude of the electric field at the position of the TMDC flake for varying thicknesses of the bottom hBN layer with the top hBN 10 nm and 20 nm thick. In both cases electric field exhibit a local maxima close to the thickness of bottom hBN layer of $\lambda_0/2n_{\text{hBN}} = 170.6$ nm (for $n_{\text{hBN}}(\lambda = 720 \text{ nm}) = 2.11$ [207]). According to the simulations for top hNB with thickness of 10 nm the maximum lays at 162 nm and for thicker covering hBN (20 nm) the bottom hBN should be thinner and equal to 156 nm.

For those optimal hBN thicknesses, electric field distributions within the cavities are shown in Fig. 5.11c,d with the corresponding reflectance spectra in Fig. 5.11b showing cavity mode positioned at λ_0 .

5.2.3 Experimental realization of a tunable microcavity

Following this design, the tunable cavity based on two dielectric DBRs was realized in practice.

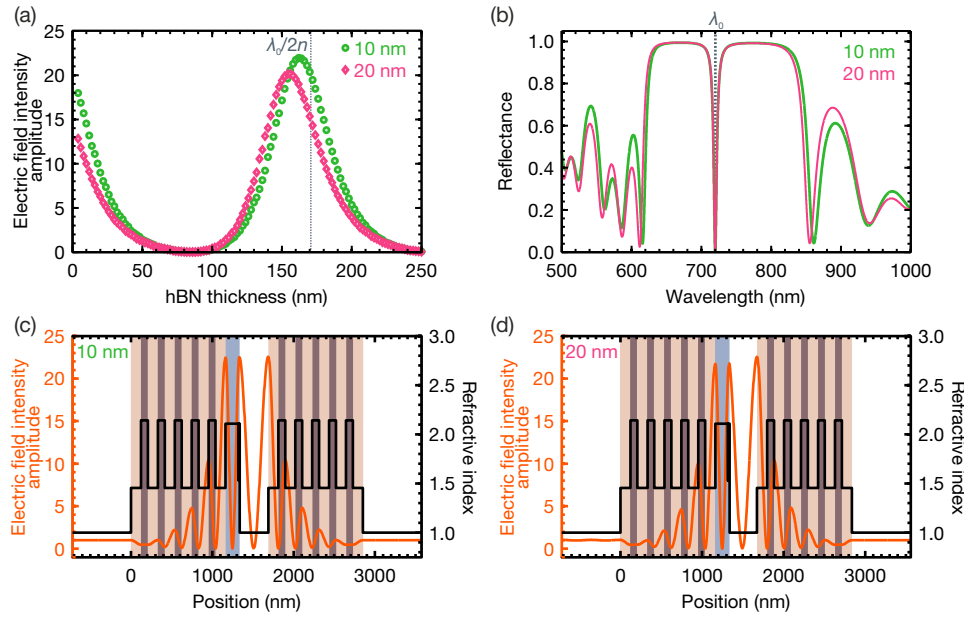


Fig. 5.11 | Dielectric cavity for encapsulated flakes. (a) Simulated electric field intensity amplitude at the position of a thin TMDC flake encapsulated in hBN for different thicknesses of bottom hBN layer with top hBN layer of 10 nm and 20 nm. (b) Calculated reflectance spectra and (c,d) electric field distribution for cavities corresponding to maximum electric field at the position of the TMDC flake with top hBN 10 nm and 20 nm thick.

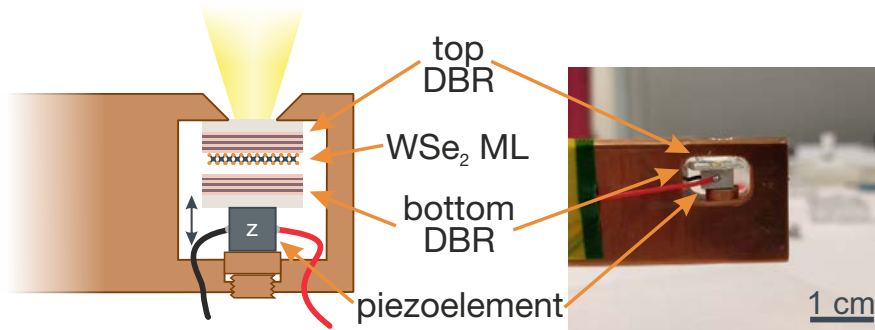


Fig. 5.12 | Tunable microcavity holder. Scheme and photo of a holder fitting inside a cryostat allowing to fabricate a tunable microcavity.

Fig. 5.12 presents a scheme and a photo of a self-made holder allowing to form such cavity inside the Oxford Instruments MicrostatHe-R cryostat.

From the top side, the holder contains an aperture with 2 mm radius allowing for optical access to the cavity. The top DBR forming the cavity was glued directly to the holder with DBR layers pointing inwards. The optical

cavity was formed by approaching with the second, bottom DBR.

There are two mechanisms to control the distance between the DBRs. The first one allows for macroscopic adjustments outside the cryostat. The bottom DBR is mounted on a piston, which can be moved by rotation of the screw at the bottom part of the holder.

The second one, for more precise control of the distance allows for adjustments during the operation at liquid helium temperature. The bottom DBR is glued to a piezoelectric chip (Thorlabs PA4FKW). By applying electric bias to the electrodes of the chip the element expands, decreasing the width of the cavity.

The holder was prepared to fit dielectric DBRs grown on transparent glass or fused silica substrates with dimensions of approx $7 \times 7 \times 1$ mm.

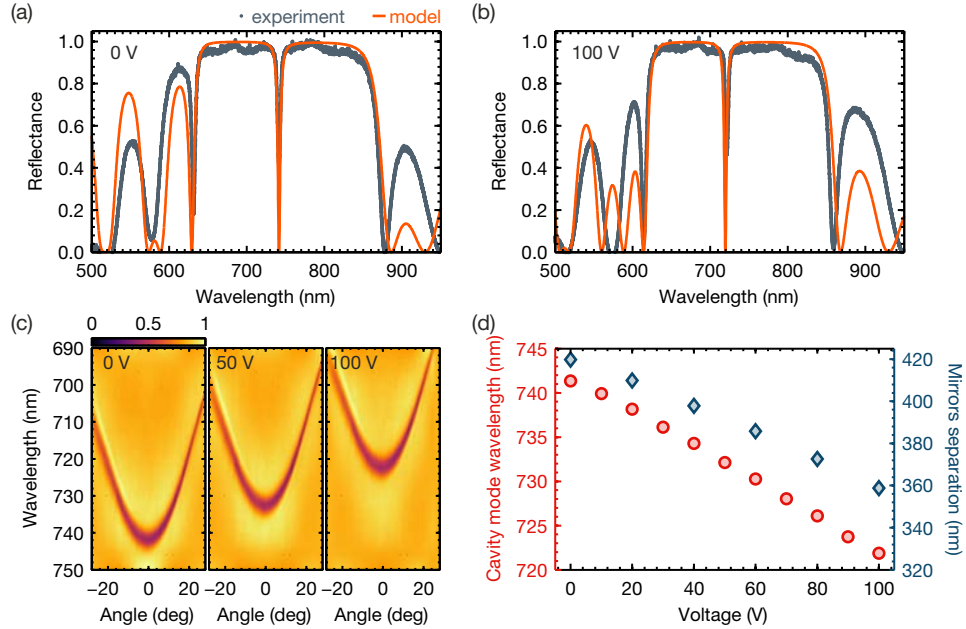


Fig. 5.13 | Tunable dielectric cavity. (a) Reflectance of cavity formed by two DBRs: measured and calculated by transfer matrix method. (b) Reflectance corresponding to 100 V applied to piezoelectric chip. (c) Experimental angle-resolved reflectance spectra at 0 V, 50 V and 100 V applied to piezoelectric chip. (d) Experimentally determined cavity mode position at different biases at piezoelectric elements with calculated distance between the DBRs obtained by transfer matrix method. All measurements performed at 4.5 K.

Operation of the full system was tested with a cavity formed by two $\text{SiO}_2/\text{TiO}_2$ DBRs with center wavelength of $\lambda_0 = 720$ nm. Holder was placed

in the cryostat. Fig. 5.13 presents results obtained with the system cooled with liquid helium.

Fig. 5.13a shows reflectance spectrum measured at normal incidence. Within the photonic stopband region the spectrum shows clearly a cavity mode with the resonant wavelength of around 740 nm and the second cavity mode at the short-wavelength edge of the stopband at around 630 nm.

With the application of 100 V of the external bias to the piezoelectric chip the width of the cavity decreases which results of in the shift of the cavity mode towards lower wavelengths at around 720 nm, and the other cavity mode at shorter wavelengths almost entirely shifts outside the photonic stopband.

The holder allows to use the high numerical aperture objective (up to approx. $NA = 0.6$ with working distance of 100 mm), which provides opportunity to collect angle-resolved reflectance spectra, as shown in Fig. 5.13c. These (measured in shorter wavelength range than Fig. 5.13a,b) clearly show both: parabolic dispersion relation of the cavity modes and the wavelength tunability with voltage applied to the piezoelectric chip.

Resonant wavelength of the cavity mode extracted from the series with increasing voltage is shown in Fig. 5.13d. In the range of 0–100 V almost perfect linear dependence between the electric bias and the position of the cavity mode is observed. In this case small tunability of the mode in approx. 740–720 nm (shift of approx. 45 meV) is connected with low thickness of the obtained cavity.

The thickness of the cavity can be estimated based on the reflectance spectra, especially those collected in wider spectral range. In Fig. 5.13a experimental spectrum (gray dots) is compared with the simulations of two $\text{SiO}_2/\text{TiO}_2$ DBRs designed for maximum reflectance at 720 nm separated between each other by a vacuum gap of 420 nm (orange line). Such theoretical curve properly describes multiple features of the experimental spectrum. The most sensitive parameter changing with the spacing between the DBRs is the wavelength of the cavity mode, which is in a good agreement between simulations and experiment at 740 nm. For wider cavities the second parameter is the energy difference between subsequent cavity modes. In the case of the spectrum in Fig. 5.13a the second cavity mode is located just at the short wavelength edge of the photonic stopband, but its position (630 nm) in simulations match the experimental results. In good agreement with the experiment is also the wavelength of the first reflectance minimum at the higher wavelength edge of the photonic stopband (in simulations shifted by approx. 10 nm towards higher wavelengths than in the experiment). Remaining oscillations of reflectance outside the stopband range are in general

well captured by simulations in regards of spectral positions of subsequent minima and maxima of reflectance. The highest discrepancy is visible around 580 nm where in the experiment a single minima can be observed but the simulations show a double structure, but the overall agreement between the experimental spectra measured at 0 V and the simulations for the cavity thickness to 420 nm is very good.

This is particularly important when the experimental results are compared with simulations for different cavity thicknesses. Fig. 5.13a presents also simulations performed for 50 nm (green) and 790 nm (pink) spacing between the DBRs. Those considered thicknesses are the closest values at which the resonant wavelength of the cavity mode matches the experiment. In both cases the agreement between simulations and experimental results is significantly worse than for 420 nm spacing, especially at the position of the cavity mode at 630 nm, where it is shifted towards shorter wavelengths for 50 nm and towards the higher wavelengths for 790 nm spacing.

Similarly, the spectra measured for 100 V electric bias can be compared with the simulations. Fig. 5.13b in solid orange lines presents the simulations performed at 360 nm distance between the DBRs. Like for the previous spectra there is a good agreement between the experimental results and the simulations in the spectral position of the cavity mode and both short- and long- wavelength minima of the reflectance. Once again only significant discrepancy can be observed around 575 nm where single wide minima in experimental results shows additional maximum in simulations.

Based on the simulations in the initial case without external bias the distance between the DBRs is around 420 nm and decreases to 360 nm for 100 V applied to the piezoelectric chip. Those values suggests that during the operation both DBRs are in contact at the other parts of the sample. This results in a limited tunability of the cavity width of the order of 60 nm., which suggests that the movement of the piezoelectric chip is blocked. Typically operation at higher initial cavity width allows for extended tunability range. Nevertheless observed in this case tunability of around 45 nm of the cavity mode is sufficient to fine-tune the cavity mode around the exciton resonance in WSe₂.

The final concern regarding the realization of the cavity containing WSe₂ monolayer is the ability to locate the position of the desired flake. As the flakes placed in the cavity are covered from both sides with the DBR structures with high reflectance the imaging of the flakes and finding desired monolayer flake is difficult. Nevertheless imaging even in such conditions is possible as is shown in Fig. 5.14. Fig. 5.14a presents area around a monolayer flake deposited on top of a dielectric DBR obtained under a standard

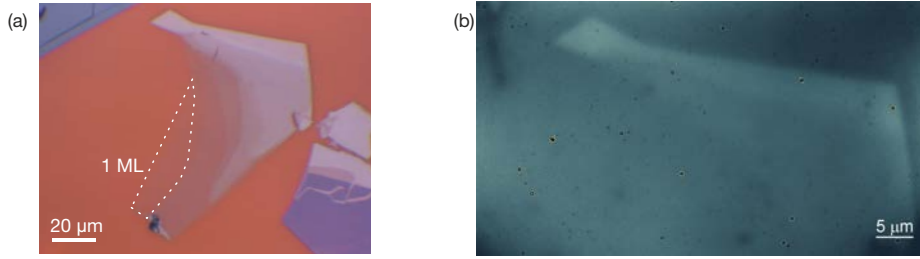


Fig. 5.14 | Imaging of WSe₂ flake. (a) Image from an optical microscope of the exfoliated WSe₂ flakes deposited on top of a DBR with the monolayer region marked by dashed white line. (b) Image (pseudocolor scale of monochromatic image) of the same flake placed inside a dielectric microcavity.

optical microscope. The monolayer area marked by a dashed white line is surrounded by thicker flakes. The same area can be found when the flakes are in the center of the cavity, as shown in the real space image taken in the experimental setup Fig. 5.14b. In this case the visibility of the flakes is reduced and the observation of the flake is more difficult. Based on such preview area, the actual monolayer cannot be directly observed, but its position can be found based on the relative orientation from the edges of the thicker flakes. To enhance contrast of the image it is beneficiary to use illumination outside the stopband region of the DBR. Fig. 5.14b illustrates the flake with the illumination by a broadband LED. The LED light was transmitted through 532 nm laser clean-up filter, but optical shortpass or longpass filters blocking the stopband region could be also used.

In overall, the results in Fig. 5.13 shows that the proposed design of two dielectric DBRs placed in the holder inside the cryostat allows for the realization of the cavity with the width below 1 μm. The use of piezoelectric chip allows for the tunability of the cavity mode and high aperture of the holder allows for collecting angle-resolved spectra in ±20 deg range.

5.3 Multilayer flakes on DBRs

After the operation of an empty cavity described in the previous section, this section concentrates on the characterization of optical properties of thin WSe₂ flakes, outside of the cavity.

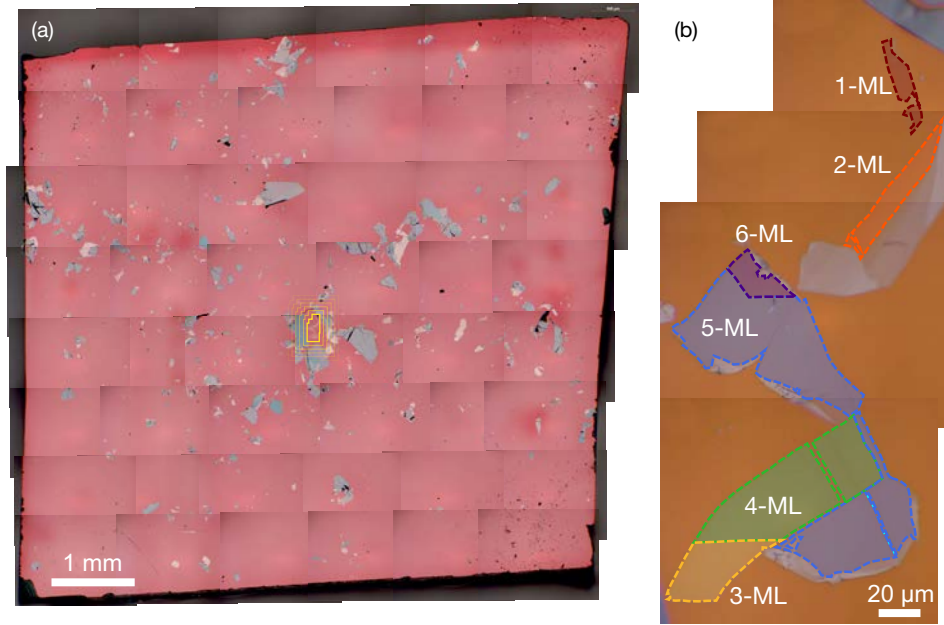


Fig. 5.15 | Thin WSe₂ flakes on DBR. Image from optical microscope of the sample with exfoliated WSe₂ flakes deposited on top of a DBR. (a) Image of the whole sample. (b) Close-up on the area containing 1 to 6 monolayer thin flakes.

5.3.1 Samples with WSe₂ deposited on DBRs

Fig. 5.15 presents image from optical microscope of the sample in which WSe₂ was deposited directly on the surface of the DBR. Fig. 5.15a covers whole area of the DBR with size of around 7×7 mm. At this scale mostly visible are only thick WSe₂ flakes remaining after the exfoliation and the transfer process. Thin flakes prepared for optical measurements were intentionally transferred at the center of the mirror, and the close-up on this fragment of the sample is presented in Fig. 5.15b showing flakes with thicknesses ranging from single monolayer to 6 ML. Regions between flakes of different thicknesses is marked by dashed lines. Thicknesses of the 1 to 4 ML flakes was confirmed by AFM measurement.

Preparation of the sample with encapsulated WSe₂ flakes was significantly more complex as it required multiple consecutive flake transfers. Fig. 5.16 presents microscopic image after the first preparation step which was the transfer of $\lambda/2$ -thick hBN flakes onto the DBR. Image shows mostly two big hBN flakes. To confirm desirable thickness of the hBN multiple height scans across different parts (marked with colored squares) of the flakes we-

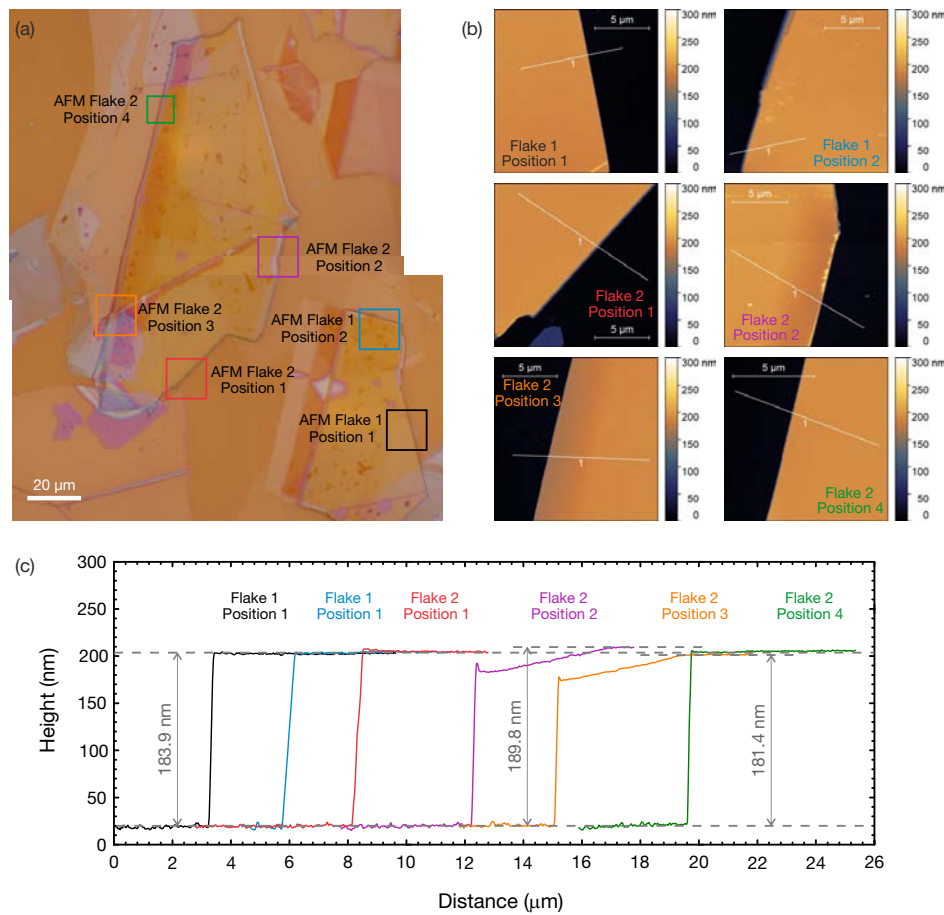


Fig. 5.16 | hBN layers on DBR. (a) Image of the sample with hBN flakes after transfer on DBR. (b) Thickness profiles measured by AFM at different parts of the hBN flakes. (c) Thickness profiles across different parts of the hBN layers.

re performed, as is shown in Fig. 5.16b. Line profiles across those scans are resented in Fig. 5.16c, showing flat areas with thicknesses between 181 nm to 190 nm.

Image from the optical microscope of the fully finished sample, after transfer of WSe₂ flakes and covering hBN, is shown in Fig. 5.17. Fig. 5.17b presents close-up for the region interesting for the optical measurements. Presented area contains flakes with thicknesses from 1 to 4 monolayers which are encapsulated in hBN. Edges of the thinner covering hBN are marked with black dotted line. Additionally, there are two flakes—monolayer marked as "1|hBN" and a fragment of a bilayer "2|hBN"—which are deposited on hBN, but without the covering layer.

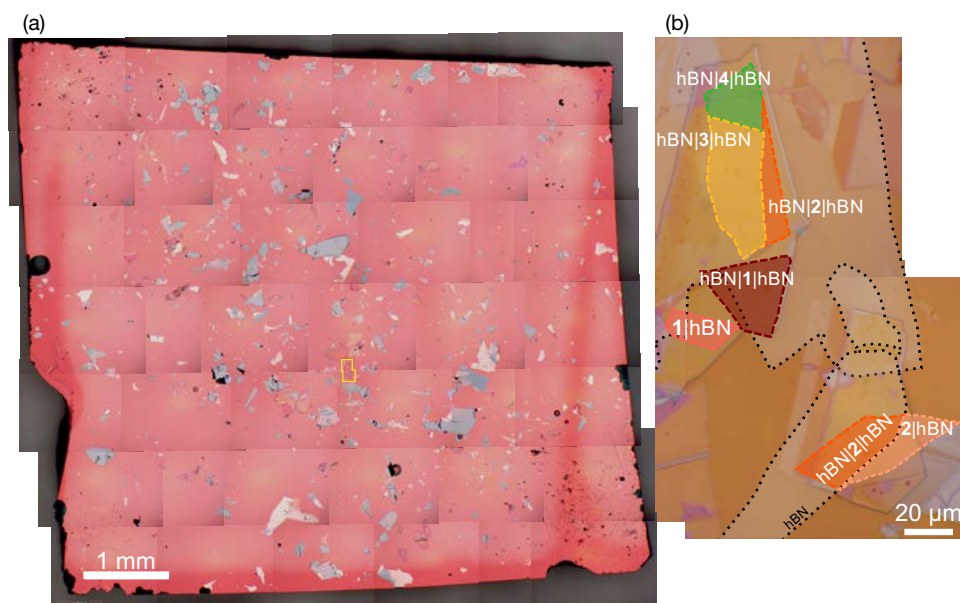


Fig. 5.17 | Thin WSe₂ flakes encapsulated in hBN deposited on DBR. Image from optical microscope of the sample with exfoliated WSe₂ flakes deposited on top of a DBR. (a) Image of the whole sample. (b) Close-up on the area containing 1 to 4 monolayer thin flakes.

Exfoliation and transfer of WSe₂ and hBN flakes and AFM measurements were performed by Dr Karol Nogajewski, and DBRs were grown in VIGO by Dr Janusz Kubrak.

5.3.2 Optical characterization of WSe₂ flakes on DBR

The characteristic optical spectra of the sample with WSe₂ deposited directly on hBN are shown in Fig. 5.18. Measurements comprised of photoluminescence spectra excited nonresonantly with 514 nm continuous wave green laser and reflectance contrast of broadband light from a halogen lamp, both performed with sample inside a cryostat at the temperature of 5 K.

For monolayer (1 ML) the photoluminescence spectra is dominated by spectrally broad emission band with energy around 1.7 eV. This band consists of multiple transition of so-called localized exciton states. Line corresponding to neutral exciton transition in the monolayer lays at the higher energy slope of this band and is shown in the inset, at the energy of 1.75 eV. Assignment of this line to an exciton transition in a direct bandgap semiconductor is consistent with reflectance contrast measurement. At the energy of

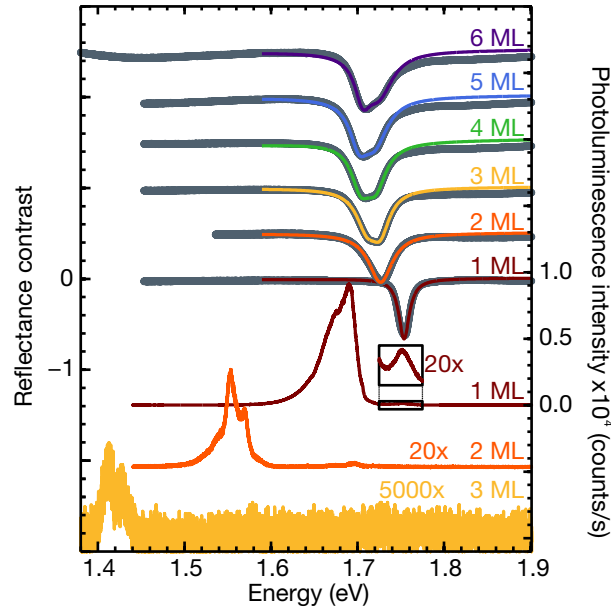


Fig. 5.18 | Multilayer WSe₂ flakes deposited on dielectric DBR. Photoluminescence (color lines) and reflectance contrast (gray lines) spectra of thin layer WSe₂ flakes (1–6 ML). Reflectance measurements were fitted with transfer matrix model (color lines). Measured at 5 K.

this energy peak in the emission spectra there is an reflectance contrast negative peak. This spectral feature corresponds to absorption of the incident light and optical creation of excitons.

For the WSe₂ bilayer (2 ML) the emission shifts to lower energy at around 1.57 eV, and becomes much weaker, as signal shown in Fig. 5.18 is multiplied 20×. This emission can be associated with lowest energy, optically indirect transitions between Γ and Q points in the Brillouin zone [203]. As for the monolayer, the emission signal consists of series of multiple lines. Comparing emission with reflectance contrast spectrum for bilayer, as expected from optically indirect transition no absorption associated with this emission band can be observed. Reflectance contrast spectrum consists of single line at 1.73 eV, much closer to the exciton energy for monolayer. This line corresponds to absorption of excitons at K points in the Brillouin zone. As this transition is optically direct, photoabsorption is efficient leading to pronounced reflectance contrast signal. But as it is not the lowest energy transition, the emission line from this state was not observed. Additionally there is a significant increase in the linewidth between the mono- and bi-layer, as

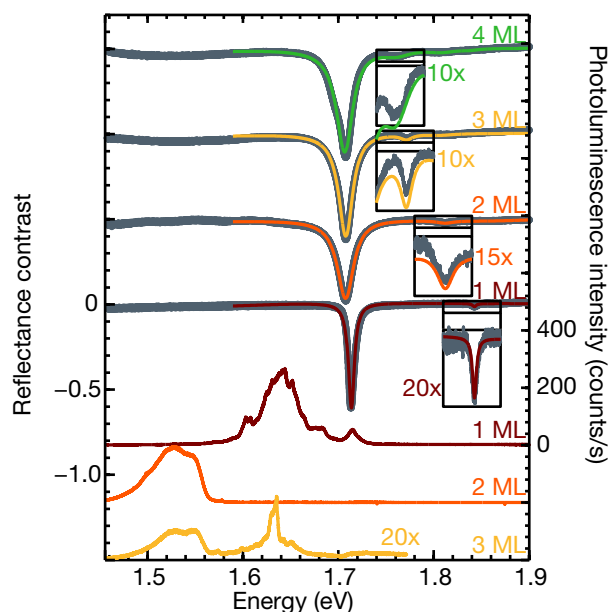


Fig. 5.19 | Multilayer WSe₂ flakes encapsulated in hBN deposited on dielectric DBR. Photoluminescence (color lines) and reflectance contrast (gray lines) spectra of thin layer WSe₂ flakes (1–4 ML). Reflectance measurements were fitted with transfer matrix model (color lines). Measured at 5 K.

observed earlier by other authors [203].

Considering thicker trilayer (3 ML), the emission spectrum follows the trend between mono- and bi-layer. Emission band shifts to even smaller energies at approx 1.42 eV. Emission intensity is extremely weak and spectrum shown in Fig. 5.18 is multiplied 5000 $\times$. But reflectance contrast spectrum still contains an absorption negative peak at the energy of 1.72 eV. The absorption line is broad and slightly asymmetric. The fine structure of *K* excitons in TMDCs what can be explained by hybridization of multiple interlayer and extralayer exciton complexes in multilayer TMDCs [153, 208]. Here, the splitting is comparable with the linewidth, so this fine structure is hard to analyze in details.

For thicker flakes from 4 ML to 6 ML only reflectance contrast was measured, as even lower emission intensity was reported [203]. Reflectance spectra for all of this flakes are very similar consisting of two deeps at 1.71 eV with energy splitting significantly smaller than their individual linewidths.

The same measurements were also conducted on the sample containing

hBN encapsulated flakes, with experimental results plotted in Fig. 5.19.

Starting from the encapsulated monolayer emission spectrum is still dominated by band of so-called localized exciton transitions with the center energy of around 1.65 eV. hBN encapsulations clearly shows that this emission band consists of multiple, spectrally narrow lines. At the higher energy side exciton emission line can be clearly observed at the energy of 1.71 eV. Energy shift of 40 meV between monolayer WSe₂ excitons is attributed to the change in dielectric environment around the flake. As expected from direct optical transition, the emission line is accompanied with absorption visible as reflection contrast deep. Apart from this transition additional weaker and spectrally narrower deep in RC spectra can be observed at the energy of 1.84 eV, which corresponds to 2s excited exciton state.

For encapsulated bilayer (2 ML) emission spectrum consists of spectrally broad band with lower energy at around 1.53 eV, which is not accompanied by any absorption signal at the reflectance contrast spectrum. Only absorption occurs at the energy slightly redshifted from the excitonic transition in the monolayer. As for unencapsulated flakes there is a significant linewidth increase between mono- and bi-layer. For bilayer the excited exciton state is visible at the RC spectrum at the energy of 1.81 eV, closer in energy to the ground state than in monolayer. This line is also spectrally broader.

Following the trilayer (3 ML), the measured emission spectrum is similar to the combined mono- and bi-layer signal, what is probably caused by spatial proximity of those layers. As shown in Fig. 5.17 trilayer region is in direct contact with a bilayer fragment and in close neighborhood with the encapsulated monolayer. Emission spectrum was collected close to the center of the trilayer region, but as both mono- and bi-layers are characterized by significantly higher emission intensity, the collected signal is dominated by their response. Clear trilayer signal measured for nonencapsulated flake was possible thanks to significantly higher distance (above 150 μm) between trilayer and closer bilayer (Fig. 5.15). Trilayer response can be still differentiated by RC spectrum. Main exciton transition energy is still very close to the bilayer one, but the excited exciton state energy (1.77 eV) comes visibly closer to the main transition.

For the tetralayer there was no attempt to measure the emission spectrum. RC is dominated by a deep close to those for bi- and tri-layer. This line is slightly more asymmetric at the lower energy side, and at the high energy slope the excited exciton transition appears with energy of 1.76 eV.

To summarize, the encapsulation of WSe₂ flakes in hBN results in significant reduction in spectral linewidth, and for investigated flakes allowed for the observation of excited states of excitons at the cost of overall decrease of

the emission intensity. Only for the direct bandgap monolayer the excitonic transition is visible in both emission and reflectance spectrum. For thicker WSe₂ layers with transition to indirect bandgap semiconductor emission intensity significantly drops and the emission from K points excitons was not observed. Nevertheless excitons at K points in the Brillouin zone can be still created optically which was observed at reflectance contrast spectra.

5.3.3 Reflectance spectra model

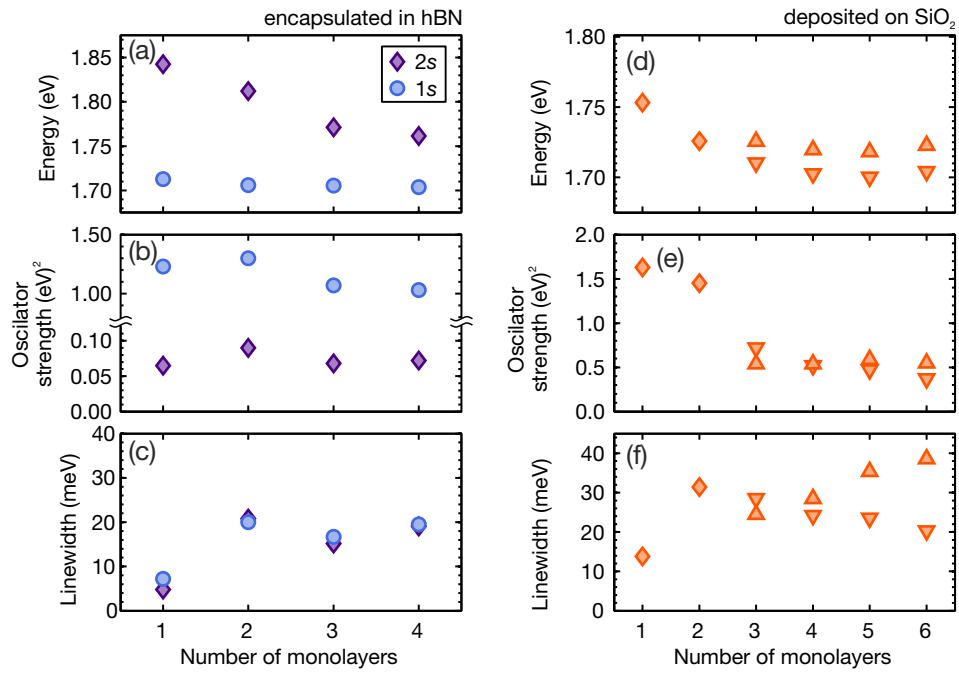


Fig. 5.20 | Multilayer WSe₂ excitons properties based on transfer matrix model. (a) Energy, (b) oscillator strength and (c) linewidth of excitons in WSe₂ multilayers encapsulated in hBN obtained by fitting of transfer matrix model. (d–f) Corresponding results for multilayer WSe₂ flakes deposited directly on DBR.

Tab. 5.1 | RC fitting parameters for WSe₂ deposited on the DBR encapsulated in hBN with modeled exciton-photon coupling strength in dielectric cavity (abbrev. model) compared with the values obtained experimentally (abbrev. exp.).

	E_{1s} (eV)	f_{1s} (eV ²)	γ_{1s} (meV)	E_{2s} (eV)	f_{2s} (eV ²)	γ_{2s} (meV)
1 ML	1.7128	1.23	7.2	1.8426	0.065	4.8
2 ML	1.7059	1.30	20.0	1.8122	0.090	20.8
3 ML	1.7055	1.07	16.7	1.7713	0.068	15.2
4 ML	1.7036	1.03	19.5	1.7632	0.072	19.4

Tab. 5.2 | RC fitting parameters for WSe₂ deposited on the DBR with modeled exciton-photon coupling strength in dielectric cavity (abbrev. model) compared with the values obtained experimentally (abbrev. exp.).

	E_{1s} (eV)	f_{1s} (eV ²)	γ_{1s} (meV)	$\Omega_{1s}^{\text{model}}$ (meV)	Ω_{1s}^{exp} (meV)
1 ML	1.7532	1.63	13.8	19.4	23.9
2 ML	1.7258	1.45	31.4	20.8	23.2
3 ML	1.7103	0.72	28.5	29.1	30.2
	1.7257	0.54	24.5		
4 ML	1.7026	0.52	24.2	31.3	37.5
	1.7196	0.54	28.5		
5 ML	1.7002	0.47	23.5	33.6	36.8
	1.7182	0.59	35.4		
6 ML	1.7041	0.37	20.3	33.2	40.7
	1.7228	0.55	38.6		

Reflectance spectra were further analyzed using transfer matrix method. It was assumed that thin WSe₂ flakes can be simulated by a layer with dielectric function given by [209]:

$$\varepsilon_{\text{WSe}_2}(E) = \varepsilon_0 - \sum_i \frac{f_i}{E_i^2 - E^2 - i\gamma_i E}, \quad (5.1)$$

where E_i is energy and γ_i linewidth of given i transition. Thickness of each layer was assumed to be a multiply of thickness of the monolayer equal to 0.65 nm [206]. Parameter f_i acts as oscillator strength per layer thickness of given optical transition. As the background, the dielectric constant $\varepsilon_0 = 16$ was used [209].

For encapsulated flakes i goes over the main and excited state transition.

For flakes placed directly on SiO₂ starting from trilayer it was assumed that spectrum consists of two lines.

Calculating reflectance contrast between the bare DBR and the structure with WSe₂ flakes allows to fit parameters of Eq. (5.1) to the experimental data. Fitted parameters are plotted in Fig. 5.20 and collected in 5.2 for flakes without and in Fig. 5.1 for flakes with hBN encapsulation.

For encapsulated flakes the main exciton energy remains almost constant with varying thickness Fig. 5.20a, with the highest change between the monolayer (1.713 eV) and the bilayer (1.706 eV). Excited exciton state energy changes significantly decreasing from 1.843 eV for monolayer to 1.763 eV for the tetralayer.

Oscillator strength per layer thickness (Fig. 5.20b) remains similar between all encapsulated flakes with value of around 1.15 (eV)² for the main excitonic transition, and significantly lower of around 0.07 (eV)² for the 2s state.

Linewidths follow the same trend for ground and excited exciton state (Fig. 5.20c). Lowest spectral width is observed for the monolayer (7.2 meV for 1s state). For bilayer linewidth increases almost three times to 20 meV, and stays similar for tri- and tetra-layer (16.7 meV and 19.5 meV correspondingly).

For flakes deposited directly on top of the DBR exciton energy change with layer thickness (Fig. 5.20d) is more visible and changes from 1.753 eV to 1.726 eV from the monolayer to the bilayer.

Oscillator strength is hard to directly compare between mono-, bi-layer with thicker flakes as for them reflectance deep was simulated with two transitions instead one for thinner flakes.

Linewidths follow the same trend as for unencapsulated flakes, but are significantly higher starting from 13.8 meV for the monolayer.

The same analysis was applied to the reflectance results of mono- and bi-layer WSe₂ which were deposited on hBN, but without covering layer. Fig. 5.21 presents photoluminescence and reflectance contrast spectra of those flakes. The overall shape of the spectrum resembles more the fully encapsulated flakes than those deposited on SiO₂. Photoluminescence spectra of the monolayer shows multiple sharp lines in monolayer luminescence. Reflectance spectra contain spectrally narrow absorption deeps, but only for the ground exciton state with no experimental evidence of excited exciton states. Obtained fitting parameters, presented in Fig. 5.3, are close to the fully encapsulated flakes in terms of transition linewidth and the oscillator strength. Exciton resonance energy for those flakes lays in between energy of fully encapsulated flakes and those deposited directly on dielectric mir-

Tab. 5.3 | Fitting parameters for WSe₂ deposited on hBN with modeled coupling strength in dielectric cavity compared with the values obtained experimentally.

	E_{1s} (eV)	f_{1s} (eV ²)	γ_{1s} (meV)	$\Omega_{1s}^{\text{model}}$ (meV)	Ω_{1s}^{exp} (meV)
1 ML	1.7366	1.10	7.6	21.7	24.5
2 ML	1.7132	1.03	20.5	27.6	

rors.

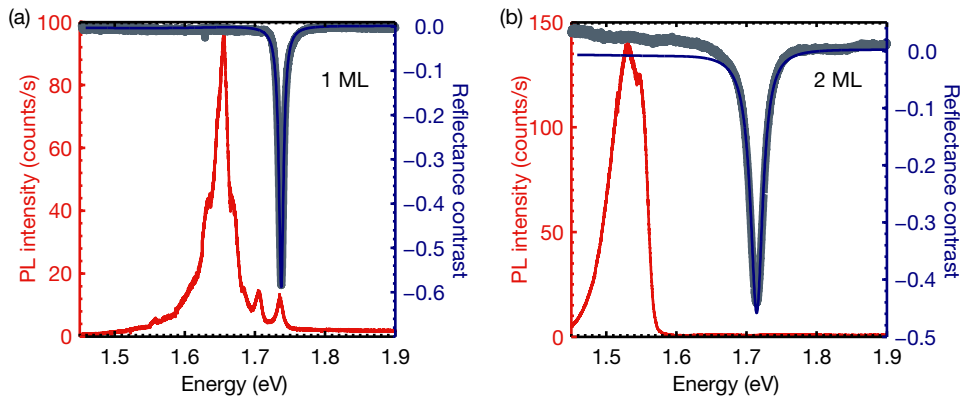


Fig. 5.21 | **Multilayer WSe₂ flakes deposited on hBN without full encapsulation.** Photoluminescence (red) and reflectance contrast (gray) spectra of (a) monolayer and (b) bilayer WSe₂. Reflectance measurements were fitted with transfer matrix model (blue lines). Measured at 5 K.

5.4 Exciton-polaritons

5.4.1 Tunable dielectric cavity with WSe₂ multilayer flakes

To investigate the coupling of excitons with cavity photons, a microcavity was formed by approaching the dielectric DBR to the second DBR containing WSe₂ flakes. Full planar cavity structure was cooled down in a cryostat to approx. 5 K and the spacing between the DBRs forming the cavity was controlled by external voltage applied to piezoelectric chip. Results of angle-resolved reflectance spectra for the encapsulated WSe₂ flakes of different thicknesses are shown in Fig. 5.22.

For all 1 to 4 ML flakes two anticrossing modes are clearly visible which confirms achievement of the strong coupling regime in which the eigensta-

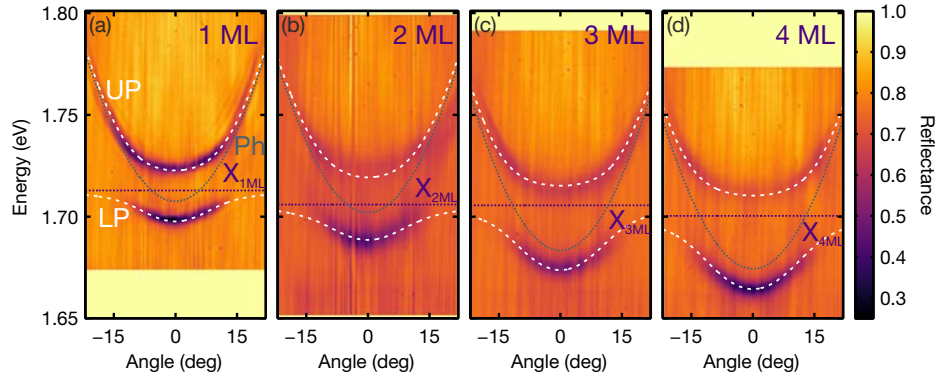


Fig. 5.22 | Strong coupling regime in multilayer WSe_2 . Angle-resolved reflectance spectra from tunable dielectric microcavity containing WSe_2 : (a) monolayer, (b) bilayer, (c) trilayer and (d) tetralayer encapsulated in hBN. Measured at the temperature of 5 K.

tes of the systems are coupled exciton-photon states: lower and upper polaritons. Results with the highest visibility were obtained for the monolayer flake

(Fig. 5.22a) with the spectrally narrowest lines. For thicker flakes the linewidth of the cavity modes increases and especially upper polariton is more pronounced at more negative exciton-photon detuning presented in Fig. 5.22b–d.

Modes observed in the reflectance were fitted with dispersion relation of exciton-polaritons given by coupled oscillator model (1.2). Energy of excitonic transition in the given flake are marked by purple horizontal dashed line, and were taken from previous fitting to RC measurements summarized in Fig. 5.1. Results of the fitting providing lower (LP) and upper (UP) polariton branches are marked in Fig. 5.22 by white dashed line. Fitting parameters were: energy and effective mass of photonic mode (marked by gray dashed line for bare cavity mode), and exciton–photon coupling strength Ω .

To estimate the distance between the DBRs forming the cavity a reflectance measurements in wider spectral range were performed. Spectra were collected near the WSe_2 flakes, at the hBN layer. Fig. 5.23a–c presents obtained results at three different electric biases applied to the piezoelectric chip. Without external voltage (Fig. 5.23a) two cavity modes are visible. As the one observed at the longest wavelengths is close to the edge of the stopband region of the DBRs it is visibly broader. With applied voltage both of the modes shift towards shorter wavelengths (higher energies). Experimental spectra are compared with the simulations by Berreman matrix method

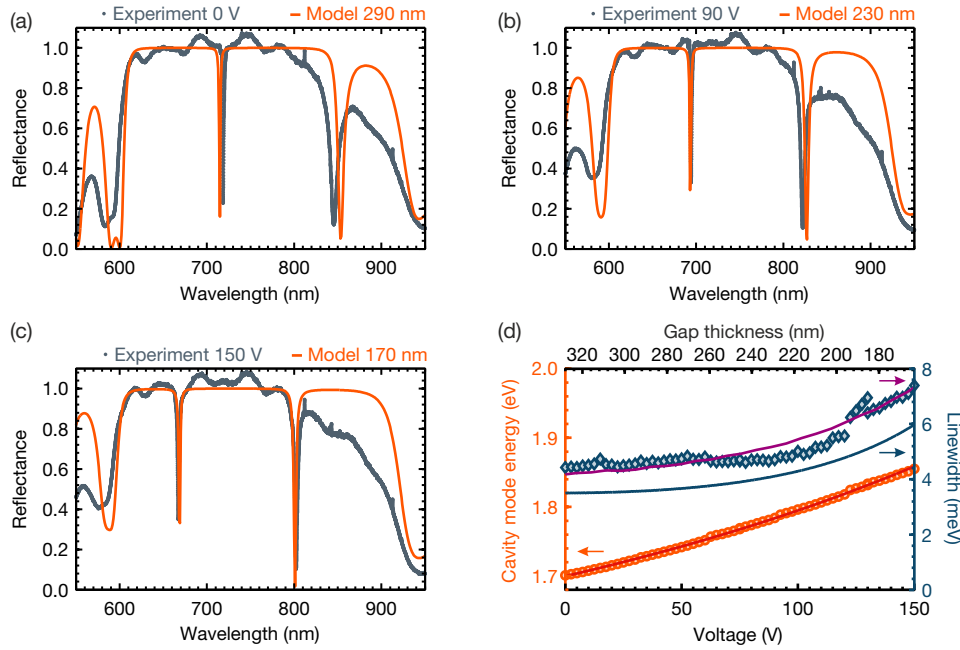


Fig. 5.23 | Reflectance of tunable microcavity with hBN flake. Measured normal incidence reflectance from the cavity with hBN flake at (a) 0 V, (b) 90 V and (c) 150 V applied to piezoelectric chip with corresponding transfer matrix simulations at different distances between the DBRs. (d) Comparison between experimental (points) energy and linewidth of the cavity mode versus applied electric bias with simulated (solid lines results at different spacing between the DBRs).

performed for such spacing between the DBRs to obtain the best match with the experiment. At 0 V this distance is equal to 290 nm and decreases with voltage to 170 nm at the highest 150 V electric bias. Simulations well recreate all main features of the experimental results. Energy of cavity modes and their spacing is the most sensitive to the cavity width, but simulations well recreate also both edges of the stopband region, and the narrowing of the cavity mode with longer wavelength with increasing voltage. Spectra in Fig. 5.23a–c were collected by a fiber-coupled spectrometer with broad spectral range, but limited spectral resolution.

Precise tunability of the cavity is shown in Fig. 5.23d presenting evolution of the energy of the cavity mode spectrally closer to WSe₂ excitonic resonance with applied electric bias. In this case external voltage allows to obtain tunability of 120 meV. Fitting energy of this single mode to the experimental results allows for extraction of the precise distance between the mirrors,

and as is shown in upper axis it varies in 170–330 nm. Simulations (blue line) reproduce also observed in the experimental data spectral broadening of the mode as it approaches the short-wavelength edge of the photonic stopband (Fig. 5.23c) with increasing voltage (decreasing spacing). Experimental linewidth below 100 V are almost constant and equal to approx. 5 meV, compared with 4 meV in simulations.

Such broadening can be explained as a result of variation in the thicknesses of individual layers as well as the whole DBR structure. In the experiment the diameter of the incident white light spot is on the order of 5 μm , so collected spectra represents an average over such area.

Purple line in Fig. 5.23d shows mean value of the simulations in which distance between the mirrors is described by a normal (Gaussian) distribution with standard deviation of $\sigma = 1$ nm, which very well matches to the experimental data.

Tunable cavity allows for crossing with the cavity mode energy through the excitonic resonances in WSe₂ flakes. Especially at zero exciton-photon detuning the energy splitting between the upper and the lower polariton branches is the smallest and equal to the coupling strength Ω .

Normal incidence reflectance spectra for increasing voltage at piezoelectric chip for 1 ML to 4 ML flakes are shown in Fig. 5.24a–d. For all the flakes the cavity mode energy crosses with the excitonic resonance resulting in a clear anticrossing behavior between two polaritonic branches. With increasing WSe₂ thickness the coupling strength, given by the smallest energy difference between the two branches, also increases. The fitting of the energy position of the modes provides experimental determination of Rabi energy which changes from 24.5 meV for 1 ML, 30.5 meV for 2 ML, 35.0 meV for 3 ML to 36.1 meV for 4 ML.

Experimental results in Fig. 5.24a–d are in a good agreement with Berreman matrix simulations presented in Fig. 5.24e–h. It is worth noting that this simulations do not include any parameters extracted from experiment in strong coupling regime, apart from the estimation of the cavity width in Fig. 5.23.

The values of the coupling strength obtained in simulations can be extracted from theoretical reflectance spectra in the same way as in the experiment by fitting energy positions of both polariton branches and finding their minimal distance. The values from both experiment and simulations are collected in Fig. 5.26a. The Rabi energy clearly increases with the number of monolayers. Simulations are in a good agreement with measurements but slightly underestimate experimental values.

Following coupling strength expected from strong coupling from N atoms

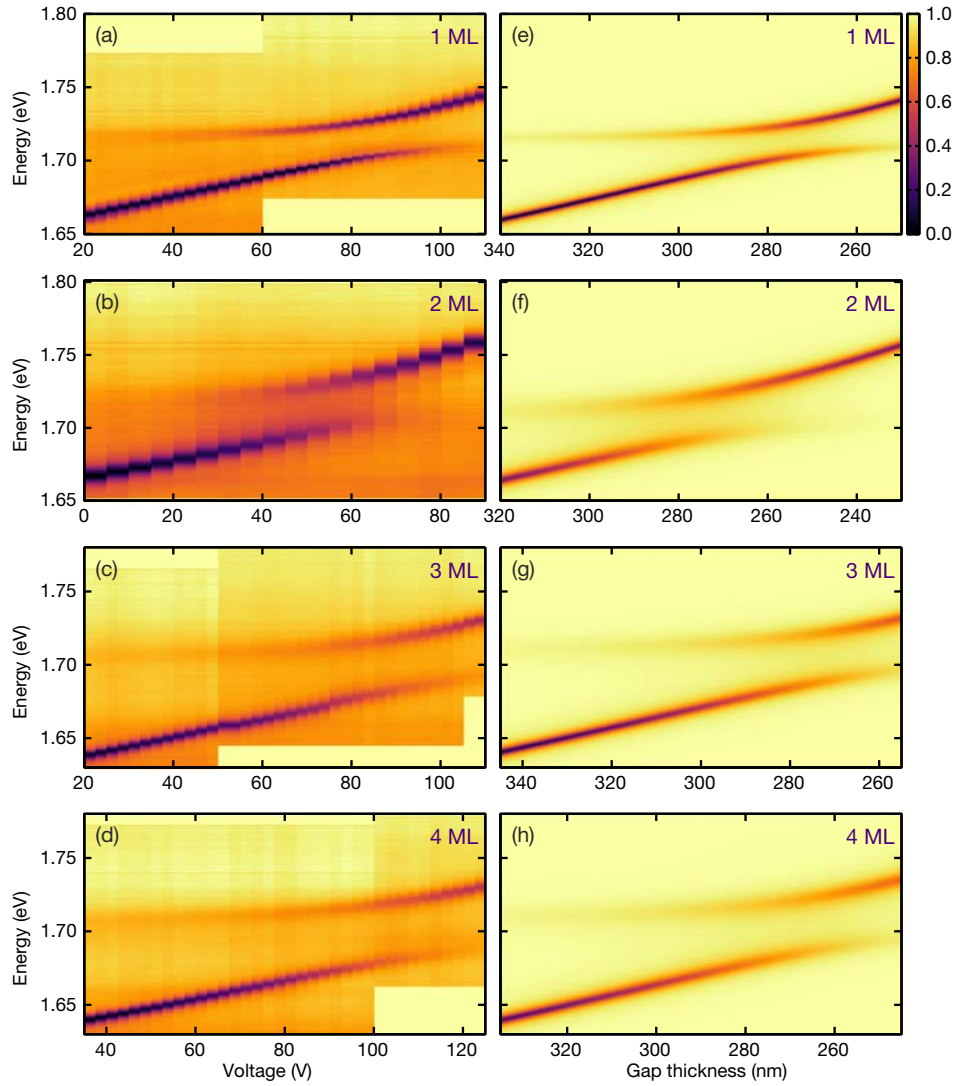


Fig. 5.24 | Strong coupling regime in multilayer WSe₂ in tunable microcavity. Normal incidence reflectance spectra from dielectric tuned by external voltage containing WSe₂ flakes: (a) monolayer, (b) bilayer, (c) trilayer and (d) tetralayer. Measured at the temperature of 5 K. (e–h) Corresponding transfer matrix simulations for varying distance between the DBRs forming the cavity.

coupled to cavity field [210]:

$$\Omega(N) = \sqrt{N\Omega_1}, \quad (5.2)$$

similar behavior was predicted for a planar microcavity incorporating a qu-

antum well [211]:

$$\Omega = 2\sqrt{\frac{2c\Gamma_0}{2n_{\text{cav}}L_{\text{eff}}} - (\gamma - \gamma_c)^2/4}, \quad (5.3)$$

where Γ_0 is the exciton radiative time [212], proportional to oscillator strength per unit area plays a role of a number of available resonators and additional term includes corrections due to spectral linewidths mismatch between the cavity mode γ_c and the excitonic transition γ .

More specifically, for a WS_2 monolayer in an tunable open cavity C. Flatten et al. [201] derived an equation for Rabi energy as:

$$\Omega = 2\sqrt{wd - (\gamma - \gamma_c)^2/4}, \quad (5.4)$$

where d is the thickness of the monolayer. Parameter w is proportional to the exciton oscillator strength per layer thickness and at the same time depends on such cavity parameters as effective cavity thickness and effective refractive index of the cavity.

The same experiment and analysis was also performed on another sample containing WSe_2 flakes deposited directly on the dielectric DBR. Obtained normal incidence reflectance for tuning the cavity mode across excitonic resonances in WSe_2 flakes are presented in Fig. 5.25.

In this case the distance between the mirrors was much higher as four consecutive cavity modes were observed over the whole photonic stop-band region, which based on simulations, corresponds to the cavity width at the order of $3\ \mu\text{m}$. For both experiment and simulations for all investigated flakes with thicknesses varying between 1 ML and 6 ML reflectance spectra consists of two branches exhibiting clear anticrossing behavior. The clearest anticrossing is observable for the monolayer flake (Fig. 5.25a), where two anticrossing branches are simultaneously clearly distinguishable in 115–130 V range. For the thicker layers (Fig. 5.25b–f) for increasing voltage the lower polariton branch disappears and the upper polariton branch emerges at higher energy. All those observations are consistent with the transfer matrix method simulations presented in (Fig. 5.25g–l), with clear transition for the monolayer and significant increase of the polariton modes linewidth around zero detuning for thicker layers.

Obtained Rabi energies, which are summarized in Fig. 5.26b, confirm the same trend as for encapsulated flakes. Rabi energy increases with the number of monolayers, but slower than can be expected from stacking of multiple isolated monolayers (square root projection). As before the experimentally observed coupling strength is better recreated with the transfer matrix simulations (red crosses) although it slightly but consistently underestimates the value of the Rabi splitting.

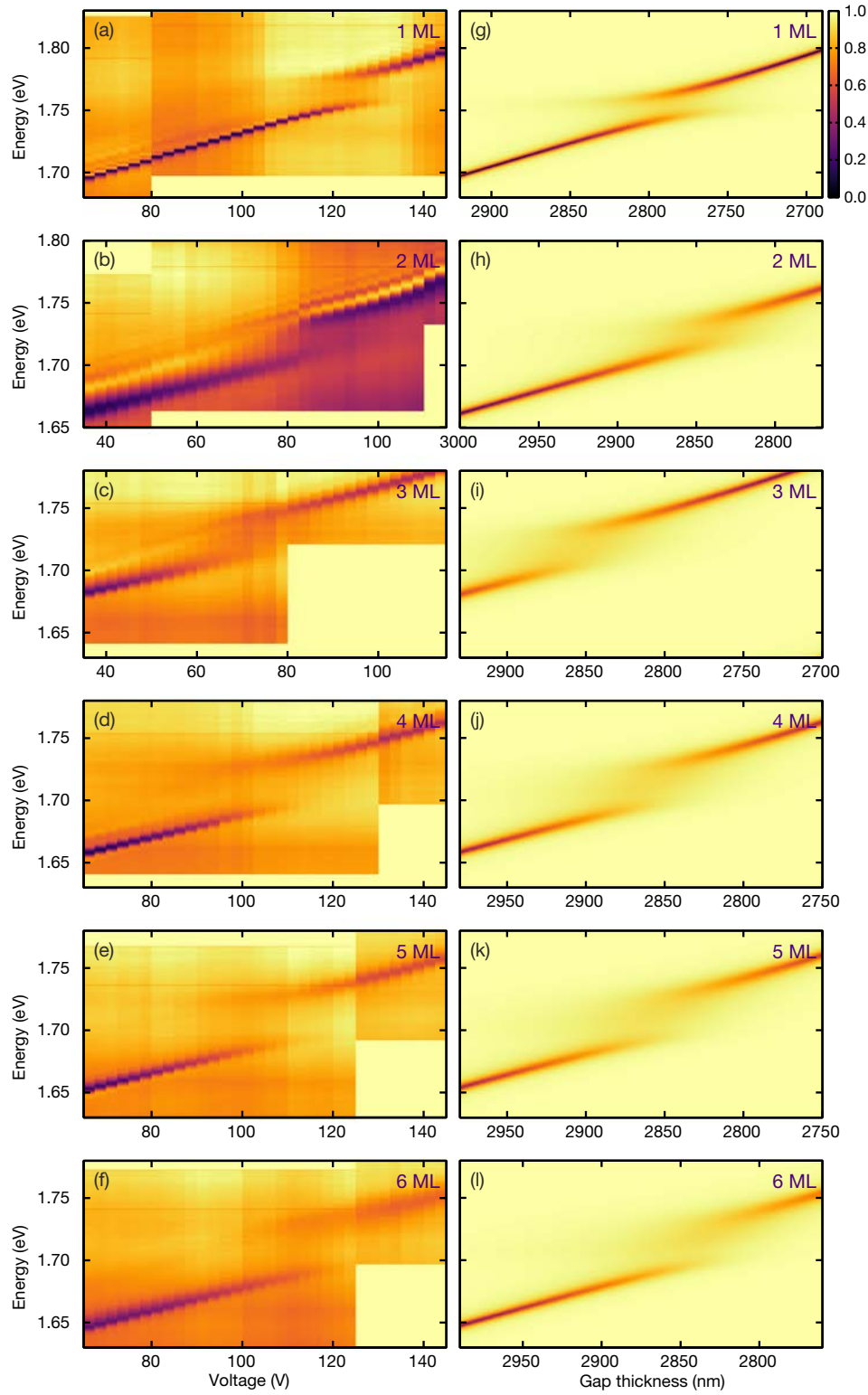


Fig. 5.25 | Strong coupling regime in multilayer WSe₂ in tunable microcavity. Normal incidence reflectance spectra from dielectric tuned by external voltage containing WSe₂ flakes: (a) monolayer, (b) bilayer, (c) trilayer and (d) tetralayer, (e) pentalayer, (f) heksalayer. Measured at the temperature of 5 K. (g–l) Corresponding transfer matrix simulations for varying distance between the DBRs forming the cavity.

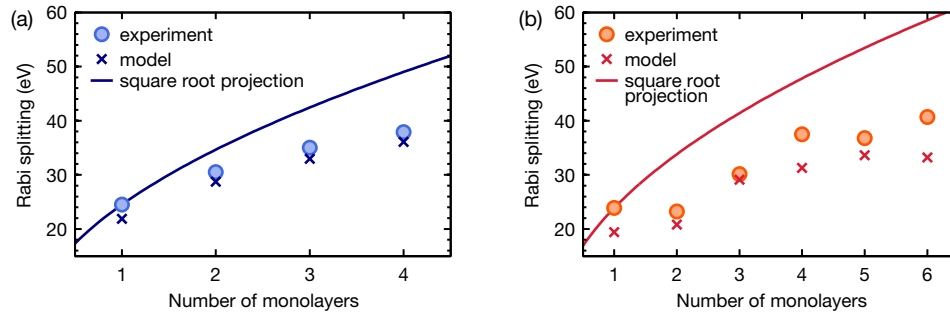


Fig. 5.26 | Rabi splitting. Dependence of Rabi splitting on thickness of WSe₂ layers extracted from experiment and transfer matrix method. (a) WSe₂ flakes encapsulated in hBN, (b) WSe₂ flakes deposited directly onto DBR

The tunability range in the experiment shown in Fig. 5.24 was insufficient to observe crossing between the cavity mode and the 2s excited state in the monolayer WSe₂ at 1.84 eV.

Coupling with the higher Rydberg states is of interests as due to their higher Bohr radius and higher values of expected optical nonlinearities [213]. Coupling between the excitons in excited states and the cavity mode was observed earlier in GaAs-based cavities [214], especially under external magnetic field, and for cavities with inorganic perovskites [215].

To investigate potential coupling with such excited state another experiment focused at this spectral range was performed with results summarized in Fig. 5.27. Fig. 5.27a presents normal incidence reflectance spectra obtained for increasing voltage applied to the piezoelectric chip, which shows crossing of the cavity mode with the energy of the excited state of hBN encapsulated monolayer, extracted from the measurements without top DBR in Fig. 5.1 (marked with dashed lines).

For both: normal incidence reflectance spectra as a function of voltage applied to the piezoelectric stage (Fig. 5.27a) and angle resolved spectra (Fig. 5.27b,c) there is no experimental evidence of coupling between the 2s excited state and the cavity mode. Neither clear anticrossing behavior nor resonance broadening is observable at the crossing of photonic and excitonic resonances.

For fabricated structure the oscillator strength of 2s exciton resonance appeared to be insufficient to achieve strong coupling regime, as can be shown in simulations. Fig. 5.27d presents simulated reflectance spectra performed for increasing value of 2s exciton oscillator strength. For high oscillator strength strong coupling regime is evinced by the appearance of two modes corresponding to the lower and the upper polariton. With decre-

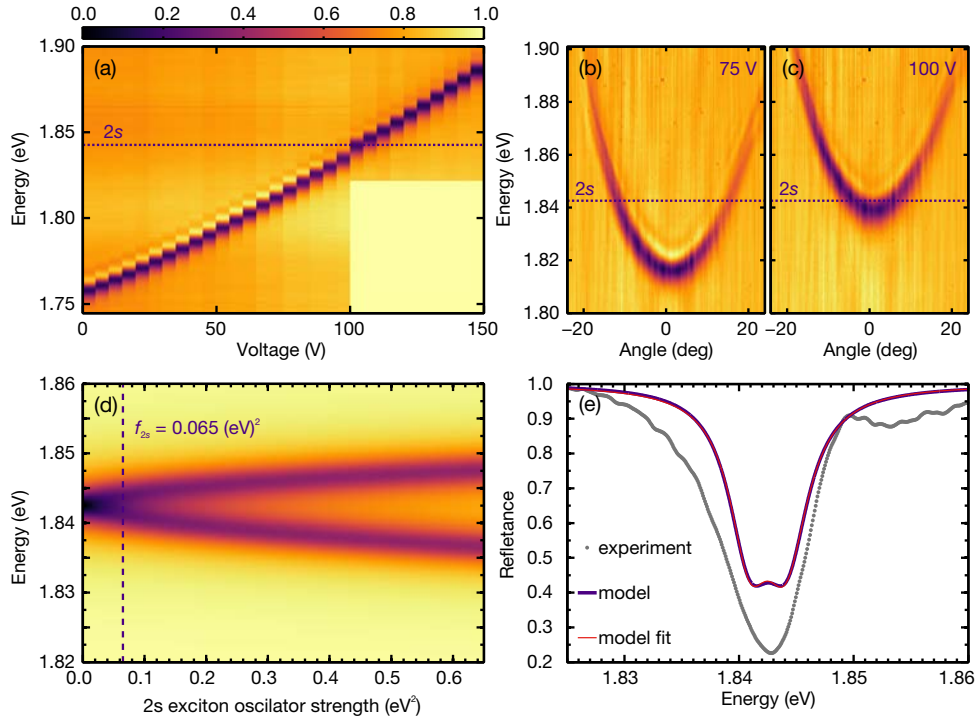


Fig. 5.27 | Potential coupling with 2s exciton excited state. (a) Normal incidence reflectance for tuning of the cavity mode across the energy of 2s exciton state in encapsulated WSe₂ monolayer. Angle-resolved reflectance spectra corresponding to (b) 75 V, (c) 100 V from panel (a). (d) simulated normal incidence reflectance for increasing value of 2s exciton oscillator strength. (e) Comparison of experimental, and simulated normal incidence reflectance.

asing oscillator strength energy the splitting between those branches decreases and especially at vanishing oscillator strength single uncoupled cavity mode can be observed. At 2s exciton oscillator strength for encapsulated monolayer extracted from fitting the reflectance spectra which is equal to $f_{2s} = 0.065 \text{ eV}^2$ and marked by a dashed vertical line in Fig. 5.27d, and corresponding reflectance spectrum in Fig. 5.27e (blue line). For those parameters simulated splitting is smaller than the linewidths and it cannot be resolved in the experimental spectrum (gray dots).

Thus in the investigated cavity the strong coupling regime between the cavity mode and the 2s excited state of the WSe₂ monolayer was not achieved, which is consistent with the theoretical predictions. Strong coupling conditions were reported in the literature in Ref. [216] in which in order to enhance the coupling condition three independent WSe₂ monolayers sepa-

rated by hBN layers were placed at the antinodes of the electric field of the cavity resulting in Rabi splitting of 7.7 meV.

5.5 Valley polarized exciton-polaritons

As it was demonstrated in Sec. 5.4.1, due to the indirect bandgap of investigated multilayer flakes, the optically direct excitonic transition was not visible in the photoluminescence and the investigation of the strong coupling was concentrated on the reflectance measurements. This is not the case for monolayer WSe₂, where both photoluminescence and reflectance spectra can be traced.

Fig. 5.28a–e presents angle-resolved reflectance spectra from the cavity with monolayer WSe₂ at different voltages applied to the piezoelectric stage. Each spectra clearly show two strongly coupled lower and upper polariton branches. Exciton-polariton dispersion relations were fitted to each spectra and are shown in white dashed lines with resulting bare cavity and exciton energies marked with gray dashed lines.

For monolayer WSe₂ the same dispersion relations can be observed in emission. Fig. 5.28f–j presents the photoluminescence spectra corresponding to the same electric biases applied to the piezoelectric stage. The sample was excited non-resonantly with a continuous wave green laser with emission wavelength of 532 nm (2.33 eV) and mean excitation power of 50 μ W. For the highest negative exciton-photon detuning in Fig. 5.28f the highest emission intensity is located at the bottom of the lower polariton branch fitted to the reflectance measurement. Emission intensity from the upper polariton branch is much weaker, but it can be observed when the emission intensity is rescaled for the energies exceeding the uncoupled exciton energy. For Fig. 5.28f with scaling factor of 182 $\times$ the upper polariton branch is clearly visible in emission and follows exactly the dispersion relation fitted to the reflectance measurement. Analogous results can be obtained for higher piezo voltages, which increase the value of exciton-photon detuning, as presented in Fig. 5.28g–j. As the cavity mode approaches the exciton resonance, the overall emission intensity decreases. The highest emission intensity is observed at the bottom of the lower polariton branch.

In order to observe the effects of valley polarization in the cavity with monolayer WSe₂, the emission intensity was detected independently in σ^+ and σ^- polarizations and the sample was excited with laser beam polarized: σ^+ , σ^- and linearly along horizontal, x axis. At first, the same nonresonant green laser beam 532 nm (2.33 eV) was used. Collected photoluminescen-

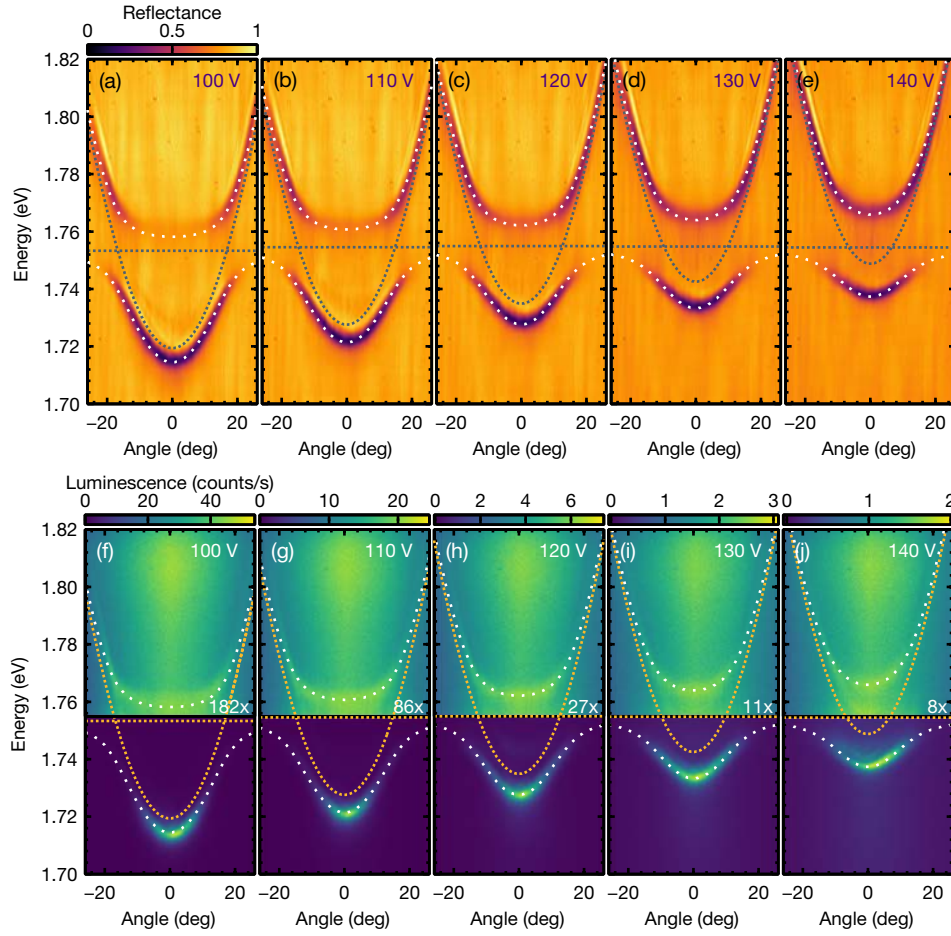


Fig. 5.28 | Reflectance and photoluminescence from monolayer WSe₂ exciton-polaritons. Angle resolved (a–e) reflectance and (f–j) photoluminescence spectra from tunable dielectric cavity containing WSe₂ monolayer at different voltages applied to piezoelectric stage. White dashed lines mark fitted exciton polariton dispersion relations. Measured at the temperature of 5 K.

ce spectra obtained at different voltages applied to the piezoelectric stage are plotted in Fig. 5.29a–l. In Fig. 5.29a for negative exciton-photon detuning emission comes mostly from the lowest energy of the lower polariton mode at around 1.70 eV with less intense signal from upper polariton at around 1.75 eV. In Fig. 5.29a the sample is excited with σ^+ polarized light with slightly stronger co-polarized emission (red line) than for the circular polarization with the opposite helicity (blue line). This slight imbalance suggests that even for this highly nonresonant excitation at least partially the angular momentum of the incident beam is preserved through orbital momentum

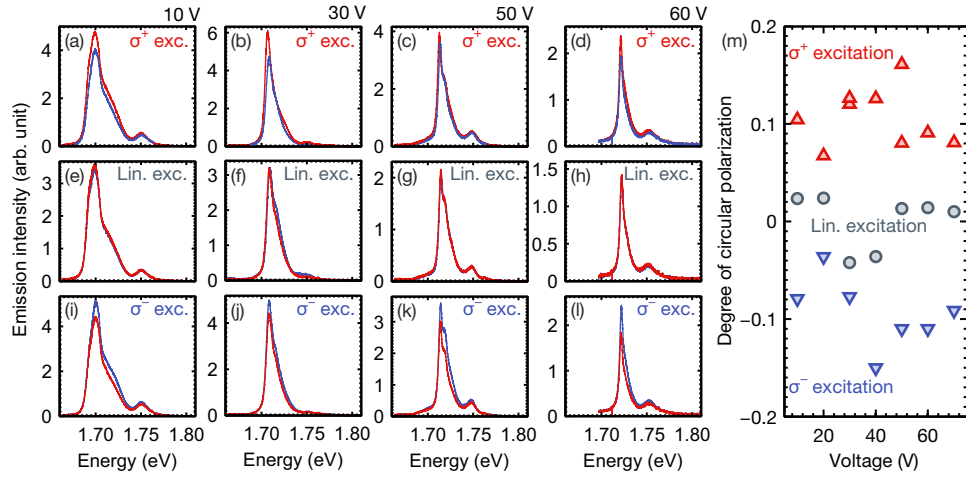


Fig. 5.29 | Valley polarization in WSe₂ exciton-polaritons. (a)–(l) Photoluminescence spectra detected in σ^+ (red lines) and σ^- (blue lines) polarizations for: σ^+ , σ^- and linearly polarized excitation at 532 nm (2.33 eV) at different voltages applied to the piezoelectric stage. (m) Degree of circular polarization of the total emitted light intensity at different voltages excited with σ^+ , σ^- and linearly polarized laser beam.

of WSe₂ excitons in final exciton-polariton state as σ^+ polarized excitation beam creates more charge carriers in the K valley than in the $-K$ one.

To check the reproducibility of this effect, at the same conditions, when the polarization of the excitation beam is changed to the linear, emission intensity detected in both circular polarizations is almost the same, as shown in Fig. 5.29e. In this case linearly polarized excitation beam equally excites carriers in both K and $-K$ valleys, and the resulting emission shows no degree of circular polarization.

For the final check, at the same voltage the polarization of the excitation beam is changed to σ^- , carriers in $-K$ valley are excited more efficiently which results in slightly stronger σ^- polarized emission Fig. 5.29i.

As the overall observed effect of valley polarization is quite small, the same experiment was repeated at different cavity widths controlled by piezo voltage, as shown in Fig. 5.29a–l. For each realization observed emission light intensity is higher in co-circularly polarized light and the degree of circular polarization of the emitted light is smallest for linearly polarized excitation. Those results are summarized in Fig. 5.29m which presents the degree of circular polarization of the total emitted light. As the degree of circular polarization of the emitted light clearly depends on the excitation

polarization, no evident dependence on the cavity detuning (voltage) is observed.

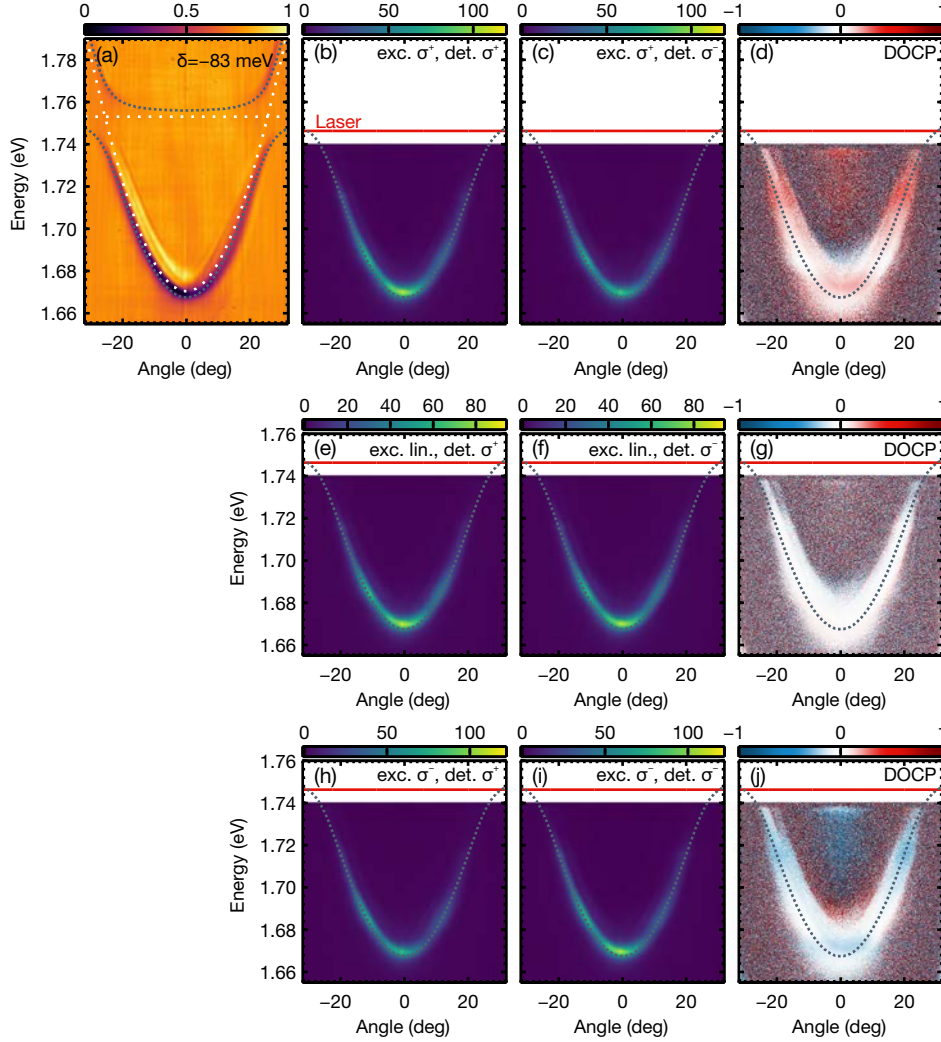


Fig. 5.30 | Valley polarization in WSe₂ exciton-polaritons. (a) Angle-resolved reflectance spectrum at negative detuning of $\delta = -83$ meV. Angle-resolved photoluminescence spectrum for quasi-resonant excitation: detection in σ^+ and σ^- polarizations with corresponding degree of circular polarization (DOCP) for (b–d) σ^+ , (e–g) linear, and (h–j) σ^- excitation polarization. Measured at the temperature of 5 K.

Second, to achieve stronger valley polarization similar experiment was repeated with quasi-resonant excitation, with laser light wavelength closer in energy to the excitonic transition. In this configuration for optical exci-

tation we used continuous wave tunable Ti:sapphire laser set to 709.3 nm (1.748 eV), close to the energy of exciton in monolayer WSe₂. Starting from highly negative exciton-photon detuning $\delta = -83$ meV angle-resolved reflectance spectra was measured in order to prove operation in strong coupling regime shown by anticrossing behavior between the polariton modes Fig. 5.30a. As the excitation laser energy is set quasi-resonantly with the exciton energy, photoluminescence signal is collected only for lower polariton branch. For σ^+ polarized excitation beam angle-resolved photoluminescence spectra detected in σ^+ and σ^- polarizations are plotted in Fig. 5.30b,c. In those spectra, the signal follows dispersion relations observed in reflectance, with higher intensity in co-polarized detection with the excitation beam (Fig. 5.30b). To compare emission intensity detected in both circular polarizations Fig. 5.30d presents the degree of circular polarization calculated from spectra Fig. 5.30b–c. The obtained degree of circular polarization is positive along the dispersion relation, but it is the highest at emission energies of around 1.72 eV and 1.67 eV, with a minimum in between around 1.685 eV.

For linearly polarized excitation beam (Fig. 5.30e–g), consistent with earlier highly nonresonant excitation, emission intensity detected in both circular polarizations is almost the same (Fig. 5.30e–g) resulting in vanishing DOCP in Fig. 5.30g.

As expected for σ^- polarized excitation beam the results resemble those obtained for opposite helicity of the incident light, but with stronger emission intensity in σ^- polarization resulting in negative DOCP. Similarly, like for σ^+ excitation, DOCP approaches zero as emitted photon energy is close to 1.685 eV.

To investigate how the observed valley polarization changes with exciton-photon detuning analogous measurements were repeated at different voltages applied to the piezoelectric stage which controls exciton-photon detuning δ . Fig. 5.31a–d presents angle-resolved photoluminescence spectra measured for increasing piezo voltage between 15 V and 60 V (corresponding to the detunings in the -68 meV to -26 meV). The emitted light follows exciton-polariton dispersion relations fitted from the corresponding reflectance spectra. The spectrally narrow feature in the emission intensity at the energy of 1.715 eV we attribute to the phonon assisted scattering of the incident laser light as the energy of phonon in monolayer WSe₂ is equal to 31.2 meV [217]. To analyze the overall emission intensity, the sum of the signals detected in σ^+ and σ^- polarizations along the dispersion relation for all of the investigated detunings are plotted in Fig. 5.31e. For the three highest detunings the emission intensity is the highest at the normal direction,

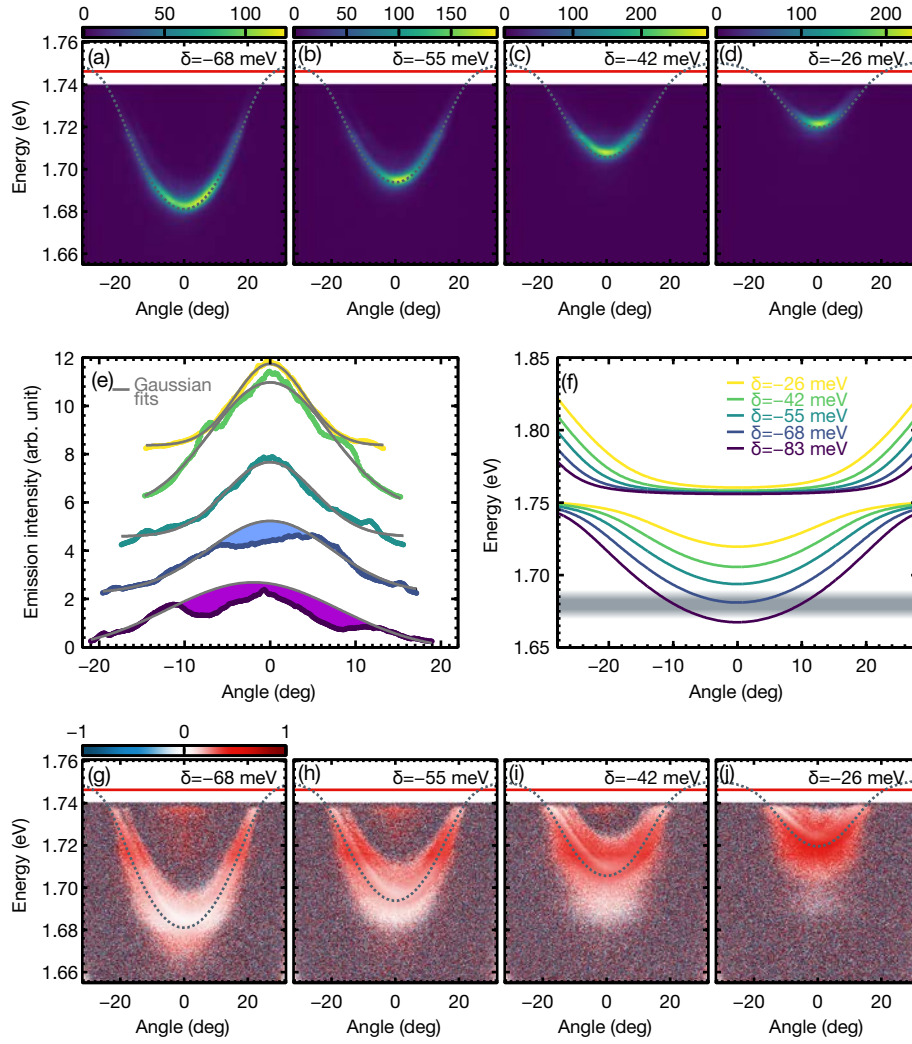


Fig. 5.31 | Valley polarization in WSe₂ exciton-polaritons at different detunings. (a)–(d) Angle-resolved photoluminescence detected in σ^+ for the same polarization of the incident pump at different voltages applied to the piezoelectric stage. (e) Total emission intensity as a function of the emission angle fitted with Gaussian relation. (f) Exciton-polariton dispersion relations at different exciton-photon detunings. (g) Angle-resolved DOCP spectra at different detunings for σ^+ polarized quasiresonant excitation.

with Gaussian-like dependence on the emission angle. For the two most negative detunings a deviation from this behavior can be observed. Fitting Gaussian relation to emission intensity dependence on the emission angle show significant discrepancy at normal incidence for the -68 meV detuning

(blue shaded region), and for the -86 meV detuning the highest discrepancy can be observed slightly below ± 10 deg of the emission angle (purple shaded region). Those discrepancies can be easier understood when plotting observed exciton-polariton dispersion relations at different detunings in a single graph, like in Fig. 5.31f. Those region corresponding to the deviations of the observed light emission and the Gaussian relation coincide with the region in which the dispersion relations crosses the resonance at around 1.68 eV, which is marked in Fig. 5.31f with a shaded line. Such behavior suggest existence of some additional resonant recombination channel which decreases total emission, but which does not have to influence the spin coherence of the system.

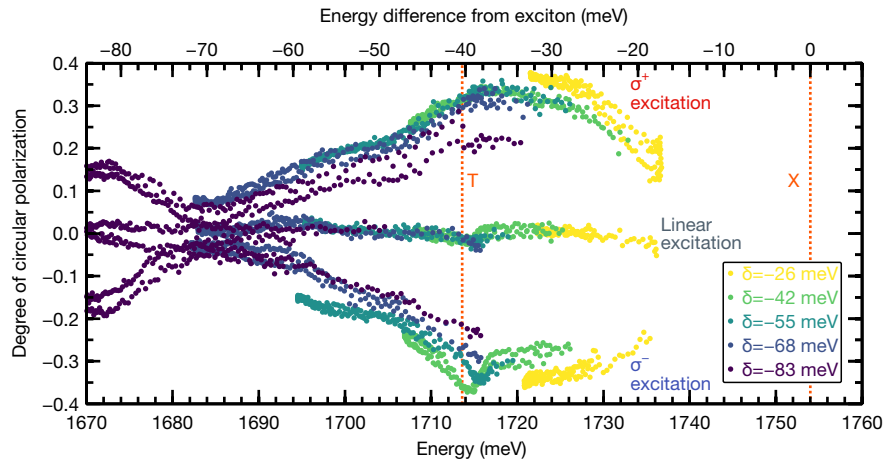


Fig. 5.32 | Energy dependence of valley polarization in WSe₂ exciton-polaritons. Degree of circular polarization as a function of emitted photon energy for σ^+ , σ^- and linearly polarized quasiresonant excitation.

DOCP spectra corresponding to Fig. 5.31a–d are plotted in Fig. 5.31g–j. For the highest detuning the emitted light is in significant degree co-polarized with excitation beam resulting in high positive DOCP. As the detuning becomes more negative, the highest values of the DOCP can be observed at high emission angles, with almost vanishing DOCP at normal emission angle for the most negative detuning -68 meV (Fig. 5.31g).

The easiest way to combine all results on valley polarization of the emitted light is to plot observed DOCP as a function of the emitted photon energy, as shown in Fig. 5.31. The plot combines data collected for all five investigated exciton-photon detunings at three excitation polarizations.

For the linearly polarized excitation beam observed DOCP of the emit-

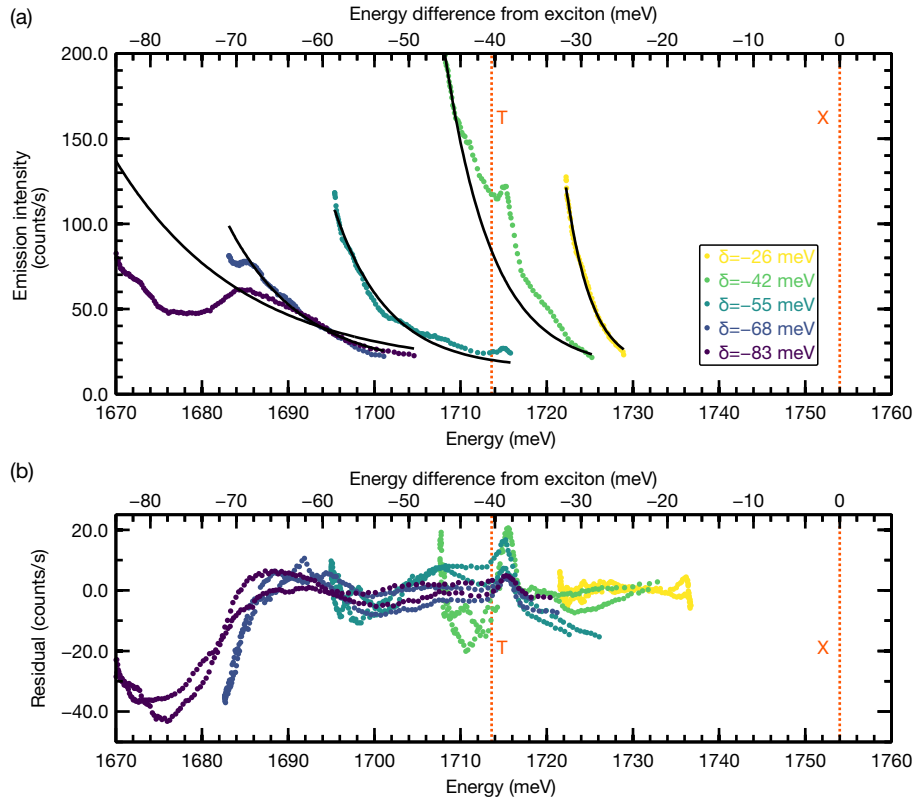


Fig. 5.33 | Polariton emission intensity (a) Emission intensity depending on the emission energy. Boltzmann distribution fits are marked with black lines with temperatures equal to: 171 K, 100 K, 72 K, 64 K, and 35 K, from the most negative detuning. (b) Difference between Boltzmann fit and the observed experimentally emission intensity.

ted light remains close to zero regardless on the detuning.

For the circularly polarized excitation, the observed polaritons DOCP is nonzero and depends on the energy of the emitted photons. Dependence on the energy is similar for both circularly polarized excitation polarizations, but the sign of the observed DOCP depends on the helicity of the pump. The highest retention of the incident polarization (highest module of DOCP) is observed at around 1.72 eV, corresponding to the energy 35 meV below the excitonic transition in the investigated monolayer WSe₂.

This maximum coincides in energy with the trion (charged exciton) resonance visible in the photoluminescence spectra, marked with dashed orange line.

Such an observation is also consistent with reported in the literature

time-resolved photoluminescence studies of bare WSe₂ monolayers, showing much slower valley depolarization time for charged excitons [218]. In the time-probe experiments for the neutral exciton the valley polarization decay time for monolayer WSe₂ was reported to be equal to 6 ps [219], which is comparable with the valley polarization time of intervalley trion of 4 ps with energy 28 meV below the neutral exciton [220]. However, in the same work for the intravalley trion located 35 meV below the excitonic transition the valley polarization time was observed to be significantly longer than 25 ps, which by resonant scattering may enhance depolarization time of the polariton state thus increase the value of the observed DOCP.

An analogous relative high value of the DOCP at the energy of trions was also observed in a similar system incorporating MoSe₂ monolayers inside a tunable microcavity cavity in Ref. [176].

For lower energies of the emitted photons the DOCP decreases reaching a local minimum at the energy of around 1.68 meV (70 meV below the excitonic resonance).

Based on the experiments reported in the literature with the detection set in-plane of a monolayer flake [221], coupling with surface plasmons [222] or brightening with magnetic field applied in Voigt configuration [223], the dark exciton resonance can be observed at energy between 40 meV and 46 meV below the excitonic resonance. Independent studies of monolayer WSe₂ subjected to high magnetic field in Voigt configuration report emergence in the photoluminescence spectra of two lines corresponding to dark exciton states at the energies 47 meV and 61 meV below the neutral exciton line [224].

To summarize, the decrease in the valley polarization observed at about 1.687 eV can be attributed to resonant scattering between the photon-coupled polarized bright excitons and other excitonic transitions such as the dark trion state [223] or some localized states. At this resonance we observe the decrease of the total emission intensity, shown in Fig. 5.31, which is expected for such a scattering mechanism. The phonon-assisted processes cannot be also excluded [217]. Fig. 5.33 illustrates the total emission intensity extracted from dispersion curves. The Maxwell-Boltzmann (MB) distribution was fitted to the intensity-energy relationship. The occupancy of polariton modes is expected to follow the MB relationship only if the relaxation along a lower polariton branch with acoustic phonons is active and no additional channel that could lead to the loss (resonant scattering) or increase (LO phonon assisted transitions) of particles is present. We do not observe any bottleneck effect, which supports the statement that the relaxation with acoustic phonons is efficient and the polariton gas can be thermal-

lized. We observe that the polariton gas can easily relax to the bottom of the dispersion curve for detuning -26 meV and -55 meV, as the polariton population follows the MB relationship very well. If polaritons encounter any of the possible resonant channels (detunings -42 meV, -68 meV, -83 meV), the system cannot be easily described by MB distribution. The highest deviation from the expected population is observed in the energy range from 1.675 eV 1.678 eV, where the increase of the DOCP is also observed. The effective temperature of polariton gas extracted from fitting MB to the polariton population depends on detuning. The temperature is the highest for large negative detunings where the polariton lifetime is the shortest (approx. 0.4 ps) and polaritons contain a low exciton fraction which results in long relaxation times. The polariton lifetime is generally much shorter than the relaxation time towards the ground state by phonon interaction.

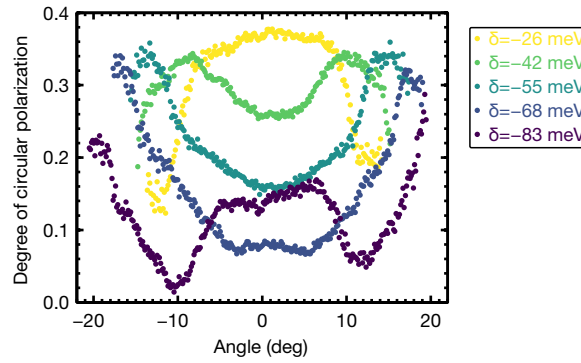


Fig. 5.34 | Angular dependence of valley polarization. Observed DOCP of the emitted light at different exciton-photon detunings as a function of the emission angle. Data obtained for σ^+ polarized quasi-resonant excitation.

It is also important to mention, that in the obtained results no evident dependence on the emission angle was observed. In the literature, the change of the degree of circular polarization was assigned as an effect of TE-TM splitting arising for emitted photons with increasing wave vector e.g. in Ref. [175]. In our case we plot DOCP of the polaritons excited with σ^+ polarized beam as a function of the emission angle like in Fig. 5.34, where DOCP undoubtedly changes with the emission angle. Nevertheless there is no clear dependence on the emission angle between measurements at different detunings, which due to quasi-resonant excitation all remain in the significantly negative regime. This is in contrast with the Fig. 5.32 in which points obtained at the same emission energy show similar values of DOCP regardless on the exciton-photon detuning. Such behavior suggests that the

DOCP observed in this work is rather an effect of wave-vector independent processes.

5.6 Exciton-polaritons with TMDCs thin layers: summary

In summary, this chapter presented possibility to control over the spin state of the cavity photons through strong coupling with valley polarized excitons in WSe_2 monolayer.

At first, based on transfer matrix simulations we proposed a realistic dielectric approach to construct a tunable microcavity, which can incorporate WSe_2 monolayers and operate at a liquid helium temperatures.

Second, the designed structure was successfully used to demonstrate strong coupling regime with clear anticrossing behavior between lower and upper polariton branches. Additionally strong coupling regime was achieved not only for monolayer WSe_2 , but also for thicker up to 6 ML flakes which were not earlier investigated towards coupling with microcavity field due to their indirect bandgap.

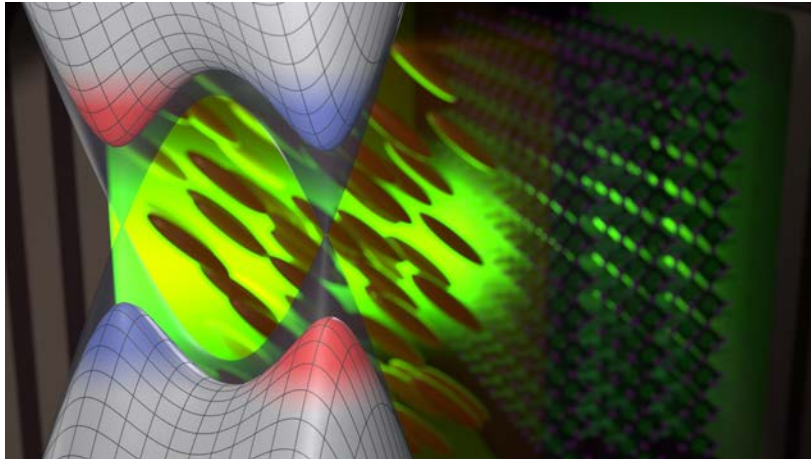
Third, the results in the strong coupling condition were in a good agreement with theoretical predictions based on the transfer matrix method. What is especially important, the analysis of the reflectance spectra of thin WSe_2 flakes deposited on dielectric flakes allowed for the extraction of optical properties of excitonic transition in this material. Obtained properties were then used to predict exciton-photon coupling strengths in fully assembled microcavities, which showed good agreement with the experimental results.

At the end we showed that spin of the exciton polaritons in monolayer WSe_2 were controlled with the polarization of the excitation laser beam due to valley polarization in TMDCs. Using tunable microcavity we showed that the maximum valley polarization of the exciton polaritons can be observed at the energies close to the trion resonance in the monolayer WSe_2 .

Chapter 6

LC microcavities with 2D perovskite and nonzero Berry curvature

Results described in a manuscript currently under review:



- [225] K. Lempicka-Mirek, M. Król et al., Electrically tunable Berry curvature and strong light-matter coupling in birefringent perovskite microcavities at room temperature. [arXiv:2203.05289](https://arxiv.org/abs/2203.05289) (2022).

In the previous chapter it was described how an excitonic transition within a material can be strongly coupled to a cavity mode resonance to form a mixed light-matter state. Especially that as a result of such mixing spin properties of the exciton can be passed to the final coupled state. In the case of investigated single WSe₂ flakes coupling strength was enough to obtain strong coupling condition but only at low temperatures. Within the same system operating at room temperature the decreasing exciton oscillator strength and increasing linewidth did not allowed for observation of the strong coupling regime in the investigated system.

In this chapter another material system exhibiting robust excitonic transition at room temperature will be used to demonstrate strong coupling regime in microcavities filled with liquid crystal. In addition by introducing diagonal-antidiagonal splitting by nonparallel rubbing layers allowed for engineering of nontrivial topology of the photonic modes exhibiting locally nonzero Berry curvature.

The first section (Sec. 6.1) of this chapter will be devoted to a brief summary of optical properties of lead halide perovskites based on review papers [226] and especially their applications in polaritonics [227, 228].

The second section (Sec. 6.2) presents experimental results on luminescence of 2D perovskite at room temperature combined with its refractive index extracted from the ellipsometry.

In the third section (Sec. 6.3) this refractive index will be used in Berreman matrix simulations to demonstrate possibility of strong light-matter coupling in the system of liquid crystal microcavity incorporating 2D perovskite. Such simulations will be compared with experimental angle-resolved photoluminescence spectra.

The final section (Sec. 6.4) will be devoted to similar system of LC microcavity with 2D perovskite in which due to intentional twisted nematic orientation of the LC resulting polariton bands show locally nonzero Berry curvature. Analytical model will be compared with Berreman matrix simulations and experimental results demonstrating tunability of the polaritonic Berry curvature with applied external electric field.

6.1 2D organic-inorganic perovskites

Metal halide perovskites combine multiple extraordinary properties which make them uniquely suitable for both light emitting and harvesting applications. Their strong and broadband absorption [229] accompanied by long carrier propagation length [230] makes them especially promising mate-

rial for photovoltaics. Their ease of preparation at room temperature with wet chemistry methods is possible thanks to their specific resilience against influence of the defects [231]. Some factors of their outstanding efficiency can be explained by polaronic effects in a soft, prone to deformation lattice [232].

On the other hand perovskies show also multiple additional characteristics desired for photoemitting applications [233]. Depending on the composition the emission spectra can be tuned over the visible range retaining narrow emission linewidths below 45 nm [234], which is especially valuable for color display technology.

Perovskite light emitting diodes can be obtained by printing [235], operate on flexible substrates [236] and in state of the art devices reach high peak external quantum efficiency exceeding 20% [237].

Despite remarkable achievements in the laboratory environment before widespread commercialization of the perovskites based photovoltaic some significant issues like limited long term stability and production scalability [238].

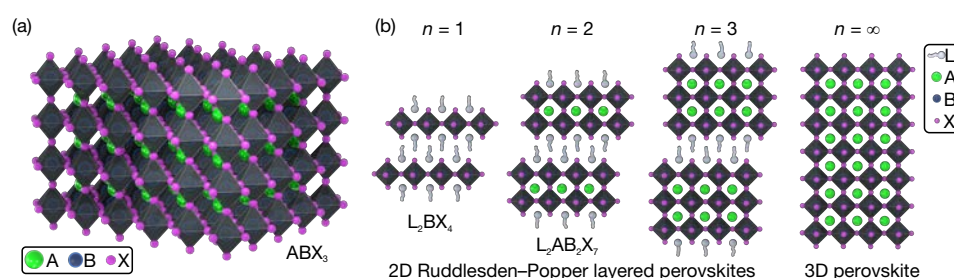


Fig. 6.1 | Perovskite crystallographic structure. (a) 3D perovskite crystallographic structure with general formula ABX₃. (b) 2D Ruddlesden-Popper layered perovskite with general formula L₂A_{n-1}B_nX_{3n+1} for increasing number n , which for $n \rightarrow \infty$ is equivalent to 3D perovskite structure.

Bulk 3D perovskites have a general formula ABX₃ and a scheme of crystallographic structure of those materials is shown in Fig. 6.1a. For lead halide perovskites B stands for Pb²⁺ ion and X is a halide. A stands for cation, which for inorganic perovskites could be a metallic ion such as Cs²⁺ or for so called hybrid perovskites small to mid-size organic ion such as e.g. methylammonium CH₃NH₃⁺ or formamidinium NH₂CHNH₂⁺. The crystallographic structure consists of octahedral [BX₆]⁴⁻ with A cation surrounded by 4 octahedra. The 3D perovskite structure is stable only for specific proportions of A, B, X ionic effective radii, which can be compared using empirical

Goldschmidt tolerance factor.

Especially when the cation is a bigger organic ion more stable crystallographic form are 2D Ruddlesden–Popper layered hybrid perovskites, schematically shown in Fig. 6.1b. They have general formula $L_2A_{n-1}B_nX_{3n+1}$, where n is an integer. The structure consists of n layers of octahedral, separated by the organic cations L^+ . In the limit of $n \rightarrow \infty$ the structure is equivalent to 3D perovskite. Contrary, for small n such Ruddlesden–Popper perovskite phases consists of stack of alternating high dielectric constant inorganic perovskite layers separated by lower dielectric constant organic layers, which can be viewed as an assembly of 3D perovskites quantum wells.

Specifically investigated in this work perovskites based on lead iodine and phenylethyloammonium iodine $C_6H_5C_2H_4NH_3I$ crystallizes in a 2D Ruddlesden–Popper phase with $n = 1$ with chemical formula $(C_6H_5C_2H_4NH_3)_2PbI_4$.

One of the biggest advantages of perovskites is the ability to engineer the optical bandgap, e.g. in $CsPbX_3$ change of the halide X allow to change the emission wavelength in whole visible range spanning 400–700 nm [234], which can be additionally tuned by quantum confinement in 2D phases [239].

Thin perovskite layers are easy to obtain by spin-coating technique in which stoichiometric solution of the lead halide and salt is spin-coated on the surface of the substrate and high quality perovskite grows during evaporation of the solvent.

Different other techniques can be used to obtain perovskites in various forms. Nanoplatelets or nanocrystals can be obtained using wet chemistry methods [240]. Hybrid 2D perovskite nanoplatelets can be also obtained using vapour-phase van der Waals epitaxy [241]. Moreover 2D perovskite crystals can be exfoliated up to the monolayer limit, with the interlayer spacing of 1.6 nm [242].

The question of a value of exciton binding energy in bulk 3D perovskites is a subject of still ongoing scientific debate, with different methods leading to contradictory results [228]. However in 2D perovskites thanks to significant quantum confinement reported values exceeding 400 meV assures they stability at room temperature [243].

The first demonstration of strong coupling regime between a perovskite exciton and a cavity mode was using so-called distributed feedback microcavities, with observed coupling strength exceeding 100 meV at room temperature [244] followed a decade later with 200 meV Rabi splitting observed in a planar metallic-dielectric microcavity [245]. In both of those cases hybrid phenylethyllummonium lead iodide was used.

The next milestone in a form of polariton lasing at room temperature was demonstrated using inorganic CsPbCl_3 grown by a vapour-phase synthesis [246] followed by analogous behavior in CsPbCl_3 [247]. Such non-equilibrium polariton condensates exhibit quantization effects in polariton lattices [248, 247] can propagate over macroscopic distances up to $60\text{ }\mu\text{m}$ [249], allows for construction of ultrafast switches [250] and undergo parametric scattering [251]. High optical nonlinearity of the perovskites in optical cavity were successfully used to construct an optical parametric oscillator (OPO) [252]. High crystallographic quality of perovskites obtained in this way is evidenced in the linear regime in HV splitting of the modes when such perovskites are placed inside a cavity, what allowed for observation of exceptional points [43]. Recent advancements allowed for observation of electrically pumped polariton emission [253].

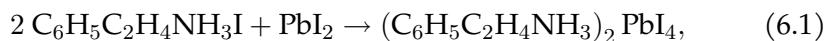
At the same time hybrid perovskites with inorganic cations are also characterized with high exciton oscillator strength that allows for observation of their strong coupling even with Bragg modes of the cavity [254]. They simultaneously can show sufficient crystallographic quality to observe cavity effects within bare monocrystal [255], and polarization splitting of the polariton modes, proving their optical birefringence [37, 38]. Thus although high optical nonlinearities at ambient conditions [256], the regime of polariton condensation was up to now obtained only at low temperatures [257].

6.2 Optical properties of 2D PEPI perovskite

All of the experimentally investigated perovskites within this study were synthesized by Karolina Lempicka-Mirek who perfected the preparation process based on initial experience of Adam Wincukiewicz.

Thin layers of polycrystalline 2D PEPI perovskites were prepared using spin-coating method directly on the surface of dielectric mirrors.

Following reaction:



phenethylammonium iodide PEAI ($\text{C}_6\text{H}_5\text{C}_2\text{H}_4\text{NH}_3\text{I}$) was mixed with lead iodide (PbI_2) in stoichiometric amount 2:1 in mole and dissolved in *N,N*-dimethylformamide (DMF) with concentration of 10%. Prepared solution was stirred for several hours. The solvent containing PEAI and PbI_2 was then spin-coated on a dielectric mirror with speed of about 3000 rpm for 30 s. After the spin coating the substrate is placed on a hot plate with the

temperature of 100 °C for about 60 s. A polycrystalline 2D perovskite layer is obtained upon solvent evaporation. To avoid exposition on air and moisture the whole procedure was performed inside a glove box under inert gas atmosphere.

For the investigated perovskite, for which broad area homogeneous layers can be obtained via spin coating method the refractive index can be obtained by the ellipsometric measurement.

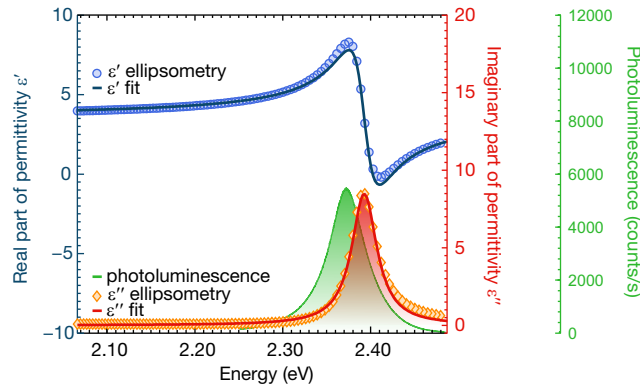


Fig. 6.2 | Dielectric function of PEPI perovskite. Obtained by ellipsometric measurement real and imaginary part of the dielectric function of thin polycrystalline PEPI layer. Results are fitted with Lorentzian model (solid lines).

Fig. 6.2 presents modeled dielectric function of PEPI perovskite based on the ellipsometric measurement performed and further on analyzed by dr. Tomasz Stefaniuk. Results of the analysis separated into real (ϵ') and imaginary part (ϵ'') of the dielectric function $\epsilon(E)$ are marked by circles in Fig. 6.2. The real part of the dielectric function exhibit characteristic dispersion shape at around 2.4 eV, which coincides with a peak in the absorption of the perovskite visible in the imaginary part of $\epsilon(E)$.

The excitonic resonance in PEPI can be also observable in photoluminescence. PEPI exhibit very intense green light emission when excited nonresonantly with blue laser. Fig. 6.2 green solid line marks photoluminescence spectrum obtained for illumination of PEPI layer spin coated on top of $\text{SiO}_2/\text{TiO}_2$ DBR with 405 nm (3.06 eV) continuous wave laser at 250 nW mean power. The emission spectra consists of a single peak at 2.37 eV. The peak is slightly asymmetric as its higher energy slope overlaps with the high absorption of the perovskite.

Similarly to the approach presented for the WSe_2 flakes [in Eq. (5.1)] the dielectric function of the material with excitonic resonance can be described

by the Lorentz model:

$$\varepsilon(E) = \varepsilon'(E) + i\varepsilon''(E) = \varepsilon_0 - \frac{fe^2}{E_{ex}^2 - E^2 - i\gamma_{ex}E}. \quad (6.2)$$

Fitting of Eq. (6.2) to the ellipsometric results gives very good agreement between the experimental data and the model with fitting parameters: $\varepsilon_0 = 3.536$, $f = 0.708 \text{ eV}^2$, $\gamma_{ex} = 34.9 \text{ meV}$ and $E_{ex} = 2.3928 \text{ eV}$ (518.15 nm). The fitted model is marked in Fig. 6.2 by solid blue and red lines.

Comparing fitted parameters with values obtained for WSe₂ monolayers shows lower exciton oscillator strength ($f \approx 0.7 \text{ eV}^2$ for the perovskite with $f \approx 1 \text{ eV}^2$ for WSe₂ layers). This is not a significantly obstacle to obtain the strong coupling regime, as the thickness of the perovskite layer at the order of 50 nm is significantly thicker than thin TMDC flakes (0.65 nm for the monolayer). Additionally as expected from the material investigated at room temperature the linewidth is significantly higher with $\gamma_{ex} = 34.9 \text{ meV}$ for the 2D perovskite.

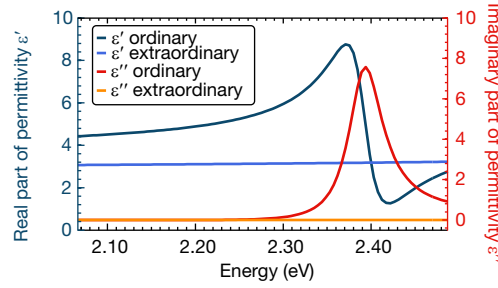


Fig. 6.3 | Dielectric function of PEPI perovskite. Obtained by ellipsometric measurement real and imaginary part of the dielectric function in ordinary and extraordinary (z) directions.

It is important to note that results of ellipsometric measurements presented in Fig. 6.2 were obtained with assumption that the perovskite layer is optically isotropic. This can be expected from polycrystalline material obtained by a spin coating technique. The more precise analysis especially based on comparison of results obtained for PEPI layer deposited on silicon wafer with specially selected interference SiO₂ layer and PEPI deposited on transparent fused silica substrate allowed to show anisotropy of the 2D perovskite in the growth z direction perpendicular to the substrate plane. Such results of the dielectric function are shown in Fig. 6.3. Observed optical anisotropy confirms that despite the polycrystalline form of the spin coated perovskite material 2D character of the layers is preserved.

As the observed anisotropy of the perovskite layer is in direction per-

pendicular to the substrate plane, its contribution to the simulations in the structure of LC cavity is not significant. For further analysis optical properties based on Fig. 6.2 will be used, as they are much better described by the Lorentz model.

6.3 Strong coupling regime in LC cavity

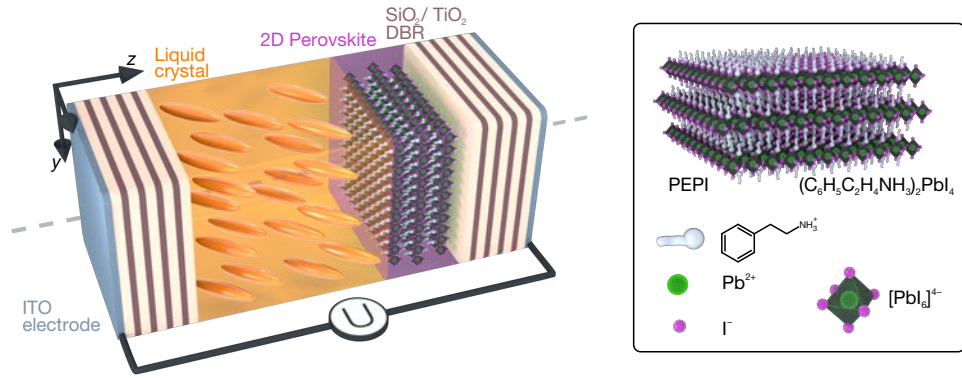


Fig. 6.4 | LC cavity with perovskite. Scheme of an optical cavity filled with liquid crystal incorporating 2D PEPI perovskite.

For known dielectric function of the perovskite the Berreman matrix method can be used to simulate optical properties of the cavity filled with LC and incorporating perovskite layer. Initial simulations were performed for thin 20 nm layer of the perovskite placed directly on top of one of the DBRs finished with lower refractive index material (SiO_2).

Fig. 6.5 presents angle resolved reflectance spectra simulated by Berreman method for cavity with extraordinary direction of LC along x axis (horizontal, H direction), corresponding to the experiment without external voltage.

Calculations were performed for precisely selected thicknesses of LC resulting in zero energy detuning between the excitonic transition in the perovskite ($E_{ex} = 2.3928$ eV from fitting dielectric function with Lorentz model) and the photonic mode at normal incidence. Energy of the photonic mode in absence of the coupling with the excitonic transition can be obtained by replacing the perovskite layer with artificial layer characterized by dielectric constant ϵ_0 . This corresponds with the Lorentz model with exciton oscillator strength $f = 0$. Such approach introduces thus an form of inactive cavity without coupling with the exciton. Simulated angle-resolved reflectance spectra for increasing thicknesses of the cavity are plotted in Fig. 6.5a–e. On

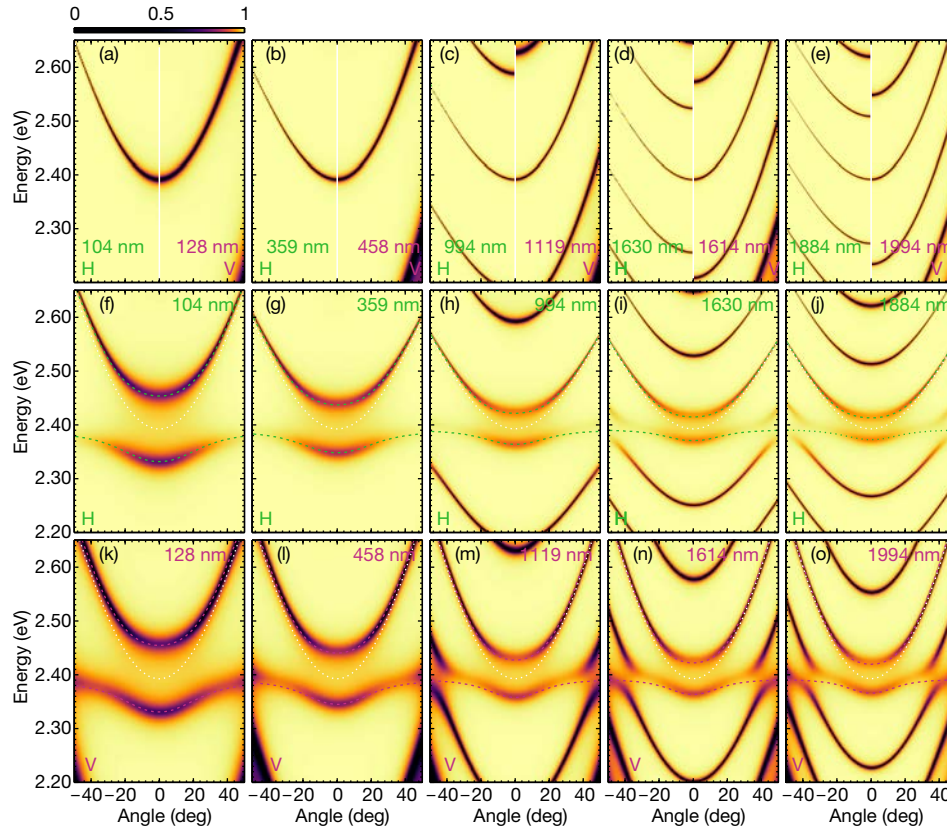


Fig. 6.5 | Strong coupling regime in LC microcavity with 2D perovskite. Simulated angle-resolved reflectance spectra. (a)–(e) Inactive perovskite layer with increasing thicknesses of LC layer selected to obtain zero exciton-photon detuning. Active perovskite layer: (f)–(j) for horizontally (H) polarized incident light, (k)–(o) for vertically (V) polarized incident light.

each of the panels the left hand side corresponds to simulations with horizontally (H) polarized light and the right hand side to the vertical (V) linear polarization of the incident light. To obtain the same energy of the cavity mode resonance for both those polarizations the thickness of the LC layer is different for different polarizations.

For the thinnest $\lambda_0/2n$ cavity (Fig. 6.5a) the dispersion relation in both polarizations is similar. As the thickness of the LC layer increases difference between both linear polarizations can be observed in the effective mass of the modes, lower for the H polarized light which corresponds to the higher effective refractive index of the LC layer (Fig. 6.5b). As the thickness of the cavity increases further to around $1 \mu\text{m}$ in Fig. 6.5c, subsequent cavity mo-

des start to appear in the investigated spectral range. For around $1.6 \mu\text{m}$ cavity thickness three subsequent cavity modes are observable in both linear polarizations in Fig. 6.5d. The energy spread between the modes is different for both polarizations. As effective refractive index for H polarized modes is higher, the energy separation for H polarized modes is lower than for V polarization at comparable cavity widths.

Using thicknesses of the LC layers corresponding to zero exciton-photon detunings shown in Fig. 6.5b the same parameter can be used with proper perovskite layer taking into account the excitonic transition.

Starting from horizontally polarized mode with the thinnest LC layer thickness allowing to obtain zero exciton-photon detuning in Fig. 6.5f, the simulated spectrum clearly realizes strong coupling regime. The spectrum contains two anticrossing branches of lower and upper polariton with characteristic anticrossing behavior. For thicker cavity Fig. 6.5g the dispersion relation also consists of only two anticrossing polariton branches, but with lower energy separation than for the thinnest case. The energy distance between the lower and upper polariton modes decreases further as the cavity width approaches $1 \mu\text{m}$ in Fig. 6.5h, which also show additional photonic modes within investigated spectral range, but without evident influence on two modes closest to the exciton. In Fig. 6.5i,j for $1.6 \mu\text{m}$ and $1.9 \mu\text{m}$ cavity width the dispersion relation significantly changes. As the observation angle increases the excitonic transition couples not only with the photonic mode at zero exciton-photon detuning but also with the one with lower mode number. As dispersion of this lower energy mode approaches the exciton energy for high observation angle also revealing strong coupling regime with additional anticrossing behavior.

Depending on the cavity width number of the cavity photonic modes which couple with the exciton changes. In order to obtain consistent description of the coupling strength with the LC layer thickness the first analysis will be based on 2×2 coupled oscillator Hamiltonian:

$$\hat{H}_{\text{H/V}} = \begin{pmatrix} E_{\text{Ex}} & \Omega_{\text{H/V}}/2 \\ \Omega_{\text{H/V}}/2 & E_{\text{Ph}}^{\text{H/V}}(\mathbf{k}) \end{pmatrix}. \quad (6.3)$$

For the investigated results the energy of the excitonic transition is known from the dielectric function ($E_{\text{ex}} = 2.3928 \text{ eV}$). Additionally the dispersion relation of the bare cavity mode $E_{\text{Ph}}^{\text{H/V}}(\mathbf{k})$ can be extracted from the corresponding spectra for inactive perovskite layer Fig. 6.5a–e. The only parameter left in Eq. (6.3) is exciton-photon coupling strength Ω_{H} . As the 2×2 model given by Eq. (6.3) takes into account only single cavity mode (espe-

cially resonant with the excitonic transition at normal incidence) fitting of the coupling strength can be restricted to only $k = 0$ case as the influence of the subsequent cavity mode appears only at high incidence angles.

The green dashed lines in Fig. 6.5f presents such approach in which exciton coupling strength Ω_H is determined based solely on the energy of the modes at normal incidence and then is plotted for all investigated detection angles based on the dispersion relation of the bare cavity mode (marked with white dotted lines) taken from Fig. 6.5a. Obtained exciton-polariton dispersion relations at $\Omega_H = 121.9$ meV are perfect match with simulated reflectance spectra in Fig. 6.5f. In Fig. 6.5g the coupling strength decreases to 89.1 meV with a great agreement between the model and simulated reflectance spectra. The disagreement between those two approaches appears in Fig. 6.5h for $1\ \mu\text{m}$ cavity width. At the high observation angles the energies of the lower polariton branch in simulated reflectance is higher than prediction of the 2×2 model, which still provides great description of the polariton modes at normal incidence at coupling strength of 57.5 meV. The same situation continues for thicker cavities (Fig. 6.5i,j) with very good agreement at $k = 0$, for which this model concentrates, and clear deviations when additional cavity modes approaches at high incidence angles. At $1.6\ \mu\text{m}$ ($1.9\ \mu\text{m}$) fitted coupling strength is equal to 44.0 meV (40.4 meV).

The same analysis can be performed for the simulations at vertical polarization of the incident light in Fig. 6.5k-o. Qualitatively simulated reflectance spectra at this polarization are similar to those for perpendicular horizontal polarization. For the lowest cavity widths (Fig. 6.5k,l) the reflectance spectra in the investigated energy range shows only two anticrossing lower and upper polariton branches. In wider cavities (Fig. 6.5m-o) the influence of the subsequent cavity modes emerges resulting in more complicated dispersion relations of the polaritonic modes.

As previously determination of the coupling strength can be at first limited to 2×2 model concentrating on the energies at the vicinity of the normal incidence conditions. As like before the dispersion of the cavity mode is taken directly from the simulations of corresponding cavity with inactive perovskite layer. For the thinnest cavity in Fig. 6.5m such approach gives a great agreement in whole range of the investigated incidence angles with coupling strength $\Omega_V = 123.5$ meV. For the $0.5\ \mu\text{m}$ in Fig. 6.5l thick cavity the influence of the subsequent cavity modes is already observable in the divergence between the 2×2 model and the reflectance spectra. This discrepancies appear earlier than for the results for horizontally polarized light. This is contrary to the prediction than for higher effective refractive index the energy separation between the modes should be smaller, thus the sho-

uld earlier affect the dispersion relation. As visible in Fig. 6.5 another effect plays more significant role. Different effective refractive indexes affect also the effective mass of the cavity modes. As for the vertically polarized light the effective mass is significantly higher, the subsequent cavity modes earlier affect the dispersion of the polaritonic modes. Nevertheless for $\Omega_V = 95.1$ meV the model very well describes energy of the polaritonic modes at normal incidence. For the wider cavities Fig. 6.5m–o the discrepancies between the simplest 2×2 model and the simulated reflectance spectra become more significant, but both are in a good agreement regarding the energies at normal incidence with coupling strengths: 69.5 meV at 1.1 μm , 58.6 meV at 1.6 μm and 53.6 meV at 2.0 μm cavity width.

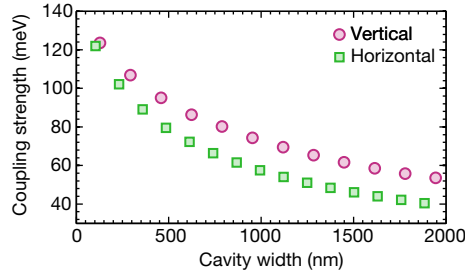


Fig. 6.6 | Strong coupling regime in LC microcavity with 2D perovskite. Exciton-photon coupling strength dependence on the cavity length for cavity modes with horizontal and vertical linear polarization.

The coupling strengths obtained in the same approach for any width of the cavity resulting in zero exciton photon detuning in 0–2 μm range are plotted in Fig. 6.6. As expected the exciton-photon coupling strength decreases for cavity modes with horizontal and vertical polarization. At the thinnest possible cavity (approx. 100 nm depending on the polarization) the coupling strength for both polarizations is similar, and at the order of 130 meV, which is significantly higher than the exciton linewidth of $\gamma_{ex} = 34.9$ meV. With the increasing cavity width the coupling strength decreases. Especially it decreases significantly faster for horizontal light polarization, which corresponds to the higher, extraordinary refractive index. E.g. at 1.12 μm (when perpendicularly polarized modes are closest to degeneracy at common cavity width) coupling strength in vertical polarization of $\Omega_V = 69.5$ meV where for the horizontal polarization $\Omega_H = 54.0$ meV, as for a fixed physical width the cavity optical widths vary significantly depending of the effective refractive index associated with a given polarization of the light.

Approach described above can be used to compare the coupling strength

for increasing thicknesses of the LC layer, but this simple 2×2 model is not enough to describe coupling with the subsequent cavity modes which clearly can be observed for the thicker LC layer widths. In order to describe theoretically observed dispersion relations multiple cavity modes have to be incorporated within the model.

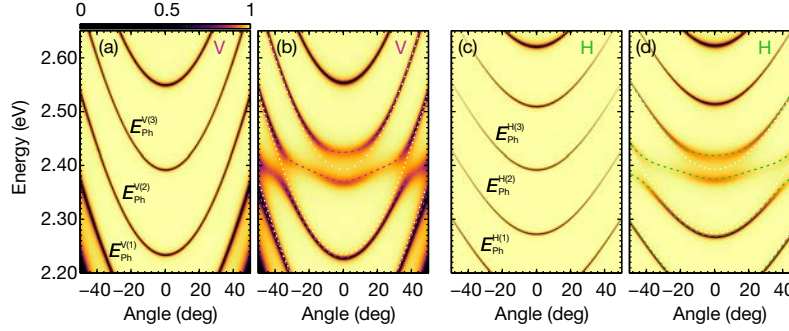


Fig. 6.7 | Strong coupling regime in LC microcavity with 2D perovskite. Simulated angle-resolved reflectance spectra: (a) for inactive perovskite layer observed in vertical polarization, (b) with active perovskite layer in vertical polarization. (c)–(d) corresponding spectra simulated for horizontally polarized light.

Simulated reflectance spectra for $2 \mu\text{m}$ thick LC cavity incorporating artificial inactive perovskite layer in vertically polarized light is presented in Fig. 6.7a. Apart from the cavity mode resonant with the energy of the exciton in 2D PEPI perovskite multiple additional cavity modes are observable within the investigated spectral range. Especially at around 30° observation angle second cavity mode with lower mode number becomes resonant with the exciton energy and at 50° mode with mode number smaller than two also approaches in energy. Those cavity resonances also could be incorporated within the coupled oscillator model in a form of 4×4 matrix:

$$\hat{H}_V = \begin{pmatrix} E_{\text{Ex}} & \Omega_{V(1)}/2 & \Omega_{V(2)}/2 & \Omega_{V(3)}/2 \\ \Omega_{V(1)}/2 & E_{\text{Ph}}^{V(1)}(\mathbf{k}) & 0 & 0 \\ \Omega_{V(2)}/2 & 0 & E_{\text{Ph}}^{V(2)}(\mathbf{k}) & 0 \\ \Omega_{V(3)}/2 & 0 & 0 & E_{\text{Ph}}^{V(3)}(\mathbf{k}) \end{pmatrix}. \quad (6.4)$$

It assumes that each of the subsequent cavity modes $E_{\text{Ph}}^{V(i)}(\mathbf{k})$ for $i = 1, 2, 3$ couples independently with the excitonic resonance E_{Ex} with the coupling strength $\Omega_{V(i)}$. Results of such model fitted were fitted to the simulated angle-resolved spectra for corresponding cavity incorporating perovskite layer with active excitonic resonance presented in Fig. 6.7b. During the fit-

ting procedure dispersion relations of the bare cavity resonances were taken directly from simulations with inactive perovskite layer in Fig. 6.7a, and plotted in Fig. 6.7b with white dashed lines. Results of the fits—marked with the purple dashed lines—perfectly follow simulated decrease of the reflectance in the spectra. Here the only fitting parameters are coupling strength which were equal to: $\Omega_{V(1)} = 60.1$ meV, $\Omega_{V(2)} = 62.0$ meV, $\Omega_{V(3)} = 54.6$ meV where resonance $V(3)$ is the closest to the excitonic transition at normal incidence.

The same approach can be employed for the perpendicular light polarization. Simulated angle-resolved reflectance map with inactive perovskite layer is presented in Fig. 6.7c. the cavity width is set to $1.88 \mu\text{m}$ to obtain degeneracy of the cavity mode with the excitonic transition energy. With analogous 4×4 model in matrix form:

$$\hat{H}_H = \begin{pmatrix} E_{\text{Ex}} & \Omega_{H(1)}/2 & \Omega_{H(2)}/2 & \Omega_{H(3)}/2 \\ \Omega_{H(1)}/2 & E_{\text{Ph}}^{H(1)}(\mathbf{k}) & 0 & 0 \\ \Omega_{H(2)}/2 & 0 & E_{\text{Ph}}^{H(2)}(\mathbf{k}) & 0 \\ \Omega_{H(3)}/2 & 0 & 0 & E_{\text{Ph}}^{H(3)}(\mathbf{k}) \end{pmatrix}, \quad (6.5)$$

the fitted dispersion relations follow the simulated reflectance spectra in Fig. 6.7d. Obtained coupling strengths: $\Omega_{H(1)} = 50.6$ meV $\Omega_{H(2)} = 48.9$ meV $\Omega_{H(3)} = 44.0$ meV are as shown previously for the simplest 2×2 model smaller in horizontal polarization, as the corresponding effective refractive index is higher.

Presented here analysis of the cavity modes in LC samples based on simulated reflectance spectra can be compared with experimental results. In order to investigate strong coupling regime at room temperature we studied LC cavity incorporating PEPI layer. The perovskite was spin coated to form approx. 50 nm thick polycrystalline layer on the top of a dielectric mirror with ITO electrode beneath. The optical cavity was formed between the second dielectric DBR with the space between filled with highly birefringent nematic liquid crystal. To ensure homogeneity of the LC before the final assembly the perovskite layer as well as the surface of the second DBR was covered with a polymer layer, which after structurization allowed for proper orientation of the LC.

Fig. 6.8a presents experimental results of angle-resolved photoluminescence spectra from such cavity. The spectra was collected upon non-resonant excitation with continuous wave laser with 405 nm wavelength (3.06 eV). The excitation power was set to $30 \mu\text{W}$ and in order to limit photo-degradation of the perovskite the excitation laser was passing through me-

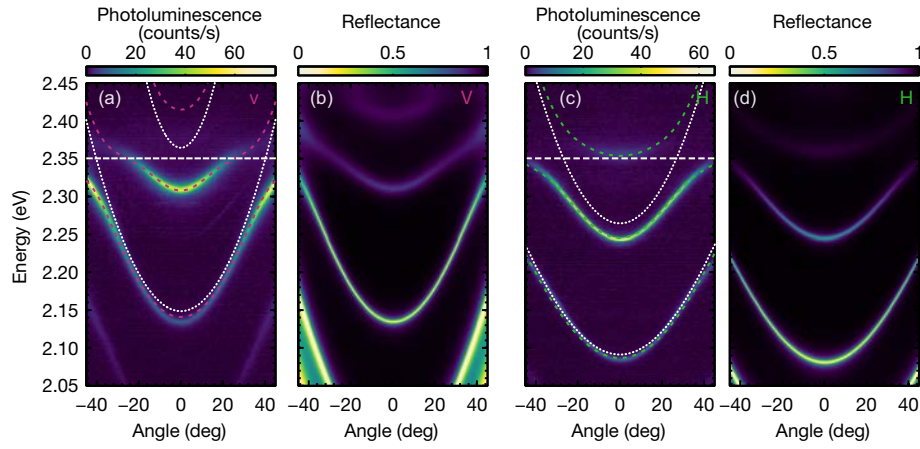


Fig. 6.8 | Strong coupling regime in LC microcavity with 2D perovskite.

(a) Experimental angle-resolved photoluminescence spectra from LC cavity incorporating PEPI layer detected in vertical polarization. (b) Corresponding reflectance spectra calculated by Berreman matrix model. (c) Photoluminescence spectra detected in linear horizontal polarization. (d) Corresponding reflectance spectra from Berreman matrix method.

chanical chopper set to 600 Hz frequency. The experiment was performed without external bias applied to the ITO electrodes.

The emission spectra in Fig. 6.8a contains two main branches with non-parabolic dispersion relations. The strongest emission comes from the highest energy branch with resonance at normal incidence at around 2.32 eV. As the energy of this mode approaches 2.4 eV the dispersion relation flattens. Similarly for the lower energy branch for the highest observation angles an anticrossing behavior is visible. At the observation angles above 30 deg and low energies below 2.15 eV traces of weak emission coming from the third mode is barely visible.

Analogous dispersion relation can be also obtained in Berreman matrix simulations. Fig. 6.8b presents results of the calculations for a structure consisting of two DBRs made of 5 pairs of $\text{SiO}_2/\text{TiO}_2$ layers centred for maximum reflectance at 530 nm separated by 120 nm of the PEPI perovskite and 1195 nm of LC. The thickness of the perovskite as well as this of LC was chosen to obtain the best match between experimentally observed dispersion relation and the simulations. Overall a good agreement between the two was achieved.

Two main branches, as well as third lower in the energy exhibit similar behavior in experimental results and simulations. Visibility of each branch

cannot be compared as in the experimental photoluminescence the occupation of each band depends on the emission properties of the perovskite whereas simulations show depth of the given mode in reflectance measurement. Additionally in the simulations a fourth mode at around 2.42 eV appears, which is not observed in the experiment probably due to high absorption of the perovskite at this spectral range.

To fit the experimental results with exciton-polariton dispersion relations the first step was to determine energy of bare cavity modes. Based on the simulations similar could be performed with inactive perovskite layer lacking the excitonic contribution to the dielectric function. Within such approach inactive cavity exhibit cavity modes resonances at $E_{\text{Ph}^0}^{V(1)} = 2.149$ eV and $E_{\text{Ph}^0}^{V(2)} = 2.364$ eV.

Those energies was used to fit the experimental dispersion relations with 3×3 coupled oscillator model in matrix form:

$$\hat{H}_V = \begin{pmatrix} E_{\text{Ex}} & \Omega_{V(1)}/2 & \Omega_{V(2)}/2 \\ \Omega_{V(1)}/2 & E_{\text{Ph}^0}^{V(1)} + \frac{\hbar^2}{2m_1^V} & 0 \\ \Omega_{V(2)}/2 & 0 & E_{\text{Ph}^0}^{V(2)} + \frac{\hbar^2}{2m_2^V} \end{pmatrix}. \quad (6.6)$$

Results of the fitting are plotted in Fig. 6.8a with purple dashed lines obtained with coupling strengths $\Omega_{V(1)} = 78.3$ meV and $\Omega_{V(2)} = 108.7$ meV with corresponding dispersions of bare cavity and excitonic modes with white lines. Here in order to obtain reasonable agreement with the experimental results both: effective masses of the bare cavity modes and perovskite exciton resonance energy were treated as fitting parameters. Overall fitted curves follow the emission signal quite well. The biggest discrepancy can be observed when comparing the highest polariton mode around 2.42 eV with reflectance simulations (as it is completely invisible in experimental emission). The curvature in the coupled oscillator model is visibly much steeper than is expected from the Berreman matrix simulations.

With the same approach to experimental results obtained for horizontal polarization of detected light was performed. Detected angle-resolved photoluminescence spectra for the same position on the sample is presented in Fig. 6.8c. Here the lowest energy branch is almost parabolic. However for the mode with higher energy at high emission angles the dispersion relation flattens with characteristic anticrossing behavior. Weak emission signal can be also observed from the next mode at around 2.37 eV. Energy dispersion of this mode is hard to evaluate as it starts to overlap with high absorption range of the perovskite.

All features of the bands visible in experiment can be also observed in simulated reflectance spectra in Fig. 6.8d. To obtain agreement the LC layer of total thickness of 1195 nm determined in vertical polarization here is divided into three regions: where only the middle one with 855 nm thickness assumes optical anisotropy of the LC. based on parameters from the simulations once again allows to determine energy of the bare cavity modes with calculations using artificial inactive perovskite. Here for fitting of experimental dispersion relations we use 3 subsequent photonic cavity modes with energies: $E_{\text{Ph}^0}^{\text{H}(1)} = 2.091 \text{ eV}$, $E_{\text{Ph}^0}^{\text{H}(2)} = 2.265 \text{ eV}$, $E_{\text{Ph}^0}^{\text{H}(3)} = 2.451 \text{ eV}$ incorporated into 4×4 coupled oscillator model:

$$\hat{H}_{\text{H}} = \begin{pmatrix} E_{\text{Ex}} & \Omega_{\text{H}(1)}/2 & \Omega_{\text{H}(2)}/2 & \Omega_{\text{H}(3)}/2 \\ \Omega_{\text{H}(1)}/2 & E_{\text{Ph}^0}^{\text{H}(1)} + \frac{\hbar^2}{2m_1^{\text{H}}} & 0 & 0 \\ \Omega_{\text{H}(2)}/2 & 0 & E_{\text{Ph}^0}^{\text{H}(2)} + \frac{\hbar^2}{2m_2^{\text{H}}} & 0 \\ \Omega_{\text{H}(3)}/2 & 0 & 0 & E_{\text{Ph}^0}^{\text{H}(3)} + \frac{\hbar^2}{2m_3^{\text{H}}} \end{pmatrix}. \quad (6.7)$$

When fixing the energy of the excitonic resonance at the same value as already determined for the perpendicular light polarization resulting coupling strength are equal to: $\Omega_{\text{H}(1)} = 67.2 \text{ meV}$, $\Omega_{\text{H}(2)} = 88.4 \text{ meV}$ and $\Omega_{\text{H}(3)} = 94.4 \text{ meV}$. As previously fitted dispersion relations very well describe dispersion relations observed in the emission from the cavity. Once again the only visible inconsistency can be observed when comparing the coupled oscillator model with significantly lower effective mass for the highest energy mode according to the Berreman matrix simulations.

In summary according to the Berreman matrix simulations based on dielectric function of PEPI perovskite it is possible to achieve strong coupling regime between exciton in the perovskite and the photonic modes in a microcavity filled with liquid crystal. For thin PEPI layer dispersion of exciton polaritons in the system is well described with coupled oscillator model. This predictions were tested experimentally. Observation of light emitted from the cavity showed nonparabolic dispersion of the modes within the cavity. Observed emission modes dependence on the observation angle was recreated in the Berreman method and described using coupled oscillator model with extracted coupling strengths at the order of 100 meV.

6.4 Engineering of nontrivial band geometry in LC cavities

The concept of a geometric phase: a phase shift dependent solely on the geometry of the path along which the system evolves spans over different areas of physics [258]. Phenomena such as the Aharonov-Bohm effect; Hannay angle; Pancharatnam, Zak phases can be interpreted as manifestations of different geometric phases [259].

One of the most general forms of a geometric phase is the so-called Berry phase. Originally conceived by Michael Berry, who discovered a gauge-invariant phase factor that arises when parameters of a Hamiltonian that describe a quantum system are adiabatically cycled around a closed path [260]. Later, analogous phase factor was extended to the case when the quantum system itself is cycled along a closed loop, rather than the parameters of a Hamiltonian [261].

For cyclic evolutions in parameter space, the accumulated Berry phase can be derived as an integral over the Berry curvature enclosed within the loop, similarly as the Aharonov-Bohm phase depends on magnetic flux.

Berry curvature directly influences the motion of the particles [262].

The Berry curvature also plays a crucial role in the topological classification of the materials. The integral of the Berry curvature over a whole band or Brillouin zone defines a topological invariant known as the Chern number [263].

In planar microcavity systems it has been predicted that for honeycomb lattice of micropillars in the presence of TE-TM spin orbit coupling and Zeeman splitting lead to photonic analogue of topological insulator with bands characterized by ± 2 Chern numbers [264]. This predictions have been demonstrated in experiment showing unidirectional transport in the edge states [32].

In spatially homogeneous cavities combined effects of the Zeeman effect, TE-TM and HV (XY) splitting, lead to two valleys in momentum space characterized with nonzero Berry curvature demonstrated by polarization of the bands [39, 38] as well as measurement of the resulting anomalous Hall drift [39]. Similarly, in the presence of optical activity, a nonzero Berry curvature can be observed without an external magnetic field [34, 38].

In this section, we demonstrate polaritonic bands exhibiting locally nonzero Berry curvature which can be tuned by external voltage within a system of LC cavity incorporating 2D perovskite.

Engineering of polaritonic band dispersion exhibiting locally nonzero Berry curvature will start from a remainder about dispersion of the cavity

modes around $(N + 1, N)$ resonance of the cavity modes within a cavity filled with liquid crystal. As described in Sec. 2.2.4 in this regime two closest cavity mode resonances can be described by the effective 2×2 Hamiltonian:

$$\hat{H}_{\text{RD}} = \left(E_0 + \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} \right) \hat{\sigma}_0 + \frac{1}{2} \Delta E_{\text{HV}} \hat{\sigma}_x + (\delta_x k_x^2 + \delta_y k_y^2) \hat{\sigma}_x - 2\alpha k_y \hat{\sigma}_z. \quad (6.8)$$

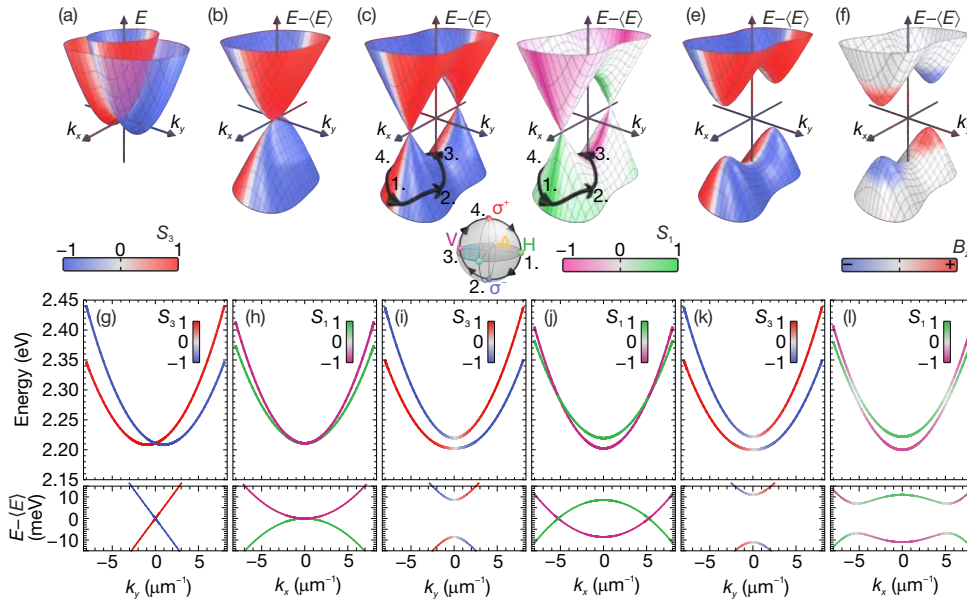


Fig. 6.9 | Cavity modes dispersion in analytical model. (a,b) Dispersion at Rashba-Dresselhauss resonance (that is, $\Delta E_{\text{HV}} = \delta = 0$). (c,d) Tunability of the LC system allows for detuning from R-D resonance by adding $\Delta E_{\text{HV}} > 0$ leading to two degeneracy points along k_x . (e,f) With additional splitting δ the degeneracy along k_x is lifted. Polarization of the modes is marked by color: red (blue) for circular σ^+ (σ^-) and green (purple) for linear along horizontal (vertical) direction.

The schematic dispersion relation resulting from Eq. (6.8) for zero HV detuning ΔE_{HV} are plotted in Fig. 6.9a, where S_3 parameter of the given bands is marked with a pseudocolor scale. Looking at the degeneracies between the bands is easier when the energy of each mode is subtracted by their mean $\langle E \rangle$, as shown in Fig. 6.9b, where single degeneracy point at $\mathbf{k} = 0$ is clearly visible. The cross sections along $k_x = 0$ and $k_y = 0$ for those dispersion relations (absolute energy as well as energy difference) is plotted in Fig. 6.9g,h, where color of the line marks S_1 or S_3 polarization compo-

nent. Along the direction of the k_y wave vector, the dispersion relation is dominated by the Rashba-Dresselhaus (RD) spin orbit coupling term $2\alpha k_y \hat{\sigma}_z$ resulting in circularly polarized modes crossing at $\mathbf{k} = 0$. Along the perpendicular wave vector direction (k_x) the RD term vanishes and the cocentric parabolic dispersion relations are split due to $\delta_x k_x^2 \hat{\sigma}_x$. As $\delta_x < 0$ mode with horizontal polarization has a lower energy than the vertically polarized one.

The ability to steer the LC director within the cavity by applying an external electric field allows for precise control over HV detuning ΔE_{HV} . When the HV detuning is positive, the dispersion relation exhibits two degeneracy points with characteristic Dirac cone-like dispersion relations around them (Fig. 6.9c,i,j). Along k_y (Fig. 6.9i) the bands never cross but exhibit a smooth change in their circular polarization helicity when passing through $k_y = 0$. Along k_x (Fig. 6.9j) the modes are strictly H/V polarized and cross when $k_x^{DP} = \sqrt{\frac{\Delta E_{HV}}{2\delta_x}}$.

We can trace the polarization state while encircling one of the Dirac points. Fig. 6.9c presents polarization state of the bands separated into S_3 (red/blue) and S_1 (green/purple) Stokes vector components. Starting at position 1. in momentum space at high positive k_x polarization state of the lower energy branch is horizontal, as S_1 component is positive and S_3 vanishes. At position 2. for high positive k_y the polarization state changes to σ^- . Moving further to the vicinity of $\mathbf{k} = 0$ at position 3. The polarization state returns to linear, but this time vertical, corresponding to the negative S_1 parameter. At position 4. for high negative k_y goes to circular σ^+ . Moving to the end from 4. to 1. which finishes the whole loop around the Dirac point in momentum space, the polarization state performed a full rotation around the meridian of the Poincaré sphere, which is an evidence that those Dirac points carry a topological charge.

It is important to point out the differences in properties of the Dirac points visible in $(N+1, N)$ from those reported already in the literature. In diabolical points observed in systems with TE-TM splitting and relatively small birefringence/XY splitting, as reported in [36, 37, 40], while encircling the DP polarization state exhibits full rotation within S_1 – S_2 . As a result, when additional splitting is provided in the remaining S_3 direction, the energy gap within the DP emerges, leading to a band structure exhibiting a nonzero Berry curvature. To obtain such a splitting in circular polarizations, one can apply an external magnetic field [39, 38] or use a medium exhibiting optical activity [34].

Here, as the polarization state rotates within the S_1 – S_3 plane, thus to open the energy gap at the Dirac points it is sufficient to introduce additio-

nal splitting distinguishing diagonal (D) and antidiagonal (A) polarized states. Such a splitting can be introduced e.g. by additional birefringent layer within the cavity oriented along diagonal direction. It can be included in the analytical model by diagonal-antidiagonal splitting ΔE_{DA} :

$$\begin{aligned} \hat{H}_{\text{RD-DA}} = & \left(E_0 + \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} \right) \hat{\sigma}_0 + \\ & + \frac{1}{2} \Delta E_{\text{HV}} \hat{\sigma}_x + \frac{1}{2} \Delta E_{\text{DA}} \hat{\sigma}_y + (\delta_x k_x^2 + \delta_y k_y^2) \hat{\sigma}_x - 2\alpha k_y \hat{\sigma}_z. \end{aligned} \quad (6.9)$$

Predictions of this model are plotted as the energy subtracted by $\langle E \rangle$ in Fig. 6.9e with S_3 polarization of the modes marked with a pseudocolor scale. Here, the dispersion relation consists of two valleys in momentum space resulting from opening the energy gap at the position of the Dirac cones. At $k_x = 0$. Changes can be observed in perpendicular wave vector direction, shown in Fig. 6.9l. At the momentum position of the crossings without diagonal-antidiagonal splitting, now the modes anticross with continuous change of the polarization of the bands from horizontal to vertical or vice versa. Based on the polarization state given by the full Stokes vector $\mathbf{S}$ Berry curvature along the dispersion can be calculated, with results plotted in pseudocolor scale in Fig. 6.9f. As expected from system with time reversal symmetry—which requires that the integral over the whole band (equal to Chern number) to be equal to zero—valleys exhibit opposite values of the Berry curvature. Additionally, for the given valley lower and upper branches are characterized with opposite Berry curvature.

A straightforward approach to engineering such additional diagonal-antidiagonal splitting inside a cavity is to add an additional layer with anisotropy axis parallel to the cavity plane, but rotated by 45 degrees around z axis from the director of the LC layer. Such an approach was tested theoretically using the Berreman method, but fabrication of such scheme turned out to be challenging.

Similarly, an earlier theoretical proposal investigating Berry curvature in a system of coupled cavities filled with liquid crystal [265] turned out to be complex to fabricate in practice, as it requires suspending a very thin DBR in between two differently oriented LC regions.

Another easier-to-make proposition that was tested in the experiment of the introduction of diagonal-antidiagonal splitting ΔE_{DA} in a realistic system was to use the twisted nematic configuration of the LC layer. In all of the LC cavities described previously, the nematic liquid crystal was oriented by parallel orienting layers, which distinguished the direction along the same

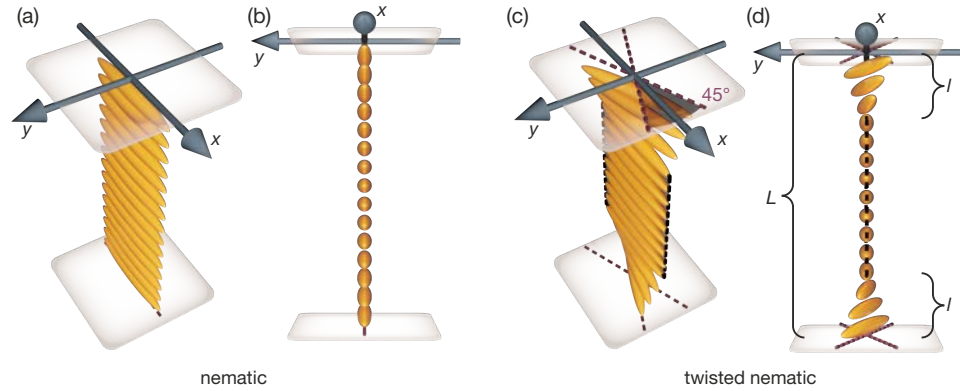


Fig. 6.10 | Nematic and twisted nematic LC configurations. (a)–(b) Nematic configuration at two different projections. (c)–(d) Twisted nematic configuration with 45 deg disorientation at two different projections.

x axis. As a result, in perfectly ordered LC all molecules are oriented in the same plane, with the long molecules oriented along the x axis, as shown in Fig. 6.10a–b presenting the same structure observed from two different perspectives.

Contrary to the twisted nematic configuration proposed here, the orienting layers are not aligned, but rotated one versus the other by 45 degrees around the z axis, as shown in Fig. 6.10c–d. The molecules of the liquid crystal closest to the orientation layers are expected to be oriented parallel to the rubbing orientation. As a result, within the cavity director orientation is expected to follow a rotation around the z axis while moving through the cavity between the first and the second DBR surface.

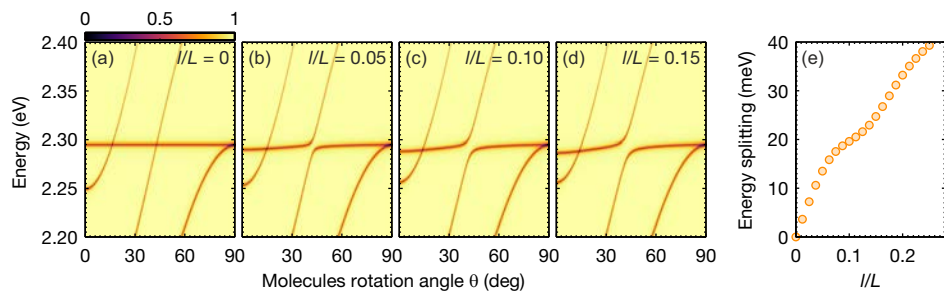


Fig. 6.11 | Voltage tuning in twisted nematic cavities. (a)–(e) Simulated normal incidence reflectance spectra for increasing the ratio of the twist distance l to the total width of the cavity L . (e) Calculated energy splitting in $(N + 1, N)$ regime for increasing l/L ratio.

To test if such twisted nematic can introduce additional splitting into $(N + 1, N)$ resonance Berreman matrix simulations were performed. The LC region of the cavity was separated into three regions, as shown in Fig. 6.10c–d. At both interfaces with the DBRs LC director close to the surfaces of the mirrors is oriented parallel to the corresponding orienting layer. With increasing distance from these interfaces, the LC director rotates around the z axis with 22.5 deg rotation at the distance l . Within the remaining central part of the cavity with thickness $L - 2l$ it is assumed that the LC director is homogeneous. Separation of the LC into such three layers was required in order to influence the value of the splitting obtained through simulations.

Under an external electric field, it is assumed that LC molecules would undergo additional rotation θ towards the z axis. Fig. 6.11 presents theoretical investigations of such a voltage tuning of a twisted nematic cell for increasing distance l for a fixed total cavity length of $L = 1362$ nm. When distance $l = 0$ nm the cavity the investigated structure constitutes a typical LC cavity with homogeneous LC orientation, thus simulated reflectance at normal incidence shows crossings when modes of the same parity [$(N + 2, N)$ regime at around 20 deg] or of the opposite parity [$(N + 1, N)$ regime at around 45 deg] approach each other with rotation of the LC director.

In a proper twisted nematic configuration in which reconfiguration, analogous simulations show the emergence of additional splitting resulting in anti-crossing behavior in the $(N + 1, N)$ regime. Starting from the simulations in which the reorientation of the LC director occurs at small length when $l = 68$ nm (which corresponds to the ratio to the total cavity length $l/L = 0.05$) presented in Fig. 6.11b reflectance spectrum at normal incidence clearly shows anticrossing behavior between the modes in the cavity at a voltage induced reorientation of around 45 deg. For increasing the twist distance l the value of the splitting visibly increases, as shown in reflectance spectra in Fig. 6.11c,d.

The energy splitting between the modes in $(N + 1, N)$ extracted from the simulations is presented in Fig. 6.11e, showing increasing of the splitting with l/L distance ratio. For $l/L = 0.1$ ratio splitting reaches 20 meV and increases to approximately 30 meV at $l/L = 0.2$. The observed increase of the splitting is non-monotonic, but this can be expected from the cavity in which strength of the effects will be affected by how the twist distance corresponds to subsequent nodes and antinodes of electric field inside the cavity.

Interestingly, the simulations presented in Fig. 6.11 shows no energy splitting in the $(N + 2, N)$ regime (corresponding to LC rotation angles of around 20 deg) where modes cross. Such behavior differs from the situation where

a single additional birefringent layer with optical axis at diagonal direction is introduced into the cavity, but those observations are outside of the scope of this work.

Most importantly the results presented in Fig. 6.11 shows that using a twisted nematic LC configuration allows generating the desired splitting around the Rashba-Dresselhaus regime in the LC cavity. Additional angle-resolved simulations were performed to test the applicability of Hamiltonian Eq. (6.9) were performed for twist distance $l = 68 \text{ nm}$ ($l/L = 0.05$).

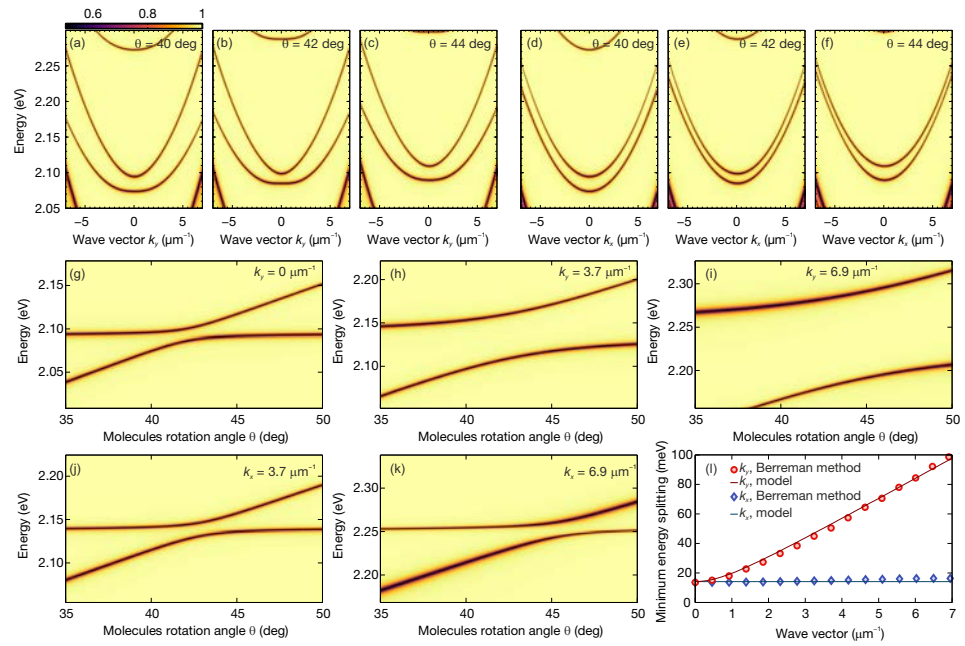


Fig. 6.12 | Voltage tuning in twisted nematic cavities simulated by Berreman method. (a)–(c) Reflectance spectra with incident light with k_y wave vector component at given LC rotation angles θ . (d)–(f) Reflectance spectra with incident light with k_x wave vector component at given LC rotation angles θ . (g)–(k) Reflectance spectra for increasing the rotation angles of LC molecules θ at given wave vector values. (l) Minimum energy distance between the cavity modes at given wave vector extracted from Berreman matrix simulations and compared with predictions of Hamiltonian Eq. (6.9).

Fig. 6.12a–f presents angle-resolved reflectance spectra along the k_y and k_x wave vector directions for molecules rotation angles below and under the $(N + 1, N)$ resonance. Along k_y direction (Fig. 6.12a–c) the dispersion relation resembles the Rashba-Dresselhaus dispersion relation, but with ad-

ditional splitting preventing resonance of the modes at normal incidence, clearly visible in the cross section for increasing the rotation angle of molecules at normal incidence Fig. 6.12g. Analogous cross section at higher values of incident light wave vector in y - z plane: $k_y = 3.7 \mu\text{m}^{-1}$ in Fig. 6.12h and $k_y = 6.9 \mu\text{m}^{-1}$ in Fig. 6.12i shows increasing with wave vector higher energy separation between the modes as Rashba-Dresselhaus term adds to splitting introduced by twisted nematic LC configuration.

Similarly for the perpendicular wave vector direction k_x (Fig. 6.12j-k) the dispersion relations resembling two parabola never cross at the presented angle-resolved spectra. Plotting reflectance spectra for increasing the rotation angle of molecules at given k_x wave vector shows a similar separation between the modes, roughly independent on the wave vector in x - z plane.

Fig. 6.12l presents plot of a minimum energy splitting between the cavity modes under varying rotation angles (like Fig. 6.12g-k) at a given wave vector. These values can be compared with predictions described by Hamiltonian Eq. (6.9), for which the energy difference between the cavity modes is equal to:

$$\Delta E = \sqrt{16\alpha^2 k_y^2 + \left(2\delta_x k_x^2 + 2\delta_y k_y^2 + \Delta E_{\text{HV}}\right)^2} + \Delta E_{\text{DA}}^2. \quad (6.10)$$

Here rotation of LC molecules corresponding to external electric bias allows for tunability of HV splitting ΔE_{HV} in wide energy range from negative to positive and for a given wave vector the minimum energy difference between the band with respect to ΔE_{HV} can be found. In particular, when $k_y = 0$ is minimal at $\Delta E_{\text{HV}} = -2\delta_x k_x^2$ energy difference

$$\Delta E_{\text{min}}(k_y = 0) = |\Delta E_{\text{DA}}|, \quad (6.11)$$

therefore, the minimal energy difference is independent of the k_x value. In the perpendicular wave vector direction when $k_x = 0$ minimal energy splitting occurs when $\Delta E_{\text{HV}} = -2\delta_y k_y^2$ and is equal to:

$$\Delta E_{\text{min}}(k_x = 0) = \sqrt{16\alpha^2 k_y^2 + \Delta E_{\text{DA}}^2}. \quad (6.12)$$

Predictions of Eqs (6.11-6.12) can be compared with the results extracted from the Berreman matrix simulations in Fig. 6.12l. For $\alpha = 3.5 \text{ meV}\mu\text{m}^{-1}$, $\Delta E_{\text{DA}} = 14.1 \text{ meV}$ analytical minimal energy difference is in very good agreement with the simulated reflectance spectra. Along the k_x wave vector direction the minimum energy splitting between the modes is almost con-

stant according to the results of the Berreman method simulations, which is consistent with a constant value $|E_{DA}|$ predicted by the analytical model in Eq. (6.11). Along k_y the energy difference which for high wave vectors tends asymptotically to linear dependence on k_y and at $k_y = 0$ is finite is well captured by theoretical Eq. (6.12) formula.

To test the validity of theoretical predictions, two similar samples with twisted nematic configurations with 30 deg and 45 deg desorientation of the rubbing layers were prepared. The experimental results will be compared with another LC cavity with a homogeneous LC structure operated in the $(N + 1, N)$ regime.

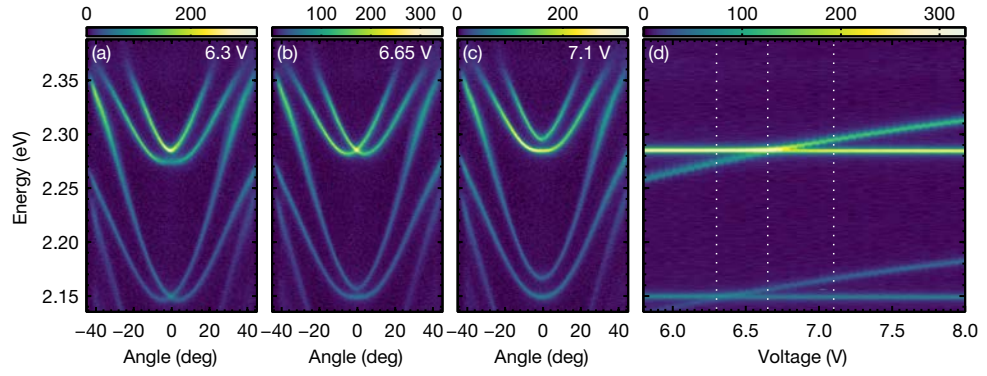


Fig. 6.13 | Rashba-Dresselhaus SOC regime in LC cavity incorporating 2D perovskite. Angle-resolved photoluminescence maps at increasing voltage applied to the structure: (a) 6.3 V, (b) 6.65 V, (c) 7.1 V. (d) Emission at normal angle as a function of the applied voltage.

Fig. 6.13a–c presents angle-resolved photoluminescence spectra for increasing voltage applied to the cavity with homogeneous LC orientation. As previously, spectra were collected with nonresonant continuous wave excitation with a laser source at 405 nm wavelength. Spectra were collected with detection in diagonal linear polarization, in which both circularly polarized bands characteristic for Rashba-Dresselhaus SOC regime are equally visible. Emission spectra were collected in the voltage range spanning from below (Fig. 6.13a) to above (Fig. 6.13b) Rashba-Dresselhaus modes resonance. For this sample, the exact resonance of $(N + 1, N)$ modes occurred at 6.65 V, at which angle-resolved photoluminescence spectra shows characteristic for the Rashba-Dresselhaus Hamiltonian dispersion relation consisting of two off centered parabolas, here additionally affected by coupling with the excitonic resonance in the perovskite layer. Corresponding emission spectra collected at normal incidence for increasing voltage applied to

the cavity (Fig. 6.13d). This image clearly shows how the horizontally polarized mode for increasing the electrical bias crosses the stationary, vertically polarized mode, as was observed for the cavities with homogeneous orientation of the LC.

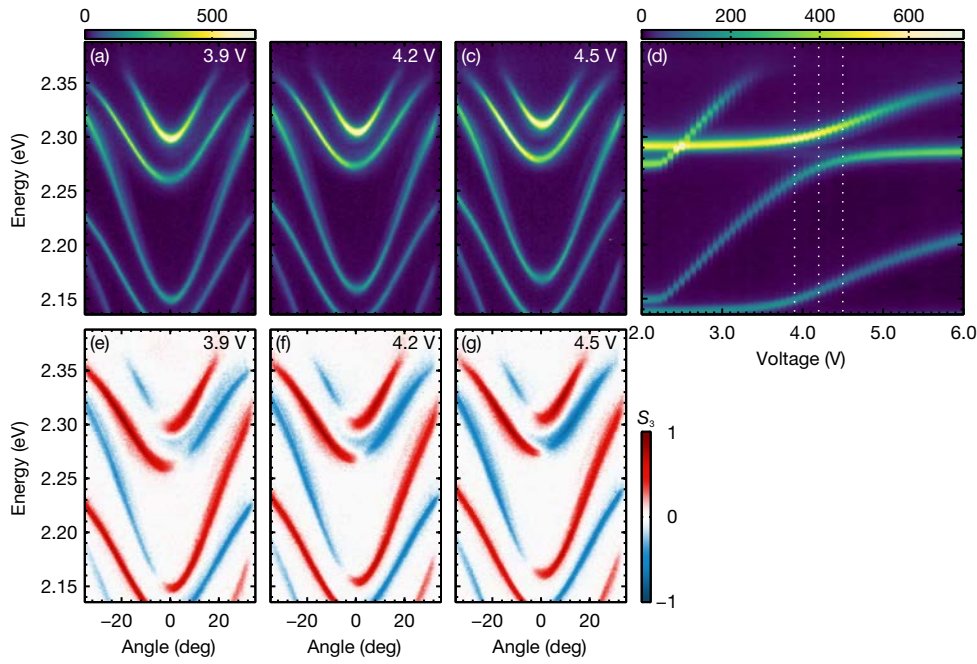


Fig. 6.14 | Rashba-Dresselhaus SOC regime in LC cavity incorporating 2D perovskite with twisted nematic configuration. Angle-resolved photoluminescence maps at increasing voltage applied to the structure: (a) 3.9 V, (b) 4.2 V, (c) 4.5 V. (d) Luminescence at normal angle as a function of applied voltage. (e)–(f) Degree of circular polarization corresponding to panels (a)–(c).

A similar photoluminescence experiment was performed on the sample with a twisted nematic LC configuration with 30 deg desorientation of the ordering layers. Angle-resolved photoluminescence spectra for increasing voltages are shown in Fig. 6.14a–c with a series at normal incidence presented in Fig. 6.14d. At low voltages around 2.5 V, first crossing of the modes occurs. In this regime corresponding to the $(N+2, N)$ resonance no anticrossing between the modes is observed in the experiment, which is consistent with the Berreman matrix simulations of the twisted nematic structure, presented earlier in Fig. 6.11. At higher voltages another mode approaches, this time exhibiting clear anticrossing behavior around 4.2 V, which was expected to emerge for twisted nematic structures. This range corresponds to the $(N+1, N)$ regime. Although the influence of the Rashba-Dresselhaus SOC

term is not evident in the angle-resolved spectra within this voltage range (Fig. 6.14a–c) it can be observed by analyzing the degree of circular polarization of the emitted light. Spectra Fig. 6.14a–c present sum of the emission intensities measured independently with the detection of σ^+ and σ^- polarized light. Such polarization-resolved data can be used to plot the S_3 parameter of the emitted light, as shown in Fig. 6.14e–g. Here the presence of the RD SOC term is evident, as for lower energy polariton branch polarization state changes from σ^+ to σ^- looking from the negative, to positive emission angles as is characteristic for Rashba-Dresselhaus Hamiltonian. For the upper energy branch, exactly opposite helicities are observed. On the basis of those observations, a desired configuration of the polaritonic bands was observed in a LC cavity incorporating 2D perovskite in which, for the non-zero Rashba-Dresselhaus term, additional splitting of the modes is introduced through the twisted nematic configuration of the LC.

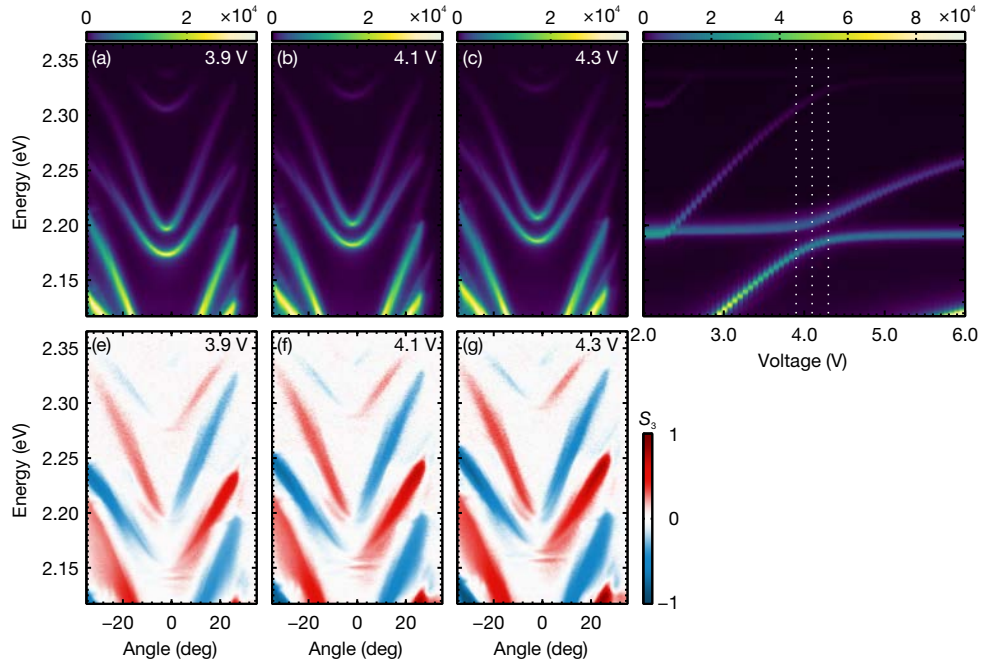


Fig. 6.15 | Rashba-Dresselhaus SOC regime in LC cavity incorporating 2D perovskite with twisted nematic configuration. Angle-resolved transmission maps at increasing voltage applied to the structure: (a) 3.9 V, (b) 4.1 V, (c) 4.3 V. (d) Transmission at normal angle as a function of applied voltage. (e)–(g) Degree of circular polarization corresponding to panels (a)–(c).

As presented above, the polarization-resolved photoluminescence experiment can be used to determine polarization of the polaritonic bands in

a cavity with perovskite. Unfortunately, photostability of the perovskite was insufficient to fully examine the two-dimensional dispersion relation of the cavity in all three Stokes parameters. Fortunately, dispersion and polarization of the bands can also be determined based on polarization-resolved transmittance measurement, for which we did not observe any significant degradation of the signal in time required to collect even multiple polarization-resolved tomographies.

In transmission configuration, we used a spectrally broad LED source. It is focused on the sample with another microscope objective ($50\times$ magnification at $\text{NA} = 0.6$), with transmitted light collected by the same optical setup as in the reflection configuration consisting of $\text{NA} = 0.75$ microscope objective and a set of lenses projecting the Fourier plane on the entrance of the spectrometer. In polarization-resolved experiments to test eigenstates of the cavity, incident light is set to desired polarization by a set of linear polarizer and half- and quarter-wave plate on rotatory stages and total transmitted light intensity collected by the spectrometer.

Measurements corresponding to Fig. 6.14, but performed in transmission configuration, are presented in Fig. 6.15. Angle-resolved spectra in Fig. 6.15a–c and normal incidence voltage tuning in Fig. 6.15d shows qualitatively the same behavior as photoluminescence result in Fig. 6.14. In the $(N + 2, N)$ regime the modes crosses each other with applied voltage and in the $(N + 1, N)$ regime an anticrossing behavior is observed. Compared with photoluminescence, transmission measurement was performed for lower photon energies because of the higher transmittance of the sample away from the absorption range of the perovskite and spectral characteristic of the illuminated light source. As previously the influence of Rashba-Dresselhaus SOC term is better visible in the polarization resolved photoluminescence spectra (Fig. 6.15e–g). As in transmission experiment we resolve polarizations rather at the incidence side contrary to the detection side in photoluminescence configuration; the S_3 parameter is reversed between those two approaches as in the $(N + 1, N)$ regime the cavity acts as a half-wave plate which switches the helicity of circularly polarized light.

To test the emergence of non-zero Berry curvature in the liquid crystal cavity with twisted nematic layer full polarization-resolved tomography in momentum space was performed on the sample with 45 deg disorientation. Applied external voltage with 3.8 V amplitude resulted in slightly positive HV detuning. The energy positions of the modes extracted from the experimental results are plotted in Fig. 6.16a. Independent illumination with H, V, D, A, σ^+ and σ^- . Especially detection in $\sigma^\pm$ polarizations allowed for determination of S_3 parameter, which marked in Fig. 6.16a by pseudocolor scale

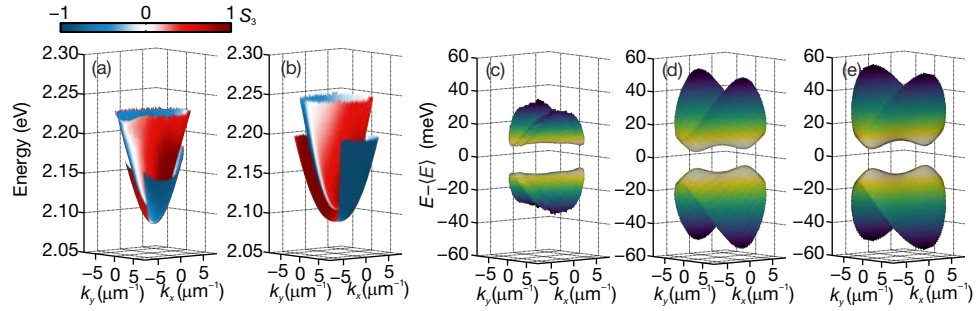


Fig. 6.16 | Band energy in LC cavity with twisted nematic LC layer. Dispersion of polaritonic modes obtained by (a) experiment and (b) Berreman matrix simulations. The pseudocolor scale denotes the S_3 component. (d) Measured polariton bands subtracted from their mean energy with corresponding to (c) experimental results, (d) Berreman matrix simulations and (e) solutions of the analytical model.

proves operation around the Rashba-Dresslehaus regime.

The experimental results were recreated with the Berreman method for the structure of the total cavity width $L = 1362$ nm. At the LC/DBR interfaces optical axis of the LC layer was desoriated by 45 deg, with the LC molecules twist width of $l = 68$ nm. Within the twist region the optical axis direction was smoothly rotated by 22.5 deg to obtain central region of 1226 nm of LC with optical axis pointing in x direction (compare with Fig. 6.10c,d). To introduce the effect of the applied external electric field, all dielectric tensors of the LC layers were additionally rotated by 42.5 deg towards the z axis. The dispersion relation of the photonic modes is plotted in Fig. 6.16b, with corresponding S_3 parameters of a given band marked with a pseudocolor scale.

For easier visualization of the gapped Dirac cones in the band structure, the energy of the modes was subtracted by their mean value $\langle E \rangle$ for experimental and Berreman matrix simulated results in Fig. 6.16c,d and the energy difference between the modes ΔE in Fig. 6.17a,b. Especially the latter approach in the simulated data shows two symmetric distinct minima of the energy difference between the modes corresponding to desired gapped Dirac cones. The same behavior can be also seen at the experimental data but close to the limit set by the limited numerical aperture of the microscopic objectives used in the experiment.

However, both of these results show that the position of the energy difference minima does not lie strictly at the $k_y = 0$ line, as was expected from the analysis of Eq. (6.9) plotted in Fig. 6.9, but are rotated slightly clockwise

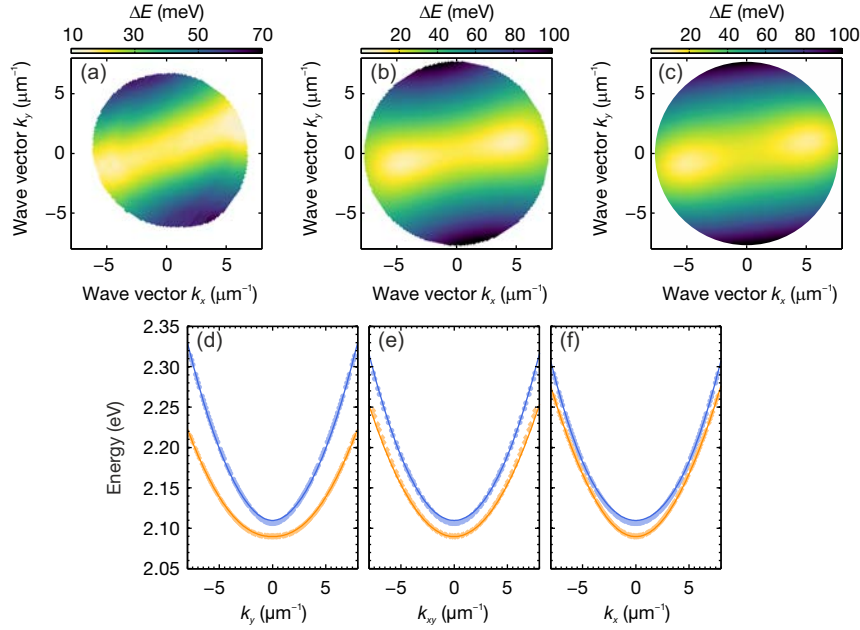


Fig. 6.17 | Energy difference between the modes in: (a) experimental tomography, (b) Berreman matrix simulations and (c) model given by Hamiltonian Eq. (6.13). Cross sections through energy dispersion in Berreman matrix simulations (points) and model Eq. (6.13) along (d) k_y , (e) $k_x = k_y$ and (f) k_x wave vector directions.

in momentum space. To recreate this behavior in the analytical model, Dr. Helgi Sigurdsson proposed to add additional $2\delta_{xy}k_xk_y\hat{\sigma}_y$ term to the Hamiltonian Eq. (6.13). Such terms appear in the Hamiltonian corresponding to the $(N+2, N)$ regime [Eq. (2.41)], and can appear in this case due to the limited symmetry of the system caused by the presence of twisted nematic layers. Physically δ_{xy} combined with the Hamiltonian for $(N+1, N)$ acts as an extended TE-TM splitting term, for which $\delta_x = \delta_{xy} = -\delta_y$.

With this additional term, total Hamiltonian reads as:

$$\begin{aligned} \hat{H}_{\text{RD-DA}} = & \left(E_0 + \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} \right) \hat{\sigma}_0 + \\ & + \frac{1}{2} \Delta E_{\text{HV}} \hat{\sigma}_x + \frac{1}{2} \Delta E_{\text{DA}} \hat{\sigma}_y - 2\alpha k_y \hat{\sigma}_z + \\ & + (\delta_x k_x^2 + \delta_y k_y^2) \hat{\sigma}_x + 2\delta_{xy} k_x k_y \hat{\sigma}_y. \end{aligned} \quad (6.13)$$

It can be fitted directly to the Berreman matrix simulations with parameters: $E_\phi^0 = 2.0997 \text{ eV}$, $\Delta E_{\text{HV}} = 13.3 \text{ meV}$, $\Delta E_{\text{AD}} = -14.9 \text{ meV}$, $m_x = 1.27 \times 10^{-5} m_e$, $m_y = 1.38 \times 10^{-5} m_e$, $\delta_x = -0.32 \text{ meV} \mu\text{m}^2$, $\delta_y = -0.53 \text{ meV} \mu\text{m}^2$,

$\delta_{xy} = 0.35 \text{ meV}\mu\text{m}^2$, $\alpha = -2.13 \times 10^{-3} \text{ eV}\mu\text{m}$ to obtain a very good agreement. Fig. 6.16e presents energies of the photonic modes subtracted by their mean value and Fig. 6.17c energy difference between the modes obtained from the analytical model Eq. (6.13) after the fitting. The model with a perfect agreement describes the energy splitting and the position of the gapped Dirac cones in the momentum space. The fitted curves can also be compared with the Berreman matrix simulations plotted in Fig. 6.17d–f for different wave vector directions.

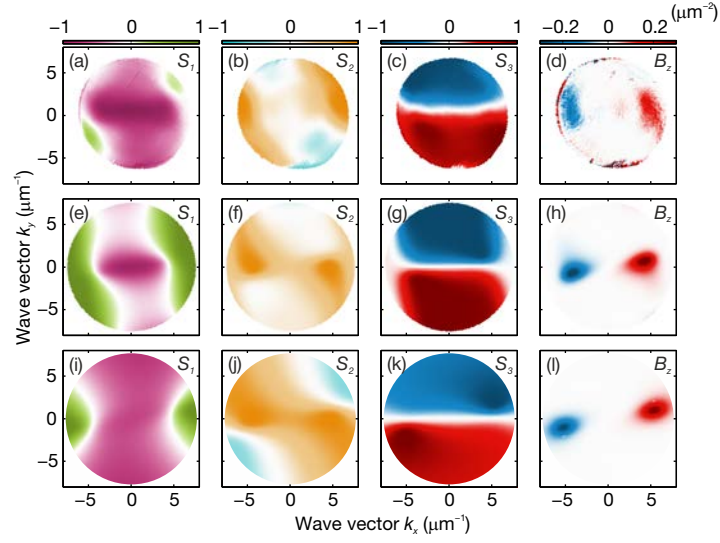


Fig. 6.18 | (a)–(d) Experimental polarization and resulting Berry curvature of the lower energy band compared with the corresponding results of (e)–(f) the Berreman matrix simulations and (i)–(l) analytical two-level model.

The presence of the energy difference minima between the bands is a strong indication of the nontrivial band geometry, but a direct demonstration of the non-zero Berry curvature polarization state of the bands has to be analyzed. Fig. 6.18a–c presents polarization state of the lower energy branch extracted from the tomography measurement.

At first energy position of two bands was determined for each wave vector by fitting two Gaussian profiles to the total transmission intensity obtained by the sum of intensities detected for horizontally and vertically polarized incident light. Based on the transmitted intensities of the H, V, D, A, σ^+ and σ^- polarized incident beam at those energy positions, the eigenpolarization of the modes can be calculated following Stokes parameters definition.

The experimentally determined polarization state in Fig. 6.18a–c are in agreement with the results of the Berreman matrix simulations (Fig. 6.18e–g) and eigenvectors of Hamiltonian Eq. (6.13) and all follows the predictions of Eq. (6.9) described in Fig. 6.9.

For the lower energy branch along the k_y wave vector direction, the polarization state switches from σ^+ to σ^- when crossing through $k_y = 0$ due to the Rashba-Dresselhaus SOC term. As HV detuning is positive, at normal incidence the mode is vertically polarized, which changes to horizontal polarization for high k_x values. At the positions in momentum space where both the S_1 and S_2 parameters are zero, S_2 has its maximum, which is the result of diagonal-antidiagonal splitting ΔE_{DA} of negative value. Based on this polarization state in momentum space described by Stokes parameters S_i the Stokes vector can be mapped into the spherical coordinate system by two angles:

$$\Theta(\mathbf{k}) = \arccos S_3(\mathbf{k}), \quad (6.14)$$

and

$$\Psi(\mathbf{k}) = \arctan \frac{S_2(\mathbf{k})}{S_1(\mathbf{k})}. \quad (6.15)$$

Based on them, the Berry curvature can be derived as [25]:

$$B_z = \frac{1}{2} \sin \Theta \left(\partial_{k_x} \Theta \partial_{k_y} \Psi - \partial_{k_y} \Theta \partial_{k_x} \Psi \right), \quad (6.16)$$

which is plotted for the experimental, Berreman matrix, and analytical results in Fig. 6.18d,h,l. All approaches lead to the dipole of the Berry curvature B_z located in proximity to the $k_y = 0$ line in the momentum space. It is concentrated around the regions corresponding to the minimum energy difference between the modes. The sign of the Berry curvature at each gapped Dirac cone is the opposite, as is expected from a time-reversal symmetric system. Overall, the Chern number of the band, which is given by integral of the Berry curvature over the whole band (or Brillouin zone for periodic systems) is equal to zero, but locally the band topology is nontrivial with nonzero Berry curvature.

Following the analytical model Eq. (6.13) it is expected that the Berry curvature of the second, upper energy branch will be the exact opposite of the lower energy branch. As both eigenvectors of any Hermitian matrix, such as the one given by Eq. (6.13), have to be orthogonal to each other, it corresponds to orthogonal polarization of the bands. This is the case when plotting the polarization state and the Berry curvature of the higher energy branch from the analytical model, as shown in Fig. 6.18i–l.

Contrary to these predictions, both experimental results (Fig. 6.19a–d) and Berreman matrix simulations (Fig. 6.19e–f) shows deviations from simple orthogonality to their counterparts for the lower energy branch. For example S_3 parameter loses its mirror antisymmetry versus the $k_y = 0$ plane, S_2 significantly more resembles a quadrupole, and the S_1 forms a positive stripe along the antidiagonal wave vector directions. All those features can be observed in both experimental and simulated data. As a result also the Berry curvature is affected. For Berreman matrix simulations, the calculated Berry curvature at Fig. 6.19h in the first approximation is of the opposite sign to the lower energy branch, but is more elongated and centered at higher wave vectors values. As a result for the experimental data with limited numerical aperture, this Berry curvature dipole is not observed.

Although some discrepancies between the analytical model given by Eq. (6.13) and the experimental results can be observed, overall the model correctly captures the general polarization of the bands and the emergence of non-zero Berry curvature in the form of a dipole oriented close to $k_y = 0$. Given the simplicity of the 2×2 model Eq. (6.13) considering only two coupling cavity modes, it is expected that the influence of the rest of the photonic resonances in the multimode cavity will affect the polarization state. Additionally, non-Hermitian contributions are also of interest in birefringent microcavities [41, 122]. Much of the discrepancies are captured by Berreman-method simulations, which provide exact solution of the Maxwell equations, but here it is only considered numerically.

Fig. 6.18 clearly demonstrate the possibility to obtain a nonzero Berry curvature in the photonic system. A significant advantage of the LC platform used in this case is the ease of tunability of the liquid crystal with an electric field. Especially, changing the voltage allows for control over the position of the Berry curvature extrema in momentum space.

Fig. 6.20a–e presents calculated with the analytical model dispersion relation ($E - \langle E \rangle$) in which the Berry curvature is overlayed on the bands with a pseudocolor scale. Calculations are performed for the same parameters as presented in Fig. 6.17 but with diagonal-antidiagonal detuning $\Delta E_{DA} = 18.9$ meV (equal to experimental realization described later). Between the panels, the HV detuning is changed in -10 meV to 20 meV range, as it is a parameter that is easily controlled in the experiment by the amplitude of the external voltage.

Starting from the negative HV detunings the bands are energy-separated with negligible Berry curvature. At zero HV detuning due to $\delta_{xy}k_xk_y$ term, already two gapped Dirac cones emerge initially with a weak concentration of the Berry curvature. At higher positive HV detunings (Fig. 6.20d–e) cle-

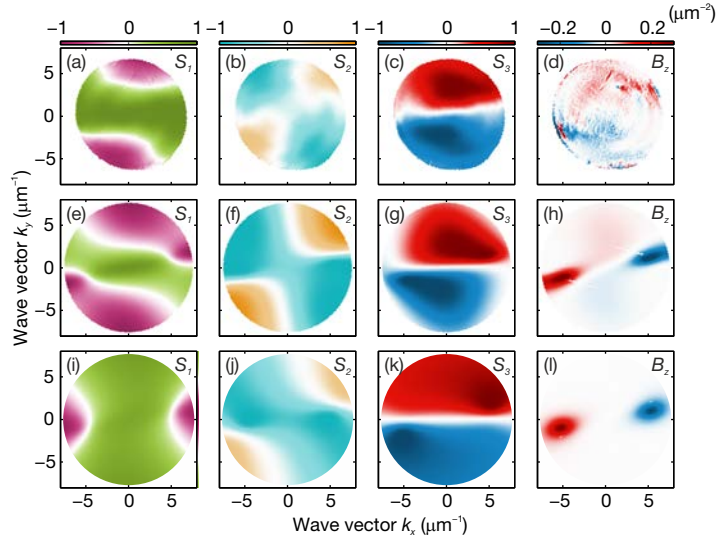


Fig. 6.19 | (a)–(d) Experimental polarization and resulting Berry curvature of the upper energy band compared with the corresponding results of (e)–(f) the Berreman matrix simulations and (i)–(l) analytical two-level model.

arly two distinct valleys in momentum space exhibiting nonzero Berry curvature are formed. Importantly, with increasing HV detuning the distance in momentum space between the Berry curvature extrema increases.

To discuss potential of LC cavities in control over photonic Berry curvature it is valuable to discuss also influence of diagonal-antidiagonal splitting on photonic modes dispersion, even if it is not adjustable during the experiment. Fig. 6.20f–j presents band structure calculated for increasing diagonal-antidiagonal splitting at fixed positive value of HV detuning of $\Delta E_{\text{HV}} = 10$ meV. As diagonal-antidiagonal splitting provides minimal energy spacing between the bands (when neglecting δ_{xy} term contribution), for small detunings it leads to a sharp energy change of the bands. This can be observed in Fig. 6.20f calculated for $\Delta E_{\text{DA}} = 2$ meV. Two valleys in the momentum space are formed with a small energy gap at each valley and a sharp dependence on the wave vector, which also leads to a very strong localization of the calculated Berry curvature. As the diagonal-antidiagonal splitting increases in Fig. 6.20g–h, the energy separation between the bands increases, the dispersion relation smooths and the Berry curvature spans over larger area in the momentum space (mind that the colorscale is adjusted for each panel individually).

Overall, the calculated momentum position and the maximum value of

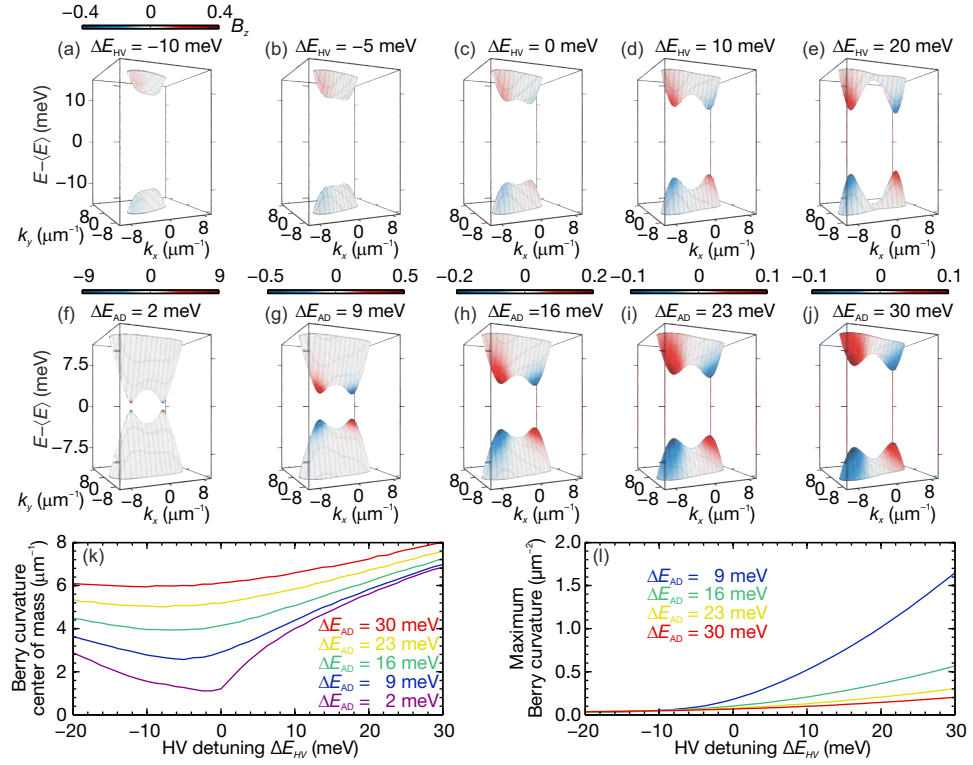


Fig. 6.20 | Electrical tuning of polariton Berry curvature. Calculated dispersion of cavity modes (energies subtracted by their mean) with B_z marked by a pseudocolor scale from the analytical model for (a)–(e) increasing HV detuning ΔE_{HV} at $\Delta E_{DA} = 18.9$ meV, (f)–(g) increasing DA detuning ΔE_{DA} at $\Delta E_{HV} = 10$ meV, (k) Resulting maximum value of B_z and B_z center of mass position in reciprocal space for varying H-V detuning calculated for $\Delta E_{DA} = 18.9$ meV. (l) Berry curvature center of mass position for varying HV detuning at different DA detunings.

the Berry curvature for varying HV detuning is plotted in Fig. 6.20k,l. The momentum position of the Berry curvature extrema in this plot is determined in terms of its center of mass. For high diagonal-antidiagonal splitting (red line for 30 meV) the minimum energy separation between the modes is relatively high, which leads to formation of the gapped Dirac cones at high wave vector, which is only slightly affected by additional HV splitting. Thus, the momentum position of the Berry curvature extrema is not constant, but also does not vary much with HV detuning. Similarly, the maximum value of the Berry curvature (Fig. 6.20l) increases with the HV detuning, but the dependence is not particularly strong. For lower diagonal-antidiagonal

splitting, Berry curvature is more localized in the momentum space, the effect of tuning with external voltage is significantly more pronounced for lower diagonal-antidiagonal splittings.

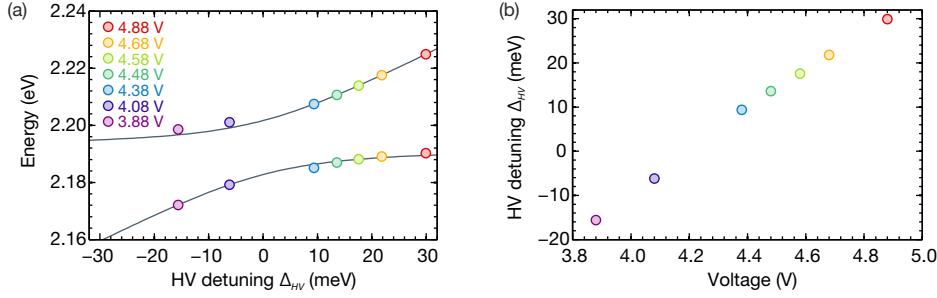


Fig. 6.21 | (a) Calculated energy of the photonic modes at normal incidence for increasing HV detuning (gray lines) compared with experimental results for varying voltage applied to the LC cavity (circles). (b) Extracted HV detuning as a function of the applied voltage.

Experimental confirmation of theoretical predictions of the voltage controllable Berry curvature was performed on the sample with 30 deg deorientation between the direction of the ordering layers. Seven polarization resolved tomographies were collected at different voltages varying in 3.88 V to 4.88 V. Based on the comparison between the measured energy positions of the modes at normal incidence plotted by dots in Fig. 6.21 with the theoretical response to the HV detuning plotted by gray line diagonal-antidiagonal splitting and value of HV detuning can be determined at each voltage. For the measured tomographies, two were performed at negative detunings, with the lowest value of -15.6 meV and the remaining four for positive detuning rising up to 29.9 meV. When the values of the extracted HV detunings are plotted as a function of the applied voltage, in this limited range the dependence is approximately linear, as shown in Fig. 6.21b

The Berry curvature for the lower energy branch extracted from the polarization resolved transmission experiment are plotted in Fig. 6.22a–f, and are compared with the corresponding results of the analytical model in Fig. 6.22g–l. For the negative detuning (Fig. 6.22a,g) the Berry curvature in both the experiment and simulations is very low. At the first measured positive HV detuning $\Delta E_{HV} = 9.3$ meV, corresponding to 4.38 V, in both the experiment (Fig. 6.22c) and model (Fig. 6.22h) a clear dipole of the Berry curvature emerges. In experiment it is almost perfectly aligned along the $k_y = 0$ line, whereas for the model the dipole is slightly rotated counterclockwise. With increasing voltage applied to the cavity Fig. 6.22c–f the mo-

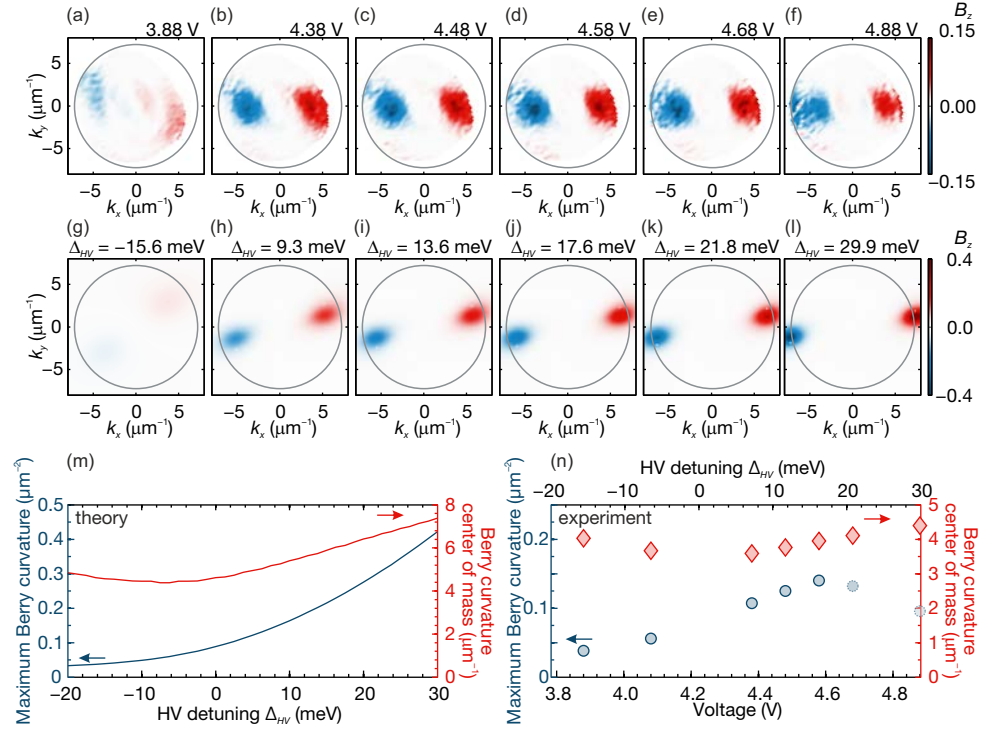


Fig. 6.22 | Electrically tunable photonic Berry curvature. (a)–(f) Berry curvature distribution for lower energy mode with increasing voltage applied to the LC cavity extracted from the polarization resolved transmission measurement. Gray circles mark the experimental region limited by the numerical aperture of the objectives. (h)–(n) Calculated Berry curvature for HV detunings corresponding to the experimental results.

momentum position of the poles of the Berry curvature dipole increases along the k_x direction. The effect is easier to observe in corresponding simulations Fig. 6.22i–l, as the experiment positions of the Berry curvature extrema is close to the limits defined by the numerical aperture of the microscopic objectives, marked with gray circles.

Theoretically calculated dependence of the position of the Berry curvature poles in momentum space and their peak values is plotted in Fig. 6.22m. As for the previously discussed results in Fig. 6.20 both maximum Berry curvature value is expected to increase with the HV detuning, especially the increase should be faster when crossing the zero detuning value. Simultaneously, the momentum position of the Berry curvature poles is calculated to achieve minimum value at slightly negative detuning of around -5 meV and increase with rising detuning. All of these trends can also be observed

in the experimental results plotted in Fig. 6.22n. The maximum value of the Berry curvature, marked with circles, clearly increases with the applied voltage. The effect is lost at the last two points marked with dashed lines, which were obtained for the highest applied electric biases. As suggested by simulations, as the Berry curvature extrema leave the observed in the experiment momentum range, it is not easy to estimate peak value. Similarly, also obtained from the experiment momentum position of the Berry curvature extrema follows the theoretical prediction.

6.5 Perovskites in LC microcavities: summary

In summary, 2D organic-inorganic PEPI perovskite was investigated experimentally towards the observation of a strong coupling regime between excitonic resonance in the perovskite with photonic modes of the cavity filled with liquid crystal. Observed in photoluminescence dispersion relation of the cavity modes was fitted with the coupled oscillator model to extract coupling strengths at the order of 100 meV, depending on the polarization of the cavity mode. The results were in a good agreement with numerical simulations of the whole cavity structure utilizing a dielectric function of the perovskite layer obtained by ellipsometry.

In addition, a scheme of desoriented orientation layers that results in a twisted nematic configuration of the liquid crystal was proposed to break diagonal-antidiagonal symmetry of the structure to obtain locally nonzero Berry curvature at the cavity bands. These predictions were confirmed experimentally, in which the ability to tune the LC cavity with external voltage was used to change the position of the Berry curvature poles in momentum space.

Chapter 7

Summary

This thesis was devoted to various polarization-dependent phenomena that arise in planar microcavities, and here we summarize the most important results described in the following chapters.

In chapter 2, we showed the rich physics of the cavities filled with a birefringent liquid crystal. Performed theoretical simulations of the dispersion relations and polarization of the photonic modes in such resonators showed a variety of different regimes arising from the possibility to tune the optical anisotropy direction in such a system. The most striking effect was emergence of the Rashba-Dresselhaus spin orbit coupling term when two modes of different parity were brought close to the resonance. Apart from that we show significant difference between the regime when two modes of the same parity, but different or the same mode numbers are spectrally tuned close to each other, which was essential for results described in the two following chapters.

In chapter 3, we investigate spatial polarization textures arising when polarized incident light is transmitted through photonic modes of LC cavities. For the case closest to isotropic cavity we showed theoretically that with changing the pumping beam polarization from linear to circular the already known meron-antimeron polarization lattice evolves into a novel second order meron polarization texture. In the former case we were able to observe experimentally using polarization resolved tomography measurement in the real space. Most importantly we demonstrated in numerical simulations and the experiment that by tuning the optical anisotropy direction of the liquid crystal layer, close to the resonance of two modes of the same parity but different mode numbers, it is possible to reverse the topological charge of the spatial polarization texture and observe second order antimeron.

In chapter 4 we investigate almost the same conditions in LC microcavity, for which the dispersion relation of the cavity modes exhibits four diabolical points. With paying closer attention to the polarization dependent losses in the system requiring non-Hermiticity corrections to the model those four points in the momentum space evolve into eight exceptional points where real and imaginary parts of the eigenvalues spectrum coalesce. Importantly, tunability of the liquid crystal cavity can be used to control the non-Hermiticity degree, which affects the momentum positions of the exceptional points. Especially for high non-Hermiticity degree the exceptional points issued from different diabolical points in the momentum space met and annihilate. This is the first demonstration of such multi-valley process in which singularity points in the reciprocal space are completely removed from the system staying in the non-Hermitian regime.

In chapter 5 we switch to optically isotropic cavities used for the observation of the strong coupling regime between the photonic modes and excitons in thin flakes of transition metal dichalcogenides, namely tungsten diselenide WSe_2 . We discuss the design of an open dielectric cavity where the electromagnetic field is confined between two independent DBRs with the distance between them controlled by a piezoelectric element. We demonstrate a strong coupling regime between the cavity mode and WSe_2 excitons using flakes with thicknesses varying between 1 and 4 monolayers at liquid helium temperature. The experimental results were confirmed by simulations based on reflectance spectra of the bare flakes. Additionally, we used characteristic for monolayer transition metal dichalcogenides spin-valley locking to obtain circularly polarized emission from the polariton state under quasi-resonant excitation.

Finally chapter 6 we combine liquid crystal microcavities with a strong coupling regime by incorporating a layer of two-dimensional perovskite within the resonator. We analyze theoretically the dispersion relation of the cavity incorporating such materials, demonstrate it experimentally based on angle-resolved photoluminescence measurements and show tunability up to the Rashba-Dresselhaus spin-orbit coupling regime. Moreover, we used twisted nematic orientation of the liquid crystal layer in order to lift the degeneracy at the diabolical points in momentum space close to the Rashba-Dresselhaus resonance in order to demonstrate polarization of the photonic bands corresponding to the nonzero Berry curvature, which we can further tune by an external voltage applied to the sample.

We believe that this work demonstrates a large number of fascinating physical effects arising from the mixing of polarization dependent photonic band structure in newly rediscovered system of cavities filled with liquid

crystals. As the system is still in its infancy, we hope that this initial research can show available possibilities and act as a foundation for further development of the system aiming towards optically nonlinear behavior, potential landscape engineering or quantum effects.

Appendix: List of scientific articles

Articles and preprints described within the Thesis

1. [50] **M. Król**, H. Sigurdsson, K. Rechcińska, P. Oliwa, K. Tyszka, W. Bardyszewski, A. Opala, M. Matuszewski, P. Morawiak, R. Mazur, W. Piecek, P. Kula, P. G. Lagoudakis, B. Piętka, J. Szczytko, Observation of second-order meron polarization textures in optical microcavities. *Optica* **8**, 255 (2021).
[doi:10.1364/OPTICA.414891](https://doi.org/10.1364/OPTICA.414891)
2. [21] K. Rechcińska, **M. Król**, R. Mazur, P. Morawiak, R. Mirek, K. Łempicka, W. Bardyszewski, M. Matuszewski, P. Kula, W. Piecek, P. G. Lagoudakis, B. Piętka, J. Szczytko, Engineering spin-orbit synthetic Hamiltonians in liquid-crystal optical cavities. *Science* **366**, 727 (2019).
[doi:10.1126/science.aay4182](https://doi.org/10.1126/science.aay4182)
3. [90] **M. Król**, I. Septembre, P. Oliwa, M. Kędziora, K. Łempicka-Mirek, M. Muszyński, R. Mazur, P. Morawiak, W. Piecek, P. Kula, W. Bardyszewski, P. G. Lagoudakis, D. D. Solnyshkov, G. Malpuech, B. Piętka, J. Szczytko, Annihilation of exceptional points from different Dirac valleys in a 2D photonic system. [arXiv:2112.06621](https://arxiv.org/abs/2112.06621) (2021).
4. [124] **M. Król**, K. Lekenta, R. Mirek, K. Łempicka, D. Stephan, K. Nogajewski, M. R. Molas, A. Babiński, M. Potemski, J. Szczytko, B. Piętka, Valley polarization of exciton–polaritons in monolayer WSe₂ in a tunable microcavity. *Nanoscale* **11**, 9574 (2019).
[doi:10.1039/c9nr02038a](https://doi.org/10.1039/c9nr02038a)
5. [123] **M. Król**, K. Rechcińska, K. Nogajewski, M. Grzeszczyk, K. Łempicka, R. Mirek, S. Piotrowska, K. Watanabe, T. Taniguchi, M. R. Molas, M. Potemski, J. Szczytko, B. Piętka, Exciton-polaritons in multilayer

WSe₂ in a planar microcavity. *2D Mater.* **7**, 015006 (2020).
[doi:10.1088/2053-1583/ab4b14](https://doi.org/10.1088/2053-1583/ab4b14)

6. [225] K. Lempicka-Mirek, **M. Król**, H. Sigurdsson, A. Wincukiewicz, P. Morawiak, R. Mazur, M. Muszyński, W. Piecek, P. Kula, T. Stefaniuk, M. Kamińska, L. De Marco, P. G. Lagoudakis, D. Ballarini, D. Sanvitto, J. Szczytko, B. Piętka, Electrically tunable Berry curvature and strong light-matter coupling in birefringent perovskite microcavities at room temperature. [arXiv:2203.05289](https://arxiv.org/abs/2203.05289) (2022).

Other scientific articles and preprints, not directly connected with the Thesis

7. [47] **M. Król**, K. Rechcińska, H. Sigurdsson, P. Oliwa, R. Mazur, P. Morawiak, W. Piecek, P. Kula, P. G. Lagoudakis, M. Matuszewski, W. Bardyszewski, B. Piętka, J. Szczytko, Realizing optical persistent spin helix and Stern-Gerlach deflection in an anisotropic liquid crystal microcavity. *Phys. Rev. Lett.* **127**, 190401 (2021).
[doi:10.1103/PhysRevLett.127.190401](https://doi.org/10.1103/PhysRevLett.127.190401)
8. [48] M. Muszyński, **M. Król**, K. Rechcińska, P. Oliwa, M. Kędziora, K. Lempicka-Mirek, R. Mazur, P. Morawiak, W. Piecek, P. Kula, P. G. Lagoudakis, B. Piętka, J. Szczytko, Realizing persistent-spin-helix lasing in the regime of Rashba-Dresselhaus spin-orbit coupling in a dye-filled liquid-crystal optical microcavity. *Phys. Rev. Applied* **17**, 014041 (2022).
[doi:10.1103/PhysRevApplied.17.014041](https://doi.org/10.1103/PhysRevApplied.17.014041)
9. [85] K. Lekenta, **M. Król**, R. Mirek, K. Lempicka, D. Stephan, R. Mazur, P. Morawiak, P. Kula, W. Piecek, P. G. Lagoudakis, B. Piętka, J. Szczytko, Tunable optical spin Hall effect in a liquid crystal microcavity. *Light Sci. Appl.* **7**, 74 (2018).
[doi:10.1038/s41377-018-0076-z](https://doi.org/10.1038/s41377-018-0076-z)
10. [266] M. Matuszewski, A. Opala, R. Mirek, M. Furman, **M. Król**, K. Tyszka, T. Liew, D. Ballarini, D. Sanvitto, J. Szczytko, B. Piętka, Energy-efficient neural network inference with microcavity exciton polaritons. *Phys. Rev. Applied* **16**, 024045 (2021).
[doi:10.1103/PhysRevApplied.16.024045](https://doi.org/10.1103/PhysRevApplied.16.024045)
11. [267] R. Mirek, A. Opala, P. Comaron, M. Furman, **M. Król**, K. Tyszka, B. Seredyński, D. Ballarini, D. Sanvitto, T. C. H. Liew, W. Pacuski, J. Suff-

- czyński, J. Szczytko, M. Matuszewski, B. Piętka, Neuromorphic binarized polariton networks. *Nano Letters* **21**, 3715 (2021).
[doi:10.1021/acs.nanolett.0c04696](https://doi.org/10.1021/acs.nanolett.0c04696)
12. [268] **M. Król**, R. Mirek, D. Stephan, K. Lekenta, J.-G. Rousset, W. Pacuski, A. V. Kavokin, M. Matuszewski, J. Szczytko, B. Piętka, Giant spin Meissner effect in a nonequilibrium exciton-polariton gas. *Phys. Rev. B* **99**, 115318 (2019).
[doi:10.1103/PhysRevB.99.115318](https://doi.org/10.1103/PhysRevB.99.115318)
13. [269] K. Tyszka, M. Furman, R. Mirek, **M. Król**, A. Opala, B. Sereďyński, J. Suffczyński, W. Pacuski, M. Matuszewski, J. Szczytko, B. Piętka, Leaky Integrate-and-Fire Mechanism in Exciton-Polariton Condensates for Photonic Spiking Neurons. [arXiv:2111.13123](https://arxiv.org/abs/2111.13123) (2021).
14. [270] K. Lempicka, M. Furman, M. Muszyński, **M. Król**, A. Wincukiewicz, K. Rechcińska, R. Mazur, W. Piecek, M. Kamińska, J. Szczytko, B. Piętka, *International Photonics and OptoElectronics Meeting 2019 (OFDA, OEDI, ISST, PE, LST, TSA)* (Optical Society of America, 2019), p. JW4A.66. [doi:10.1364/ISST.2019.JW4A.66](https://doi.org/10.1364/ISST.2019.JW4A.66)
15. [271] **M. Król**, R. Mirek, K. Lekenta, J.-G. Rousset, D. Stephan, M. Nawrocki, M. Matuszewski, J. Szczytko, W. Pacuski, B. Piętka, Spin polarized semimagnetic exciton-polariton condensate in magnetic field. *Scientific Reports* **8**, 6694 (2018).
[doi:10.1038/s41598-018-25018-2](https://doi.org/10.1038/s41598-018-25018-2)
16. [272] B. Sereďyński, **M. Król**, P. Starzyk, R. Mirek, M. Ściesiek, K. Sobczak, J. Borysiuk, D. Stephan, J.-G. Rousset, J. Szczytko, B. Piętka, W. Pacuski, (Cd,Zn,Mg)Te-based microcavity on MgTe sacrificial buffer: Growth, lift-off, and transmission studies of polaritons. *Phys. Rev. Materials* **2**, 043406 (2018).
[doi:10.1103/PhysRevMaterials.2.043406](https://doi.org/10.1103/PhysRevMaterials.2.043406)
17. [273] J.-G. Rousset, B. Piętka, **M. Król**, R. Mirek, K. Lekenta, J. Szczytko, W. Pacuski, M. Nawrocki, Magnetic field effect on the lasing threshold of a semimagnetic polariton condensate. *Phys. Rev. B* **96**, 125403 (2017).
[doi:10.1103/PhysRevB.96.125403](https://doi.org/10.1103/PhysRevB.96.125403)
18. [274] R. Mirek, **M. Król**, K. Lekenta, J.-G. Rousset, M. Nawrocki, M. Kulczykowski, M. Matuszewski, J. Szczytko, W. Pacuski, B. Piętka, Angular

dependence of giant Zeeman effect for semimagnetic cavity polaritons. *Phys. Rev. B* **95**, 085429 (2017).

[doi:10.1103/PhysRevB.95.085429](https://doi.org/10.1103/PhysRevB.95.085429)

19. [275] B. Piętka, N. Bobrovska, D. Stephan, M. Teich, **M. Król**, S. Winnerl, A. Pashkin, R. Mirek, K. Lekenta, F. Morier-Genoud, H. Schneider, B. Deveaud, M. Helm, M. Matuszewski, J. Szczytko, Doubly dressed bosons: Exciton polaritons in a strong terahertz field. *Phys. Rev. Lett.* **119**, 077403 (2017).
[doi:10.1103/PhysRevLett.119.077403](https://doi.org/10.1103/PhysRevLett.119.077403)
20. [214] B. Piętka, M. R. Molas, N. Bobrovska, **M. Król**, R. Mirek, K. Lekenta, P. Stępnicki, F. Morier-Genoud, J. Szczytko, B. Deveaud, M. Matuszewski, M. Potemski, $2s$ exciton-polariton revealed in an external magnetic field. *Phys. Rev. B* **96**, 081402 (2017).
[doi:10.1103/PhysRevB.96.081402](https://doi.org/10.1103/PhysRevB.96.081402)
21. [276] K. Łempicka, K. Norowski, M. Grzeszczyk, **M. Król**, K. Lekenta, A. Babiński, B. Piętka, J. Szczytko, Relative reflection difference as a method for measuring the thickness of the exfoliated MoSe₂ layers. *Acta Physica Polonica A* **132**, 316 (2017).
[doi:10.12693/aphyspola.132.316](https://doi.org/10.12693/aphyspola.132.316)
22. [277] J.-G. Rousset, B. Piętka, **M. Król**, R. Mirek, K. Lekenta, J. Szczytko, J. Borysiuk, J. Suffczyński, T. Kazimierczuk, M. Goryca, T. Smoleński, P. Kossacki, M. Nawrocki, W. Pacuski, Strong coupling and polariton lasing in Te based microcavities embedding (Cd,Zn)Te quantum wells. *Applied Physics Letters* **107**, 201109 (2015).
[doi:10.1063/1.4935791](https://doi.org/10.1063/1.4935791)
23. [278] B. Piętka, D. Zygmunt, **M. Król**, M. R. Molas, A. A. L. Nicolet, F. Morier-Genoud, J. Szczytko, J. Łusakowski, P. Zięba, I. Tralle, P. Stępnicki, M. Matuszewski, M. Potemski, B. Deveaud, Magnetic field tuning of exciton-polaritons in a semiconductor microcavity. *Phys. Rev. B* **91**, 075309 (2015).
[doi:10.1103/PhysRevB.91.075309](https://doi.org/10.1103/PhysRevB.91.075309)
24. [279] R. Mirek, A. Opala, M. Furman, **M. Król**, K. Tyszka, B. Seredyński, W. Pacuski, J. Suffczyński, J. Szczytko, M. Matuszewski, B. Piętka, Neural networks based on ultrafast time-delayed effects in exciton-polaritons. [arXiv:2201.00225](https://arxiv.org/abs/2201.00225) (2022).

References

- [1] K. J. Vahala, Optical microcavities. *Nature* **424**, 839 (2003). [doi:10.1038/nature01939](https://doi.org/10.1038/nature01939)
- [2] D. K. Armani, T. J. Kippenberg, S. M. Spillane, K. J. Vahala, Ultra-high-Q toroid microcavity on a chip. *Nature* **421**, 925 (2003). [doi:10.1038/nature01371](https://doi.org/10.1038/nature01371)
- [3] Y. Akahane, T. Asano, B.-S. Song, S. Noda, High-Q photonic nanocavity in a two-dimensional photonic crystal. *Nature* **425**, 944 (2003). [doi:10.1038/nature02063](https://doi.org/10.1038/nature02063)
- [4] J. J. Baumberg, J. Aizpurua, M. H. Mikkelsen, D. R. Smith, Extreme nanophotonics from ultrathin metallic gaps. *Nat. Mater.* **18**, 668 (2019). [doi:10.1038/s41563-019-0290-y](https://doi.org/10.1038/s41563-019-0290-y)
- [5] A. V. Kavokin, J. J. Baumberg, G. Malpuech, F. P. Laussy, *Microcavities* (Oxford University Press, 2017).
- [6] N. D. Mermin, The topological theory of defects in ordered media. *Rev. Mod. Phys.* **51**, 591 (1979). [doi:10.1103/RevModPhys.51.591](https://doi.org/10.1103/RevModPhys.51.591)
- [7] V. Savona, *Linear optical properties of semiconductor microcavities with embedded quantum wells* (Springer, Berlin, Heidelberg, 1999), pp. 173–242. [doi:10.1007/bfb0104383](https://doi.org/10.1007/bfb0104383).
- [8] D. W. Berreman, Optics in stratified and anisotropic media: 4×4-matrix formulation. *J. Opt. Soc. Am.* **62**, 502 (1972). [doi:10.1364/JOSA.62.000502](https://doi.org/10.1364/JOSA.62.000502)
- [9] P. Yeh, Optics of anisotropic layered media: A new 4 × 4 matrix algebra. *Surf. Sci.* **96**, 41 (1980). [doi:10.1016/0039-6028\(80\)90293-9](https://doi.org/10.1016/0039-6028(80)90293-9)
- [10] M. Schubert, Polarization-dependent optical parameters of arbitrarily anisotropic homogeneous layered systems. *Phys. Rev. B* **53**, 4265 (1996). [doi:10.1103/PhysRevB.53.4265](https://doi.org/10.1103/PhysRevB.53.4265)

- [11] N. C. Passler, A. Paarmann, Generalized 4×4 matrix formalism for light propagation in anisotropic stratified media: study of surface phonon polaritons in polar dielectric heterostructures. *J. Opt. Soc. Am. B* **34**, 2128 (2017). doi:10.1364/josab.34.002128
- [12] I. H. Malitson, Interspecimen comparison of the refractive index of fused silica. *J. Opt. Soc. Am.* **55**, 1205 (1965). doi:10.1364/josa.55.001205, numerical data after: <https://refractiveindex.info/?shelf=main&book=SiO2&page=Malitson>
- [13] S. Sarkar, V. Gupta, M. Kumar, J. Schubert, P. T. Probst, J. Joseph, T. A. König, Hybridized guided-mode resonances via colloidal plasmonic self-assembled grating. *ACS Appl. Mater. Interfaces* **11**, 13752 (2019). doi:10.1021/acsami.8b20535, numerical data after: <https://refractiveindex.info/?shelf=main&book=TiO2&page=Sarkar>
- [14] Y. Sun, P. Wen, Y. Yoon, G. Liu, M. Steger, L. N. Pfeiffer, K. West, D. W. Snoke, K. A. Nelson, Bose-einstein condensation of long-lifetime polaritons in thermal equilibrium. *Phys. Rev. Lett.* **118**, 016602 (2017). doi:10.1103/PhysRevLett.118.016602
- [15] S. Flågan, D. Riedel, A. Javadi, T. Jakubczyk, P. Maletinsky, R. J. Warburton, High quality-factor diamond-confined open microcavity. [arXiv:2105.08736](https://arxiv.org/abs/2105.08736) (2021).
- [16] <https://www.olympus-lifescience.com/en/objectives/detail/0-DIRECTORY%3A%3ADirFrontend-itemId.511708120.html>.
- [17] I.-C. Khoo, *Liq. Cryst.* (John Wiley & Sons, Inc., Hoboken, NJ, 2022).
- [18] I. Dierking, *Textures of Liquid Crystals* (Wiley, 2003).
- [19] J. W. Goodby, C. Tschierske, P. Raynes, H. Gleeson, T. Kato, P. J. Collings, eds., *Handbook of Liquid Crystals* (Wiley-VCH Verlag GmbH & Co. KGaA, 2014).
- [20] D. Andrienko, Introduction to liquid crystals. *J. Mol. Liq.* **267**, 520 (2018). doi:10.1016/j.molliq.2018.01.175

- [21] K. Rechcińska, M. Król, R. Mazur, P. Morawiak, R. Mirek, K. Łempicka, W. Bardyszewski, M. Matuszewski, P. Kula, W. Piecek, P. G. Lagoudakis, B. Piętka, J. Szczytko, Engineering spin-orbit synthetic Hamiltonians in liquid-crystal optical cavities. *Science* **366**, 727 (2019). [doi:10.1126/science.aay4182](https://doi.org/10.1126/science.aay4182)
- [22] G. Panzarini, L. C. Andreani, A. Armitage, D. Baxter, M. S. Skolnick, V. N. Astratov, J. S. Roberts, A. V. Kavokin, M. R. Vladimirova, M. A. Kaliteevski, Exciton-light coupling in single and coupled semiconductor microcavities: Polariton dispersion and polarization splitting. *Phys. Rev. B* **59**, 5082 (1999). [doi:10.1103/PhysRevB.59.5082](https://doi.org/10.1103/PhysRevB.59.5082)
- [23] K. V. Kavokin, I. A. Shelykh, A. V. Kavokin, G. Malpuech, P. Bigenwald, Quantum theory of spin dynamics of exciton-polaritons in microcavities. *Phys. Rev. Lett.* **92**, 017401 (2004). [doi:10.1103/PhysRevLett.92.017401](https://doi.org/10.1103/PhysRevLett.92.017401)
- [24] A. Kavokin, G. Malpuech, M. Glazov, Optical spin Hall effect. *Phys. Rev. Lett.* **95**, 136601 (2005). [doi:10.1103/PhysRevLett.95.136601](https://doi.org/10.1103/PhysRevLett.95.136601)
- [25] O. Bleu, D. D. Solnyshkov, G. Malpuech, Measuring the quantum geometric tensor in two-dimensional photonic and exciton-polariton systems. *Phys. Rev. B* **97**, 195422 (2018). [doi:10.1103/PhysRevB.97.195422](https://doi.org/10.1103/PhysRevB.97.195422)
- [26] M. Berry, Diabolical points in the spectra of triangles. *Proc. R. Soc. Lond. A* **392**, 15 (1984). [doi:10.1098/rspa.1984.0022](https://doi.org/10.1098/rspa.1984.0022)
- [27] D. Dubbers, H.-J. Stöckmann, *Diabolic Points, Geometric Phases, and Quantum Chaos* (Springer, Berlin, Heidelberg, 2013). [doi:10.1007/978-3-642-31060-7](https://doi.org/10.1007/978-3-642-31060-7).
- [28] D. R. Yarkony, Diabolical conical intersections. *Rev. Mod. Phys.* **68**, 985 (1996). [doi:10.1103/RevModPhys.68.985](https://doi.org/10.1103/RevModPhys.68.985)
- [29] M. Maragkou, C. E. Richards, T. Ostatnický, A. J. D. Grundy, J. Zajac, M. Hugues, W. Langbein, P. G. Lagoudakis, Optical analogue of the spin Hall effect in a photonic cavity. *Opt. Lett.* **36**, 1095 (2011). [doi:10.1364/ol.36.001095](https://doi.org/10.1364/ol.36.001095)
- [30] C. Leyder, M. Romanelli, J. P. Karr, E. Giacobino, T. C. H. Liew, M. M. Glazov, A. V. Kavokin, G. Malpuech, A. Bramati, Observation of the optical spin Hall effect. *Nat. Phys.* **3**, 628 (2007). [doi:10.1038/nphys676](https://doi.org/10.1038/nphys676)

- [31] R. Hivet, H. Flayac, D. D. Solnyshkov, D. Tanese, T. Boulier, D. Andreoli, E. Giacobino, J. Bloch, A. Bramati, G. Malpuech, A. Amo, Half-solitons in a polariton quantum fluid behave like magnetic monopoles. *Nat. Phys.* **8**, 724 (2012). [doi:10.1038/nphys2406](https://doi.org/10.1038/nphys2406)
- [32] S. Klemmt, T. H. Harder, O. A. Egorov, K. Winkler, R. Ge, M. A. Bandres, M. Emmerling, L. Worschech, T. C. H. Liew, M. Segev, C. Schneider, S. Höfling, Exciton-polariton topological insulator. *Nature* **562**, 552 (2018). [doi:10.1038/s41586-018-0601-5](https://doi.org/10.1038/s41586-018-0601-5)
- [33] S. Dufferwiel, F. Li, E. Cancellieri, L. Giriunas, A. Trichet, D. Whittaker, P. Walker, F. Fras, E. Clarke, J. Smith, M. Skolnick, D. Krizhanovskii, Spin textures of exciton-polaritons in a tunable microcavity with large TE-TM splitting. *Phys. Rev. Lett.* **115**, 246401 (2015). [doi:10.1103/physrevlett.115.246401](https://doi.org/10.1103/physrevlett.115.246401)
- [34] J. Ren, Q. Liao, F. Li, Y. Li, O. Bleu, G. Malpuech, J. Yao, H. Fu, D. Solnyshkov, Nontrivial band geometry in an optically active system. *Nat. Commun.* **12**, 689 (2021). [doi:10.1038/s41467-020-20845-2](https://doi.org/10.1038/s41467-020-20845-2)
- [35] H. Terças, H. Flayac, D. D. Solnyshkov, G. Malpuech, Non-Abelian gauge fields in photonic cavities and photonic superfluids. *Phys. Rev. Lett.* **112**, 066402 (2014). [doi:10.1103/PhysRevLett.112.066402](https://doi.org/10.1103/PhysRevLett.112.066402)
- [36] M. S. Spencer, Y. Fu, A. P. Schlaus, D. Hwang, Y. Dai, M. D. Smith, D. R. Gamelin, X.-Y. Zhu, Spin-orbit-coupled exciton-polariton condensates in lead halide perovskites. *Sci. Adv.* **7**, eabj7667 (2021). [doi:10.1126/sciadv.abj7667](https://doi.org/10.1126/sciadv.abj7667)
- [37] L. Polimeno, A. Fieramosca, G. Lerario, L. D. Marco, M. D. Giorgi, D. Ballarini, L. Dominici, V. Ardizzone, M. Pugliese, C. T. Prontera, V. Maiorano, G. Gigli, C. Leblanc, G. Malpuech, D. D. Solnyshkov, D. Sanvitto, Experimental investigation of a non-Abelian gauge field in 2D perovskite photonic platform. *Optica* **8**, 1442 (2021). [doi:10.1364/OPTICA.427088](https://doi.org/10.1364/OPTICA.427088)
- [38] L. Polimeno, G. Lerario, M. D. Giorgi, L. D. Marco, L. Dominici, F. Todisco, A. Coriolano, V. Ardizzone, M. Pugliese, C. T. Prontera, V. Maiorano, A. Moliterni, C. Giannini, V. Olieric, G. Gigli, D. Ballarini, Q. Xiong, A. Fieramosca, D. D. Solnyshkov, G. Malpuech, D. Sanvitto, Tuning of the Berry curvature in 2D perovskite polaritons. *Nat. Nanotechnol.* **16**, 1349 (2021). [doi:10.1038/s41565-021-00977-2](https://doi.org/10.1038/s41565-021-00977-2)

- [39] A. Gianfrate, O. Bleu, L. Dominici, V. Ardizzone, M. D. Giorgi, D. Ballarini, G. Lerario, K. W. West, L. N. Pfeiffer, D. D. Solnyshkov, D. Sanvitto, G. Malpuech, Measurement of the quantum geometric tensor and of the anomalous Hall drift. *Nature* **578**, 381 (2020). [doi:10.1038/s41586-020-1989-2](https://doi.org/10.1038/s41586-020-1989-2)
- [40] D. Biegańska, M. Pieczarka, E. Estrecho, M. Steger, D. W. Snoke, K. West, L. N. Pfeiffer, M. Syperek, A. G. Truscott, E. A. Ostrovskaya, Collective excitations of exciton-polariton condensates in a synthetic gauge field. *Phys. Rev. Lett.* **127**, 185301 (2021). [doi:10.1103/PhysRevLett.127.185301](https://doi.org/10.1103/PhysRevLett.127.185301)
- [41] S. Richter, T. Michalsky, C. Sturm, B. Rosenow, M. Grundmann, R. Schmidt-Grund, Exceptional points in anisotropic planar microcavities. *Phys. Rev. A* **95**, 023836 (2017). [doi:10.1103/PhysRevA.95.023836](https://doi.org/10.1103/PhysRevA.95.023836)
- [42] S. Richter, H.-G. Zirnstein, J. Zúñiga Pérez, E. Krüger, C. Deparis, L. Trefflich, C. Sturm, B. Rosenow, M. Grundmann, R. Schmidt-Grund, Voigt exceptional points in an anisotropic ZnO-based planar microcavity: Square-root topology, polarization vortices, and circularity. *Phys. Rev. Lett.* **123**, 227401 (2019). [doi:10.1103/PhysRevLett.123.227401](https://doi.org/10.1103/PhysRevLett.123.227401)
- [43] R. Su, E. Estrecho, D. Biegańska, Y. Huang, M. Wurdack, M. Pieczarka, A. G. Truscott, T. C. H. Liew, E. A. Ostrovskaya, Q. Xiong, Direct measurement of a non-Hermitian topological invariant in a hybrid light-matter system. *Sci. Adv.* **7**, eabj8905 (2021). [doi:10.1126/sciadv.abj8905](https://doi.org/10.1126/sciadv.abj8905)
- [44] B. A. Bernevig, J. Orenstein, S.-C. Zhang, Exact SU(2) symmetry and persistent spin helix in a spin-orbit coupled system. *Phys. Rev. Lett.* **97**, 236601 (2006). [doi:10.1103/PhysRevLett.97.236601](https://doi.org/10.1103/PhysRevLett.97.236601)
- [45] J. D. Koralek, C. P. Weber, J. Orenstein, B. A. Bernevig, S.-C. Zhang, S. Mack, D. D. Awschalom, Emergence of the persistent spin helix in semiconductor quantum wells. *Nature* **458**, 610 (2009). [doi:10.1038/nature07871](https://doi.org/10.1038/nature07871)
- [46] M. Kohda, G. Salis, Physics and application of persistent spin helix state in semiconductor heterostructures. *Semicond. Sci. Technol.* **32**, 073002 (2017). [doi:10.1088/1361-6641/aa5dd6](https://doi.org/10.1088/1361-6641/aa5dd6)

- [47] M. Król, K. Rechcińska, H. Sigurdsson, P. Oliwa, R. Mazur, P. Morawiak, W. Piecek, P. Kula, P. G. Lagoudakis, M. Matuszewski, W. Bardyszewski, B. Piętka, J. Szczytko, Realizing optical persistent spin helix and Stern-Gerlach deflection in an anisotropic liquid crystal microcavity. *Phys. Rev. Lett.* **127**, 190401 (2021). [doi:10.1103/PhysRevLett.127.190401](https://doi.org/10.1103/PhysRevLett.127.190401)
- [48] M. Muszyński, M. Król, K. Rechcińska, P. Oliwa, M. Kędziora, K. Łempicka-Mirek, R. Mazur, P. Morawiak, W. Piecek, P. Kula, P. G. Lagoudakis, B. Piętka, J. Szczytko, Realizing persistent-spin-helix lasing in the regime of Rashba-Dresselhaus spin-orbit coupling in a dye-filled liquid-crystal optical microcavity. *Phys. Rev. Applied* **17**, 014041 (2022). [doi:10.1103/PhysRevApplied.17.014041](https://doi.org/10.1103/PhysRevApplied.17.014041)
- [49] J. Ren, Q. Liao, X. Ma, S. Schumacher, J. Yao, H. Fu, Realization of exciton-mediated optical spin-orbit interaction in organic microcrystalline resonators. *Laser Photonics Rev.* **16**, 2100252 (2022). [doi:https://doi.org/10.1002/lpor.202100252](https://doi.org/10.1002/lpor.202100252)
- [50] M. Król, H. Sigurdsson, K. Rechcińska, P. Oliwa, K. Tyszka, W. Bardyszewski, A. Opala, M. Matuszewski, P. Morawiak, R. Mazur, W. Piecek, P. Kula, P. G. Lagoudakis, B. Piętka, J. Szczytko, Observation of second-order meron polarization textures in optical microcavities. *Optica* **8**, 255 (2021). [doi:10.1364/OPTICA.414891](https://doi.org/10.1364/OPTICA.414891)
- [51] N. Nagaosa, Y. Tokura, Topological properties and dynamics of magnetic skyrmions. *Nat. Nanotechnol.* **8**, 899 (2013). [doi:10.1038/nnano.2013.243](https://doi.org/10.1038/nnano.2013.243)
- [52] X. Z. Yu, W. Koshibae, Y. Tokunaga, K. Shibata, Y. Taguchi, N. Nagaosa, Y. Tokura, Transformation between meron and skyrmion topological spin textures in a chiral magnet. *Nature* **564**, 95 (2018). [doi:10.1038/s41586-018-0745-3](https://doi.org/10.1038/s41586-018-0745-3)
- [53] H.-B. Braun, Topological effects in nanomagnetism: from superparamagnetism to chiral quantum solitons. *Adv. Phys.* **61**, 1 (2012). [doi:10.1080/00018732.2012.663070](https://doi.org/10.1080/00018732.2012.663070)
- [54] B. Göbel, I. Mertig, O. A. Tretiakov, Beyond skyrmions: Review and perspectives of alternative magnetic quasiparticles. *Phys. Rep.* **895**, 1 (2021). [doi:10.1016/j.physrep.2020.10.001](https://doi.org/10.1016/j.physrep.2020.10.001)
- [55] T. H. R. Skyrme, A unified field theory of mesons and baryons. *Nucl. Phys.* **31**, 556 (1962). [doi:10.1016/0029-5582\(62\)90775-7](https://doi.org/10.1016/0029-5582(62)90775-7)

- [56] P. J. Ackerman, T. Boyle, I. I. Smalyukh, Squirring motion of baby skyrmions in nematic fluids. *Nat. Commun.* **8**, 673 (2017). [doi:10.1038/s41467-017-00659-5](https://doi.org/10.1038/s41467-017-00659-5)
- [57] R. Wiesendanger, Nanoscale magnetic skyrmions in metallic films and multilayers: a new twist for spintronics. *Nat. Rev. Mater.* **1**, 16044 (2016). [doi:10.1038/natrevmats.2016.44](https://doi.org/10.1038/natrevmats.2016.44)
- [58] A. Fert, N. Reyren, V. Cros, Magnetic skyrmions: advances in physics and potential applications. *Nat. Rev. Mater.* **2**, 17031 (2017). [doi:10.1038/natrevmats.2017.31](https://doi.org/10.1038/natrevmats.2017.31)
- [59] A. N. Bogdanov, C. Panagopoulos, Physical foundations and basic properties of magnetic skyrmions. *Nat. Rev. Phys.* **2**, 492 (2020). [doi:10.1038/s42254-020-0203-7](https://doi.org/10.1038/s42254-020-0203-7)
- [60] S. Mühlbauer, D. Honecker, E. A. Périgo, F. Bergner, S. Disch, A. Heinemann, S. Erokhin, D. Berkov, C. Leighton, M. R. Eskildsen, A. Michels, Magnetic small-angle neutron scattering. *Rev. Mod. Phys.* **91**, 015004 (2019). [doi:10.1103/RevModPhys.91.015004](https://doi.org/10.1103/RevModPhys.91.015004)
- [61] N. Romming, C. Hanneken, M. Menzel, J. E. Bickel, B. Wolter, K. von Bergmann, A. Kubetzka, R. Wiesendanger, Writing and deleting single magnetic skyrmions. *Science* **341**, 636 (2013). [doi:10.1126/science.1240573](https://doi.org/10.1126/science.1240573)
- [62] W. Koshibae, N. Nagaosa, Creation of skyrmions and anti-skyrmions by local heating. *Nat. Commun.* **5**, 5148 (2014). [doi:10.1038/ncomms6148](https://doi.org/10.1038/ncomms6148)
- [63] S.-G. Je, P. Vallobra, T. Srivastava, J.-C. Rojas-Sánchez, T. H. Pham, M. Hehn, G. Malinowski, C. Baraduc, S. Auffret, G. Gaudin, S. Mangin, H. Béa, O. Boulle, Creation of magnetic skyrmion bubble lattices by ultrafast laser in ultrathin films. *Nano Lett.* **18**, 7362 (2018). [doi:10.1021/acs.nanolett.8b03653](https://doi.org/10.1021/acs.nanolett.8b03653)
- [64] J. Sampaio, V. Cros, S. Rohart, A. Thiaville, A. Fert, Nucleation, stability and current-induced motion of isolated magnetic skyrmions in nanostructures. *Nat. Nanotechnol.* **8**, 839 (2013). [doi:10.1038/nnano.2013.210](https://doi.org/10.1038/nnano.2013.210)
- [65] X. Zhang, M. Ezawa, Y. Zhou, Magnetic skyrmion logic gates: conversion, duplication and merging of skyrmions. *Sci. Rep.* **5**, 9400 (2015). [doi:10.1038/srep09400](https://doi.org/10.1038/srep09400)

- [66] A. N. Bogdanov, Y. D. A., Thermodynamically stable ‘vortices’ in magnetically ordered crystals. the mixed state of magnets. *Sov. Phys. JETP* **68**, 101 (1989)
- [67] U. K. Rößler, A. N. Bogdanov, C. Pfleiderer, Spontaneous skyrmion ground states in magnetic metals. *Nature* **442**, 797 (2006). [doi:10.1038/nature05056](https://doi.org/10.1038/nature05056)
- [68] S. Mühlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, P. Böni, Skyrmion lattice in a chiral magnet. *Science* **323**, 915 (2009). [doi:10.1126/science.1166767](https://doi.org/10.1126/science.1166767)
- [69] I. Kézsmárki, S. Bordács, P. Milde, E. Neuber, L. M. Eng, J. S. White, H. M. Rønnow, C. D. Dewhurst, M. Mochizuki, K. Yanai, H. Nakamura, D. Ehlers, V. Tsurkan, A. Loidl, Néel-type skyrmion lattice with confined orientation in the polar magnetic semiconductor GaV₄S₈. *Nat. Mater.* **14**, 1116 (2015). [doi:10.1038/nmat4402](https://doi.org/10.1038/nmat4402)
- [70] A. K. Nayak, V. Kumar, T. Ma, P. Werner, E. Pippel, R. Sahoo, F. Dammay, U. K. Rößler, C. Felser, S. S. P. Parkin, Magnetic antiskyrmions above room temperature in tetragonal Heusler materials. *Nature* **548**, 561 (2017). [doi:10.1038/nature23466](https://doi.org/10.1038/nature23466)
- [71] X. Z. Yu, Y. Tokunaga, Y. Kaneko, W. Z. Zhang, K. Kimoto, Y. Matsui, Y. Taguchi, Y. Tokura, Biskyrmion states and their current-driven motion in a layered manganite. *Nat. Commun* **5**, 3198 (2014). [doi:10.1038/ncomms4198](https://doi.org/10.1038/ncomms4198)
- [72] M. Heigl, S. Koraltan, M. Vanatka, R. Kraft, C. Abert, C. Vogler, A. Semisalova, P. Che, A. Ullrich, T. Schmidt, J. Hintermayr, D. Grundler, M. Farle, M. Urbanek, D. Suess, M. Albrecht, Dipolar-stabilized first and second-order antiskyrmions in ferrimagnetic multilayers. [arXiv:2010.06555](https://arxiv.org/abs/2010.06555) (2020).
- [73] T. Shinjo, T. Okuno, R. Hassdorf, K. Shigeto, T. Ono, Magnetic vortex core observation in circular dots of permalloy. *Science* **289**, 930 (2000). [doi:10.1126/science.289.5481.930](https://doi.org/10.1126/science.289.5481.930)
- [74] C. Phatak, A. K. Petford-Long, O. Heinonen, Direct observation of unconventional topological spin structure in coupled magnetic discs. *Phys. Rev. Lett.* **108**, 067205 (2012). [doi:10.1103/PhysRevLett.108.067205](https://doi.org/10.1103/PhysRevLett.108.067205)

- [75] N. Gao, S.-G. Je, M.-Y. Im, J. W. Choi, M. Yang, Q. Li, T. Y. Wang, S. Lee, H.-S. Han, K.-S. Lee, W. Chao, C. Hwang, J. Li, Z. Q. Qiu, Creation and annihilation of topological meron pairs in in-plane magnetized films. *Nat. Commun.* **10**, 5603 (2019). doi:10.1038/s41467-019-13642-z
- [76] S. Tsesses, E. Ostrovsky, K. Cohen, B. Gjonaj, N. H. Lindner, G. Bartal, Optical skyrmion lattice in evanescent electromagnetic fields. *Science* **361**, 993 (2018). doi:10.1126/science.aau0227
- [77] T. J. Davis, D. Janoschka, P. Dreher, B. Frank, F.-J. Meyer zu Heringdorf, H. Giessen, Ultrafast vector imaging of plasmonic skyrmion dynamics with deep subwavelength resolution. *Science* **368**, eaba6415 (2020). doi:10.1126/science.aba6415
- [78] Q. Zhang, Z. Xie, L. Du, P. Shi, X. Yuan, Bloch-type photonic skyrmions in optical chiral multilayers. *arXiv:2012.01015* (2020).
- [79] L. Du, A. Yang, A. V. Zayats, X. Yuan, Deep-subwavelength features of photonic skyrmions in a confined electromagnetic field with orbital angular momentum. *Nat. Phys.* **15**, 650 (2019). doi:10.1038/s41567-019-0487-7
- [80] C. Guo, M. Xiao, Y. Guo, L. Yuan, S. Fan, Meron spin textures in momentum space. *Phys. Rev. Lett.* **124**, 106103 (2020). doi:10.1103/PhysRevLett.124.106103
- [81] P. Cilibrizzi, H. Sigurdsson, T. C. H. Liew, H. Ohadi, A. Askitopoulos, S. Brodbeck, C. Schneider, I. A. Shelykh, S. Höfling, J. Ruostekoski, P. Lagoudakis, Half-skyrmion spin textures in polariton microcavities. *Phys. Rev. B* **94**, 045315 (2016). doi:10.1103/PhysRevB.94.045315
- [82] S. Donati, L. Dominici, G. Dagvadorj, D. Ballarini, M. D. Giorgi, A. Bramati, G. Gigli, Y. G. Rubo, M. H. Szymańska, D. Sanvitto, Twist of generalized skyrmions and spin vortices in a polariton superfluid. *Proc. Natl. Acad. Sci. U.S.A.* **113**, 14926 (2016). doi:10.1073/pnas.1610123114
- [83] L. Marrucci, C. Manzo, D. Paparo, Optical spin-to-orbital angular momentum conversion in inhomogeneous anisotropic media. *Phys. Rev. Lett.* **96**, 163905 (2006). doi:10.1103/PhysRevLett.96.163905
- [84] H. Flayac, D. D. Solnyshkov, I. A. Shelykh, G. Malpuech, Transmutation of skyrmions to half-solitons driven by the nonlinear

- ar optical spin Hall effect. *Phys. Rev. Lett.* **110**, 016404 (2013). [doi:10.1103/PhysRevLett.110.016404](https://doi.org/10.1103/PhysRevLett.110.016404)
- [85] K. Lekenta, M. Król, R. Mirek, K. Łempicka, D. Stephan, R. Mazur, P. Morawiak, P. Kula, W. Piecek, P. G. Lagoudakis, B. Piętka, J. Szczytko, Tunable optical spin Hall effect in a liquid crystal microcavity. *Light Sci. Appl.* **7**, 74 (2018). [doi:10.1038/s41377-018-0076-z](https://doi.org/10.1038/s41377-018-0076-z)
- [86] W. Langbein, I. Shelykh, D. Solnyshkov, G. Malpuech, Y. Rubo, A. Kavokin, Polarization beats in ballistic propagation of exciton-polaritons in microcavities. *Phys. Rev. B* **75**, 075323 (2007). [doi:10.1103/PhysRevB.75.075323](https://doi.org/10.1103/PhysRevB.75.075323)
- [87] A. Amo, T. C. H. Liew, C. Adrados, E. Giacobino, A. V. Kavokin, A. Bramati, Anisotropic optical spin Hall effect in semiconductor microcavities. *Phys. Rev. B* **80**, 165325 (2009). [doi:10.1103/PhysRevB.80.165325](https://doi.org/10.1103/PhysRevB.80.165325)
- [88] G. Panzarini, L. C. Andreani, A. Armitage, D. Baxter, M. S. Skolnick, V. N. Astratov, J. S. Roberts, A. V. Kavokin, M. R. Vladimirova, M. A. Kaliteevski, Exciton-light coupling in single and coupled semiconductor microcavities: Polariton dispersion and polarization splitting. *Phys. Rev. B* **59**, 5082 (1999). [doi:10.1103/PhysRevB.59.5082](https://doi.org/10.1103/PhysRevB.59.5082)
- [89] P. Cilibrizzi, H. Sigurdsson, T. C. H. Liew, H. Ohadi, S. Wilkinson, A. Askitopoulos, I. A. Shelykh, P. G. Lagoudakis, Polariton spin whirls. *Phys. Rev. B* **92**, 155308 (2015). [doi:10.1103/PhysRevB.92.155308](https://doi.org/10.1103/PhysRevB.92.155308)
- [90] M. Król, I. Settembre, P. Oliwa, M. Kędziora, K. Łempicka Mirek, M. Muszyński, R. Mazur, P. Morawiak, W. Piecek, P. Kula, W. Bardyszewski, P. G. Lagoudakis, D. D. Solnyshkov, G. Malpuech, B. Piętka, J. Szczytko, Annihilation of exceptional points from different Dirac valleys in a 2D photonic system. [arXiv:2112.06621](https://arxiv.org/abs/2112.06621) (2021).
- [91] C. M. Bender, S. Boettcher, Real spectra in non-Hermitian Hamiltonians having $\mathcal{PT}$ symmetry. *Phys. Rev. Lett.* **80**, 5243 (1998). [doi:10.1103/PhysRevLett.80.5243](https://doi.org/10.1103/PhysRevLett.80.5243)
- [92] Y. Ashida, Z. Gong, M. Ueda, Non-Hermitian physics. *Adv. Phys.* **69**, 249 (2020). [doi:10.1080/00018732.2021.1876991](https://doi.org/10.1080/00018732.2021.1876991)

- [93] O. N. Kirillov, A. A. Mailybaev, A. P. Seyranian, Unfolding of eigenvalue surfaces near a diabolic point due to a complex perturbation. *J. Phys. A: Math. Gen.* **38**, 5531 (2005). doi:10.1088/0305-4470/38/24/007
- [94] H. Zhou, C. Peng, Y. Yoon, C. W. Hsu, K. A. Nelson, L. Fu, J. D. Joannopoulos, M. Soljačić, B. Zhen, Observation of bulk Fermi arc and polarization half charge from paired exceptional points. *Science* **359**, 1009 (2018). doi:10.1126/science.aap9859
- [95] C. Shi, M. Dubois, Y. Chen, L. Cheng, H. Ramezani, Y. Wang, X. Zhang, Accessing the exceptional points of parity-time symmetric acoustics. *Nat. Commun.* **7**, 11110 (2016). doi:10.1038/ncomms11110
- [96] J. Schindler, A. Li, M. C. Zheng, F. M. Ellis, T. Kottos, Experimental study of active lrc circuits with $\mathcal{PT}$ symmetries. *Phys. Rev. A* **84**, 040101 (2011). doi:10.1103/PhysRevA.84.040101
- [97] C. Dembowski, H.-D. Gräf, H. L. Harney, A. Heine, W. D. Heiss, H. Rehfeld, A. Richter, Experimental observation of the topological structure of exceptional points. *Phys. Rev. Lett.* **86**, 787 (2001). doi:10.1103/PhysRevLett.86.787
- [98] C. M. Bender, B. K. Berntson, D. Parker, E. Samuel, Observation of $\mathcal{PT}$ phase transition in a simple mechanical system. *Am. J. Phys.* **81**, 173 (2013). doi:10.1119/1.4789549
- [99] Z. Zhang, Y. Zhang, J. Sheng, L. Yang, M.-A. Miri, D. N. Christodoulides, B. He, Y. Zhang, M. Xiao, Observation of parity-time symmetry in optically induced atomic lattices. *Phys. Rev. Lett.* **117**, 123601 (2016). doi:10.1103/PhysRevLett.117.123601
- [100] B. Zhen, C. W. Hsu, Y. Igarashi, L. Lu, I. Kaminer, A. Pick, S.-L. Chua, J. D. Joannopoulos, M. Soljačić, Spawning rings of exceptional points out of dirac cones. *Nature* **525**, 354 (2015). doi:10.1038/nature14889
- [101] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, D. Kip, Observation of parity-time symmetry in optics. *Nat. Phys.* **6**, 192 (2010). doi:10.1038/nphys1515
- [102] Ş. K. Özdemir, S. Rotter, F. Nori, L. Yang, Parity-time symmetry and exceptional points in photonics. *Nat. Mater.* **18**, 783 (2019). doi:10.1038/s41563-019-0304-9

- [103] W. Chen, Ş. K. Özdemir, G. Zhao, J. Wiersig, L. Yang, Exceptional points enhance sensing in an optical microcavity. *Nature* **548**, 192 (2017). [doi:10.1038/nature23281](https://doi.org/10.1038/nature23281)
- [104] H. Hodaei, A. U. Hassan, S. Wittek, H. Garcia-Gracia, R. El-Ganainy, D. N. Christodoulides, M. Khajavikhan, Enhanced sensitivity at higher-order exceptional points. *Nature* **548**, 187 (2017). [doi:10.1038/nature23280](https://doi.org/10.1038/nature23280)
- [105] H. Wang, Y.-H. Lai, Z. Yuan, M.-G. Suh, K. Vahala, Petermann-factor sensitivity limit near an exceptional point in a Brillouin ring laser gyroscope. *Nat. Commun.* **11**, 1610 (2020). [doi:10.1038/s41467-020-15341-6](https://doi.org/10.1038/s41467-020-15341-6)
- [106] J. Wiersig, Review of exceptional point-based sensors. *Photon. Res.* **8**, 1457 (2020). [doi:10.1364/PRJ.396115](https://doi.org/10.1364/PRJ.396115)
- [107] M. Liertzer, L. Ge, A. Cerjan, A. D. Stone, H. E. Türeci, S. Rotter, Pump-induced exceptional points in lasers. *Phys. Rev. Lett.* **108**, 173901 (2012). [doi:10.1103/PhysRevLett.108.173901](https://doi.org/10.1103/PhysRevLett.108.173901)
- [108] B. Peng, Ş. K. Özdemir, S. Rotter, H. Yilmaz, M. Liertzer, F. Monifi, C. M. Bender, F. Nori, L. Yang, Loss-induced suppression and revival of lasing. *Science* **346**, 328 (2014). [doi:10.1126/science.1258004](https://doi.org/10.1126/science.1258004)
- [109] H. Hodaei, M.-A. Miri, M. Heinrich, D. N. Christodoulides, M. Khajavikhan, Parity-time-symmetric microring lasers. *Science* **346**, 975 (2014). [doi:10.1126/science.1258480](https://doi.org/10.1126/science.1258480)
- [110] L. Feng, Z. J. Wong, R.-M. Ma, Y. Wang, X. Zhang, Single-mode laser by parity-time symmetry breaking. *Science* **346**, 972 (2014). [doi:10.1126/science.1258479](https://doi.org/10.1126/science.1258479)
- [111] H.-M. Lauber, P. Weidenhammer, D. Dubbers, Geometric phases and hidden symmetries in simple resonators. *Phys. Rev. Lett.* **72**, 1004 (1994). [doi:10.1103/PhysRevLett.72.1004](https://doi.org/10.1103/PhysRevLett.72.1004)
- [112] S.-B. Lee, J. Yang, S. Moon, S.-Y. Lee, J.-B. Shim, S. W. Kim, J.-H. Lee, K. An, Observation of an exceptional point in a chaotic optical microcavity. *Phys. Rev. Lett.* **103**, 134101 (2009). [doi:10.1103/PhysRevLett.103.134101](https://doi.org/10.1103/PhysRevLett.103.134101)
- [113] T. Gao, E. Estrecho, K. Y. Bliokh, T. C. H. Liew, M. D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y. S.

- Kivshar, A. G. Truscott, R. G. Dall, E. A. Ostrovskaya, Observation of non-Hermitian degeneracies in a chaotic exciton-polariton billiard. *Nature* **526**, 554 (2015). [doi:10.1038/nature15522](https://doi.org/10.1038/nature15522)
- [114] J. Doppler, A. A. Mailybaev, J. Böhm, U. Kuhl, A. Girschik, F. Libisch, T. J. Milburn, P. Rabl, N. Moiseyev, S. Rotter, Dynamically encircling an exceptional point for asymmetric mode switching. *Nature* **537**, 76 (2016). [doi:10.1038/nature18605](https://doi.org/10.1038/nature18605)
- [115] H. Xu, D. Mason, L. Jiang, J. G. E. Harris, Topological energy transfer in an optomechanical system with exceptional points. *Nature* **537**, 80 (2016). [doi:10.1038/nature18604](https://doi.org/10.1038/nature18604)
- [116] A. Schumer, Y. G. N. Liu, J. Leshin, L. Ding, Y. Alahmadi, A. U. Hassan, H. Nasari, S. Rotter, D. N. Christodoulides, P. LiKamWa, M. Khajavikhan, Topological modes in a laser cavity through exceptional state transfer. *Science* **375**, 884 (2022). [doi:10.1126/science.abl6571](https://doi.org/10.1126/science.abl6571)
- [117] L. Feng, X. Zhu, S. Yang, H. Zhu, P. Zhang, X. Yin, Y. Wang, X. Zhang, Demonstration of a large-scale optical exceptional point structure. *Opt. Express* **22**, 1760 (2014). [doi:10.1364/OE.22.001760](https://doi.org/10.1364/OE.22.001760)
- [118] A. U. Hassan, B. Zhen, M. Soljačić, M. Khajavikhan, D. N. Christodoulides, Dynamically encircling exceptional points: Exact evolution and polarization state conversion. *Phys. Rev. Lett.* **118**, 093002 (2017). [doi:10.1103/PhysRevLett.118.093002](https://doi.org/10.1103/PhysRevLett.118.093002)
- [119] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, D. N. Christodoulides, Non-hermitian physics and PT symmetry. *Nat. Phys.* **14**, 11 (2018). [doi:10.1038/nphys4323](https://doi.org/10.1038/nphys4323)
- [120] M.-A. Miri, A. Alù, Exceptional points in optics and photonics. *Science* **363**, eaar7709 (2019). [doi:10.1126/science.aar7709](https://doi.org/10.1126/science.aar7709)
- [121] E. J. Bergholtz, J. C. Budich, F. K. Kunst, Exceptional topology of non-Hermitian systems. *Rev. Mod. Phys.* **93**, 015005 (2021). [doi:10.1103/RevModPhys.93.015005](https://doi.org/10.1103/RevModPhys.93.015005)
- [122] Q. Liao, C. Leblanc, J. Ren, F. Li, Y. Li, D. Solnyshkov, G. Malpuech, J. Yao, H. Fu, Experimental measurement of the divergent quantum metric of an exceptional point. *Phys. Rev. Lett.* **127**, 107402 (2021). [doi:10.1103/PhysRevLett.127.107402](https://doi.org/10.1103/PhysRevLett.127.107402)

- [123] M. Król, K. Rechcińska, K. Nogajewski, M. Grzeszczyk, K. Łempicka, R. Mirek, S. Piotrowska, K. Watanabe, T. Taniguchi, M. R. Molas, M. Potemski, J. Szczytko, B. Piętka, Exciton-polaritons in multilayer WSe₂ in a planar microcavity. *2D Mater.* **7**, 015006 (2020). [doi:10.1088/2053-1583/ab4b14](https://doi.org/10.1088/2053-1583/ab4b14)
- [124] M. Król, K. Lekenta, R. Mirek, K. Łempicka, D. Stephan, K. Nogajewski, M. R. Molas, A. Babiński, M. Potemski, J. Szczytko, B. Piętka, Valley polarization of exciton-polaritons in monolayer WSe₂ in a tunable microcavity. *Nanoscale* **11**, 9574 (2019). [doi:10.1039/c9nr02038a](https://doi.org/10.1039/c9nr02038a)
- [125] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva, A. A. Firsov, Electric field effect in atomically thin carbon films. *Science* **306**, 666 (2004). [doi:10.1126/science.1102896](https://doi.org/10.1126/science.1102896)
- [126] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, A. K. Geim, The electronic properties of graphene. *Rev. Mod. Phys.* **81**, 109 (2009). [doi:10.1103/RevModPhys.81.109](https://doi.org/10.1103/RevModPhys.81.109)
- [127] B. Radisavljevic, A. Radenovic, J. Brivio, V. Giacometti, A. Kis, Single-layer MoS₂ transistors. *Nat. Nanotechnol.* **6**, 147 (2011). [doi:10.1038/nnano.2010.279](https://doi.org/10.1038/nnano.2010.279)
- [128] A. Splendiani, L. Sun, Y. Zhang, T. Li, J. Kim, C.-Y. Chim, G. Galli, F. Wang, Emerging photoluminescence in monolayer MoS₂. *Nano Lett.* **10**, 1271 (2010). [doi:10.1021/nl903868w](https://doi.org/10.1021/nl903868w)
- [129] K. F. Mak, C. Lee, J. Hone, J. Shan, T. F. Heinz, Atomically thin MoS₂: A new direct-gap semiconductor. *Phys. Rev. Lett.* **105**, 136805 (2010). [doi:10.1103/PhysRevLett.105.136805](https://doi.org/10.1103/PhysRevLett.105.136805)
- [130] R. A. Bromley, R. B. Murray, A. D. Yoffe, The band structures of some transition metal dichalcogenides. III. Group VIA: trigonal prism materials. *Journal of Physics C Solid State Physics* **5**, 759 (1972). [doi:10.1088/0022-3719/5/7/007](https://doi.org/10.1088/0022-3719/5/7/007)
- [131] A. K. Geim, I. V. Grigorieva, Van der Waals heterostructures. *Nature* **499**, 419 (2013). [doi:10.1038/nature12385](https://doi.org/10.1038/nature12385)
- [132] H. Li, S. Ruan, Y.-J. Zeng, Intrinsic van der Waals magnetic materials from bulk to the 2D limit: New frontiers of spintronics. *Adv. Mater.* **31**, 1900065 (2019). [doi:10.1002/adma.201900065](https://doi.org/10.1002/adma.201900065)

- [133] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, P. Jarillo-Herrero, Unconventional superconductivity in magic-angle graphene superlattices. *Nature* **556**, 43 (2018). [doi:10.1038/nature26160](https://doi.org/10.1038/nature26160)
- [134] A. Ciarrocchi, F. Tagarelli, A. Avsar, A. Kis, Excitonic devices with van der waals heterostructures: valleytronics meets twistronics. *Nat. Rev. Mater.* (2022). [doi:10.1038/s41578-021-00408-7](https://doi.org/10.1038/s41578-021-00408-7)
- [135] D. Xiao, W. Yao, Q. Niu, Valley-contrasting physics in graphene: Magnetic moment and topological transport. *Phys. Rev. Lett.* **99**, 236809 (2007). [doi:10.1103/PhysRevLett.99.236809](https://doi.org/10.1103/PhysRevLett.99.236809)
- [136] G. Wang, A. Chernikov, M. M. Glazov, T. F. Heinz, X. Marie, T. Amand, B. Urbaszek, Colloquium: Excitons in atomically thin transition metal dichalcogenides. *Rev. Mod. Phys.* **90**, 021001 (2018). [doi:10.1103/RevModPhys.90.021001](https://doi.org/10.1103/RevModPhys.90.021001)
- [137] D. Xiao, G.-B. Liu, W. Feng, X. Xu, W. Yao, Coupled spin and valley physics in monolayers of MoS₂ and other group-VI dichalcogenides. *Phys. Rev. Lett.* **108**, 196802 (2012). [doi:10.1103/PhysRevLett.108.196802](https://doi.org/10.1103/PhysRevLett.108.196802)
- [138] A. Chernikov, T. C. Berkelbach, H. M. Hill, A. Rigosi, Y. Li, O. B. Aslan, D. R. Reichman, M. S. Hybertsen, T. F. Heinz, Exciton binding energy and nonhydrogenic Rydberg series in monolayer WS₂. *Phys. Rev. Lett.* **113**, 076802 (2014). [doi:10.1103/PhysRevLett.113.076802](https://doi.org/10.1103/PhysRevLett.113.076802)
- [139] M. R. Molas, A. O. Slobodeniuk, K. Nogajewski, M. Bartos, L. Balala, A. Babiński, K. Watanabe, T. Taniguchi, C. Faugeras, M. Potemski, Energy spectrum of two-dimensional excitons in a nonuniform dielectric medium. *Phys. Rev. Lett.* **123**, 136801 (2019). [doi:10.1103/PhysRevLett.123.136801](https://doi.org/10.1103/PhysRevLett.123.136801)
- [140] M. Goryca, J. Li, A. V. Stier, T. Taniguchi, K. Watanabe, E. Courtade, S. Shree, C. Robert, B. Urbaszek, X. Marie, S. A. Crooker, Revealing exciton masses and dielectric properties of monolayer semiconductors with high magnetic fields. *Nat. Commun.* **10**, 4172 (2019). [doi:10.1038/s41467-019-12180-y](https://doi.org/10.1038/s41467-019-12180-y)
- [141] A. Raja, A. Chaves, J. Yu, G. Arefe, H. M. Hill, A. F. Rigosi, T. C. Berkelbach, P. Nagler, C. Schüller, T. Korn, C. Nuckolls, J. Hone, L. E. Brus, T. F. Heinz, D. R. Reichman, A. Chernikov, Coulomb engineering of

- the bandgap and excitons in two-dimensional materials. *Nat. Commun.* **8**, 15251 (2017). doi:10.1038/ncomms15251
- [142] K. F. Mak, K. He, J. Shan, T. F. Heinz, Control of valley polarization in monolayer MoS₂ by optical helicity. *Nat. Nanotechnol.* **7**, 494 (2012). doi:10.1038/nnano.2012.96
- [143] H. Zeng, J. Dai, W. Yao, D. Xiao, X. Cui, Valley polarization in MoS₂ monolayers by optical pumping. *Nat. Nanotechnol.* **7**, 490 (2012). doi:10.1038/nnano.2012.95
- [144] A. M. Jones, H. Yu, N. J. Ghimire, S. Wu, G. Aivazian, J. S. Ross, B. Zhao, J. Yan, D. G. Mandrus, D. Xiao, W. Yao, X. Xu, Optical generation of excitonic valley coherence in monolayer WSe₂. *Nat. Nanotechnol.* **8**, 634 (2013). doi:10.1038/nnano.2013.151
- [145] R. Schmidt, A. Arora, G. Plechinger, P. Nagler, A. Granados del Águila, M. V. Ballottin, P. C. M. Christianen, S. Michaelis de Vasconcellos, C. Schüller, T. Korn, R. Bratschitsch, Magnetic-field-induced rotation of polarized light emission from monolayer WS₂. *Phys. Rev. Lett.* **117**, 077402 (2016). doi:10.1103/PhysRevLett.117.077402
- [146] A. Rycerz, J. Tworzydło, C. W. J. Beenakker, Valley filter and valley valve in graphene. *Nat. Phys.* **3**, 172 (2007). doi:10.1038/nphys547
- [147] J. R. Schaibley, H. Yu, G. Clark, P. Rivera, J. S. Ross, K. L. Seyler, W. Yao, X. Xu, Valleytronics in 2D materials. *Nat. Rev. Mater.* **1**, 16055 (2016). doi:10.1038/natrevmats.2016.55
- [148] K. F. Mak, K. He, C. Lee, G. H. Lee, J. Hone, T. F. Heinz, J. Shan, Tightly bound trions in monolayer MoS₂. *Nat. Mater.* **12**, 207 (2012). doi:10.1038/nmat3505
- [149] J. Shang, X. Shen, C. Cong, N. Peimyoo, B. Cao, M. Eginligil, T. Yu, Observation of excitonic fine structure in a 2D transition-metal dichalcogenide semiconductor. *ACS Nano* **9**, 647 (2015). doi:10.1021/nn5059908
- [150] M. Barbone, A. R.-P. Montblanch, D. M. Kara, C. Palacios-Berraquero, A. R. Cadore, D. D. Fazio, B. Pingault, E. Mostaani, H. Li, B. Chen, K. Watanabe, T. Taniguchi, S. Tongay, G. Wang, A. C. Ferrari, M. Atatüre, Charge-tuneable biexciton complexes in monolayer WSe₂. *Nat. Commun.* **9**, 3721 (2018). doi:10.1038/s41467-018-05632-4

- [151] M. Sidler, P. Back, O. Cotlet, A. Srivastava, T. Fink, M. Kroner, E. Demler, A. Imamoglu, Fermi polaron-polaritons in charge-tunable atomically thin semiconductors. *Nat. Phys.* **13**, 255 (2016). [doi:10.1038/nphys3949](https://doi.org/10.1038/nphys3949)
- [152] P. Rivera, H. Yu, K. L. Seyler, N. P. Wilson, W. Yao, X. Xu, Interlayer valley excitons in heterobilayers of transition metal dichalcogenides. *Nat. Nanotechnol.* **13**, 1004 (2018). [doi:10.1038/s41565-018-0193-0](https://doi.org/10.1038/s41565-018-0193-0)
- [153] I. C. Gerber, E. Courtade, S. Shree, C. Robert, T. Taniguchi, K. Watanabe, A. Balocchi, P. Renucci, D. Lagarde, X. Marie, B. Urbaszek, Interlayer excitons in bilayer MoS₂ with strong oscillator strength up to room temperature. *Phys. Rev. B* **99**, 035443 (2019). [doi:10.1103/PhysRevB.99.035443](https://doi.org/10.1103/PhysRevB.99.035443)
- [154] Z. Sun, J. Beaumariage, Q. Wan, H. Alnatah, N. Hougland, J. Chisholm, Q. Cao, K. Watanabe, T. Taniguchi, B. M. Hunt, I. V. Bondarev, D. Snoke, Charged bosons made of fermions in bilayer structures with strong metallic screening. *Nano Lett.* **21**, 7669 (2021). [doi:10.1021/acs.nanolett.1c02422](https://doi.org/10.1021/acs.nanolett.1c02422)
- [155] K. S. Novoselov, D. Jiang, F. Schedin, T. J. Booth, V. V. Khotkevich, S. V. Morozov, A. K. Geim, Two-dimensional atomic crystals. *Proceedings of the National Academy of Sciences* **102**, 10451 (2005). [doi:10.1073/pnas.0502848102](https://doi.org/10.1073/pnas.0502848102)
- [156] J. Zhou, J. Lin, X. Huang, Y. Zhou, Y. Chen, J. Xia, H. Wang, Y. Xie, H. Yu, J. Lei, D. Wu, F. Liu, Q. Fu, Q. Zeng, C.-H. Hsu, C. Yang, L. Lu, T. Yu, Z. Shen, H. Lin, B. I. Yakobson, Q. Liu, K. Suenaga, G. Liu, Z. Liu, A library of atomically thin metal chalcogenides. *Nature* **556**, 355 (2018). [doi:10.1038/s41586-018-0008-3](https://doi.org/10.1038/s41586-018-0008-3)
- [157] Y. Zhang, T.-R. Chang, B. Zhou, Y.-T. Cui, H. Yan, Z. Liu, F. Schmitt, J. Lee, R. Moore, Y. Chen, H. Lin, H.-T. Jeng, S.-K. Mo, Z. Hussain, A. Bansil, Z.-X. Shen, Direct observation of the transition from indirect to direct bandgap in atomically thin epitaxial MoSe₂. *Nat. Nanotechnol.* **9**, 111 (2014). [doi:10.1038/nnano.2013.277](https://doi.org/10.1038/nnano.2013.277)
- [158] W. Pacuski, M. Grzeszczyk, K. Nogajewski, A. Bogucki, K. Oreszczuk, J. Kucharek, K. E. Połczyńska, B. Seredyński, A. Rodek, R. Bożek, T. Taniguchi, K. Watanabe, S. Kret, J. Sadowski, T. Kazimierczuk, M. Potemski, P. Kossacki, Narrow excitonic lines and large-scale homogeneity of transition-metal dichalcogenide monolayers grown by

- molecular beam epitaxy on hexagonal boron nitride. *Nano Lett.* **20**, 3058 (2020). doi:10.1021/acs.nanolett.9b04998
- [159] T. H. Choudhury, X. Zhang, Z. Y. Al Balushi, M. Chubarov, J. M. Redwing, Epitaxial growth of two-dimensional layered transition metal dichalcogenides. *Annu. Rev. Mater. Res.* **50**, 155 (2020). doi:10.1146/annurev-matsci-090519-113456
- [160] A. Castellanos-Gomez, M. Buscema, R. Molenaar, V. Singh, L. Janssen, H. S. J. van der Zant, G. A. Steele, Deterministic transfer of two-dimensional materials by all-dry viscoelastic stamping. *2D Mater.* **1**, 011002 (2014). doi:10.1088/2053-1583/1/1/011002
- [161] S. Masubuchi, M. Morimoto, S. Morikawa, M. Onodera, Y. Asakawa, K. Watanabe, T. Taniguchi, T. Machida, Autonomous robotic searching and assembly of two-dimensional crystals to build van der Waals superlattices. *Nat. Commun.* **9**, 1413 (2018). doi:10.1038/s41467-018-03723-w
- [162] J.-S. Rhyee, J. Kwon, P. Dak, J. H. Kim, S. M. Kim, J. Park, Y. K. Hong, W. G. Song, I. Omkaram, M. A. Alam, S. Kim, High-mobility transistors based on large-area and highly crystalline CVD-grown MoSe₂ films on insulating substrates. *Adv. Mater.* **28**, 2316 (2016). doi:10.1002/adma.201504789
- [163] D.-H. Lien, J. S. Kang, M. Amani, K. Chen, M. Tosun, H.-P. Wang, T. Roy, M. S. Eggleston, M. C. Wu, M. Dubey, S.-C. Lee, J.-H. He, A. Javey, Engineering light outcoupling in 2D materials. *Nano Lett.* **15**, 1356 (2015). doi:10.1021/nl504632u
- [164] F. Cadiz, E. Courtade, C. Robert, G. Wang, Y. Shen, H. Cai, T. Taniguchi, K. Watanabe, H. Carrere, D. Lagarde, M. Manca, T. Amand, P. Renucci, S. Tongay, X. Marie, B. Urbaszek, Excitonic linewidth approaching the homogeneous limit in MoS₂-based van der Waals heterostructures. *Phys. Rev. X* **7**, 021026 (2017). doi:10.1103/PhysRevX.7.021026
- [165] H. H. Fang, B. Han, C. Robert, M. A. Semina, D. Lagarde, E. Courtade, T. Taniguchi, K. Watanabe, T. Amand, B. Urbaszek, M. M. Glazov, X. Marie, Control of the exciton radiative lifetime in van der Waals heterostructures. *Phys. Rev. Lett.* **123**, 067401 (2019). doi:10.1103/PhysRevLett.123.067401

- [166] C. Lee, H. Yan, L. E. Brus, T. F. Heinz, J. Hone, S. Ryu, Anomalous lattice vibrations of single- and few-layer MoS₂. *ACS Nano* **4**, 2695 (2010). doi:10.1021/nn1003937
- [167] C. R. Zhu, G. Wang, B. L. Liu, X. Marie, X. F. Qiao, X. Zhang, X. X. Wu, H. Fan, P. H. Tan, T. Amand, B. Urbaszek, Strain tuning of optical emission energy and polarization in monolayer and bilayer MoS₂. *Phys. Rev. B* **88**, 121301 (2013). doi:10.1103/PhysRevB.88.121301
- [168] M. Koperski, M. R. Molas, A. Arora, K. Nogajewski, A. O. Slobodeniuk, C. Faugeras, M. Potemski, Optical properties of atomically thin transition metal dichalcogenides: observations and puzzles. *Nanophotonics* **6**, 1289 (2017). doi:10.1515/nanoph-2016-0165
- [169] S. Shree, I. Paradisanos, X. Marie, C. Robert, B. Urbaszek, Guide to optical spectroscopy of layered semiconductors. *Nat. Rev. Phys.* **3**, 39 (2020). doi:10.1038/s42254-020-00259-1
- [170] X. Xu, W. Yao, D. Xiao, T. F. Heinz, Spin and pseudospins in layered transition metal dichalcogenides. *Nat. Phys.* **10**, 343 (2014). doi:10.1038/nphys2942
- [171] K. F. Mak, D. Xiao, J. Shan, Light–valley interactions in 2D semiconductors. *Nat. Photonics* **12**, 451 (2018). doi:10.1038/s41566-018-0204-6
- [172] S. Dufferwiel, S. Schwarz, F. Withers, A. A. P. Trichet, F. Li, M. Sich, O. D. Pozo-Zamudio, C. Clark, A. Nalitov, D. D. Solnyshkov, G. Malpuech, K. S. Novoselov, J. M. Smith, M. S. Skolnick, D. N. Krizhanovskii, A. I. Tartakovskii, Exciton–polaritons in van der Waals heterostructures embedded in tunable microcavities. *Nat. Commun.* **6**, 8579 (2015). doi:10.1038/ncomms9579
- [173] N. Lundt, S. Klemmt, E. Cherotchenko, S. Betzold, O. Iff, A. V. Nalitov, M. Klaas, C. P. Dietrich, A. V. Kavokin, S. Höfling, C. Schneider, Room-temperature Tamm-plasmon exciton-polaritons with a WSe₂ monolayer. *Nat. Commun.* **7**, 13328 (2016). doi:10.1038/ncomms13328
- [174] Y.-J. Chen, J. D. Cain, T. K. Stanev, V. P. Dravid, N. P. Stern, Valley-polarized exciton–polaritons in a monolayer semiconductor. *Nat. Photonics* **11**, 431 (2017). doi:10.1038/nphoton.2017.86
- [175] Z. Sun, J. Gu, A. Ghazaryan, Z. Shotan, C. R. Considine, M. Dollar, B. Chakraborty, X. Liu, P. Ghaemi, S. Kéna-Cohen, V. M. Menon, Opti-

- cal control of room-temperature valley polaritons. *Nat. Photonics* **11**, 491 (2017). doi:10.1038/nphoton.2017.121
- [176] S. Dufferwiel, T. P. Lyons, D. D. Solnyshkov, A. A. P. Trichet, F. Withers, S. Schwarz, G. Malpuech, J. M. Smith, K. S. Novoselov, M. S. Skolnick, D. N. Krizhanovskii, A. I. Tartakovskii, Valley-addressable polaritons in atomically thin semiconductors. *Nat. Photonics* **11**, 497 (2017). doi:10.1038/nphoton.2017.125
- [177] N. Lundt, S. Stoll, P. Nagler, A. Nalitov, S. Klemmt, S. Betzold, J. Goddard, E. Frieling, A. V. Kavokin, C. Schüller, T. Korn, S. Höfling, C. Schneider, Observation of macroscopic valley-polarized monolayer exciton-polaritons at room temperature. *Phys. Rev. B* **96**, 241403 (2017). doi:10.1103/PhysRevB.96.241403
- [178] N. Lundt, P. Nagler, A. Nalitov, S. Klemmt, M. Wurdack, S. Stoll, T. H. Harder, S. Betzold, V. Baumann, A. V. Kavokin, C. Schüller, T. Korn, S. Höfling, C. Schneider, Valley polarized relaxation and upconversion luminescence from Tamm-plasmon trion-polaritons with a MoSe₂ monolayer. *2D Mater.* **4**, 025096 (2017). doi:10.1088/2053-1583/aa6ef2
- [179] S. Dufferwiel, T. P. Lyons, D. D. Solnyshkov, A. A. P. Trichet, A. Catanzaro, F. Withers, G. Malpuech, J. M. Smith, K. S. Novoselov, M. S. Skolnick, D. N. Krizhanovskii, A. I. Tartakovskii, Valley coherent exciton-polaritons in a monolayer semiconductor. *Nat. Commun.* **9**, 4797 (2018). doi:10.1038/s41467-018-07249-z
- [180] L. Qiu, C. Chakraborty, S. Dhara, A. N. Vamivakas, Room-temperature valley coherence in a polaritonic system. *Nat. Commun.* **10**, 1513 (2019). doi:10.1038/s41467-019-09490-6
- [181] O. Bleu, D. D. Solnyshkov, G. Malpuech, Optical valley Hall effect based on transitional metal dichalcogenide cavity polaritons. *Phys. Rev. B* **96**, 165432 (2017). doi:10.1103/PhysRevB.96.165432
- [182] N. Lundt, Ł. Dusanowski, E. Sedov, P. Stepanov, M. M. Glazov, S. Klemmt, M. Klaas, J. Beierlein, Y. Qin, S. Tongay, M. Richard, A. V. Kavokin, S. Höfling, C. Schneider, Optical valley Hall effect for highly valley-coherent exciton-polaritons in an atomically thin semiconductor. *Nat. Nanotechnol.* **14**, 770 (2019). doi:10.1038/s41565-019-0492-0
- [183] B. Chakraborty, J. Gu, Z. Sun, M. Khatoniar, R. Bushati, A. L. Boehmke, R. Koots, V. M. Menon, Control of strong light-matter interaction

- in monolayer WS₂ through electric field gating. *Nano Lett.* **18**, 6455 (2018). doi:10.1021/acs.nanolett.8b02932
- [184] J. Gu, B. Chakraborty, M. Khatoniar, V. M. Menon, A room-temperature polariton light-emitting diode based on monolayer WS₂. *Nat. Nanotechnol.* **14**, 1024 (2019). doi:10.1038/s41565-019-0543-6
- [185] M. Sliotsky, X. Liu, V. M. Menon, S. R. Forrest, Room temperature frenkel-wannier-mott hybridization of degenerate excitons in a strongly coupled microcavity. *Phys. Rev. Lett.* **112**, 076401 (2014). doi:10.1103/PhysRevLett.112.076401
- [186] L. C. Flatten, D. M. Coles, Z. He, D. G. Lidzey, R. A. Taylor, J. H. Warner, J. M. Smith, Electrically tunable organic–inorganic hybrid polaritons with monolayer WS₂. *Nat. Commun.* **8**, 14097 (2017). doi:10.1038/ncomms14097
- [187] M. Waldherr, N. Lundt, M. Klaas, S. Betzold, M. Wurdack, V. Baumann, E. Estrecho, A. Nalitov, E. Cherotchenko, H. Cai, E. A. Ostrovskaya, A. V. Kavokin, S. Tongay, S. Klemmt, S. Höfling, C. Schneider, Observation of bosonic condensation in a hybrid monolayer MoSe₂-GaAs microcavity. *Nat. Commun.* **9**, 3286 (2018). doi:10.1038/s41467-018-05532-7
- [188] C. Anton-Solanas, M. Waldherr, M. Klaas, H. Suchomel, T. H. Harder, H. Cai, E. Sedov, S. Klemmt, A. V. Kavokin, S. Tongay, K. Watanabe, T. Taniguchi, S. Höfling, C. Schneider, Bosonic condensation of exciton–polaritons in an atomically thin crystal. *Nat. Mater.* **20**, 1233 (2021). doi:10.1038/s41563-021-01000-8
- [189] J. Zhao, R. Su, A. Fieramosca, W. Zhao, W. Du, X. Liu, C. Diederichs, D. Sanvitto, T. C. H. Liew, Q. Xiong, Ultralow threshold polariton condensate in a monolayer semiconductor microcavity at room temperature. *Nano Lett.* **21**, 3331 (2021). doi:10.1021/acs.nanolett.1c01162
- [190] H. Shan, L. Lackner, B. Han, E. Sedov, C. Rupperecht, H. Knopf, F. Eilenberger, J. Beierlein, N. Kunte, M. Esmann, K. Yumigeta, K. Watanabe, T. Taniguchi, S. Klemmt, S. Höfling, A. V. Kavokin, S. Tongay, C. Schneider, C. Antón-Solanas, Spatial coherence of room-temperature monolayer WSe₂ exciton-polaritons in a trap. *Nat. Commun.* **12**, 6406 (2021). doi:10.1038/s41467-021-26715-9
- [191] J. Zhao, A. Fieramosca, R. Bao, W. Du, K. Dini, R. Su, J. Feng, Y. Luo, D. Sanvitto, T. C. H. Liew, Q. Xiong, Nonlinear polariton parametric

- emission in an atomically thin semiconductor based microcavity. *Nat. Nanotechnol.* (2022). doi:10.1038/s41565-022-01073-9
- [192] Y. Ye, Z. J. Wong, X. Lu, X. Ni, H. Zhu, X. Chen, Y. Wang, X. Zhang, Monolayer excitonic laser. *Nat. Photonics* **9**, 733 (2015). doi:10.1038/nphoton.2015.197
- [193] O. Salehzadeh, M. Djavid, N. H. Tran, I. Shih, Z. Mi, Optically pumped two-dimensional MoS₂ lasers operating at room-temperature. *Nano Lett.* **15**, 5302 (2015). doi:10.1021/acs.nanolett.5b01665
- [194] S. Wu, S. Buckley, J. R. Schaibley, L. Feng, J. Yan, D. G. Mandrus, F. Hatami, W. Yao, J. Vučković, A. Majumdar, X. Xu, Monolayer semiconductor nanocavity lasers with ultralow thresholds. *Nature* **520**, 69 (2015). doi:10.1038/nature14290
- [195] E. Y. Paik, L. Zhang, G. W. Burg, R. Gogna, E. Tutuc, H. Deng, Interlayer exciton laser of extended spatial coherence in atomically thin heterostructures. *Nature* **576**, 80 (2019). doi:10.1038/s41586-019-1779-x
- [196] W. Liu, Z. Ji, Y. Wang, G. Modi, M. Hwang, B. Zheng, V. J. Sorger, A. Pan, R. Agarwal, Generation of helical topological exciton-polaritons. *Science* **370**, 600 (2020). doi:10.1126/science.abc4975
- [197] M. Li, I. Sinev, F. Benimetskiy, T. Ivanova, E. Khestanova, S. Kiriushechkina, A. Vakulenko, S. Guddala, M. Skolnick, V. M. Menon, D. Krizhanovskii, A. Alù, A. Samusev, A. B. Khanikaev, Experimental observation of topological Z₂ exciton-polaritons in transition metal dichalcogenide monolayers. *Nat. Commun.* **12**, 4425 (2021). doi:10.1038/s41467-021-24728-y
- [198] X. Liu, T. Galfsky, Z. Sun, F. Xia, E. chen Lin, Y.-H. Lee, S. Kéna-Cohen, V. M. Menon, Strong light-matter coupling in two-dimensional atomic crystals. *Nat. Photonics* **9**, 30 (2014). doi:10.1038/nphoton.2014.304
- [199] X. Liu, W. Bao, Q. Li, C. Ropp, Y. Wang, X. Zhang, Control of coherently coupled exciton polaritons in monolayer tungsten disulphide. *Phys. Rev. Lett.* **119**, 027403 (2017). doi:10.1103/PhysRevLett.119.027403
- [200] S. Schwarz, S. Dufferwiel, P. M. Walker, F. Withers, A. A. P. Trichet, M. Sich, F. Li, E. A. Chekhovich, D. N. Borisenko, N. N. Kolesnikov, K. S. Novoselov, M. S. Skolnick, J. M. Smith, D. N. Kri-

- zhanovskii, A. I. Tartakovskii, Two-dimensional metal–chalcogenide films in tunable optical microcavities. *Nano Lett.* **14**, 7003 (2014). [doi:10.1021/nl503312x](https://doi.org/10.1021/nl503312x)
- [201] L. C. Flatten, Z. He, D. M. Coles, A. A. P. Trichet, A. W. Powell, R. A. Taylor, J. H. Warner, J. M. Smith, Room-temperature exciton-polaritons with two-dimensional WS₂. *Sci. Rep.* **6**, 33134 (2016). [doi:10.1038/srep33134](https://doi.org/10.1038/srep33134)
- [202] M. Wurdack, N. Lundt, M. Klaas, V. Baumann, A. V. Kavokin, S. Höfling, C. Schneider, Observation of hybrid Tamm-plasmon exciton-polaritons with GaAs quantum wells and a MoSe₂ monolayer. *Nat. Commun.* **8**, 259 (2017). [doi:10.1038/s41467-017-00155-w](https://doi.org/10.1038/s41467-017-00155-w)
- [203] A. Arora, M. Koperski, K. Nogajewski, J. Marcus, C. Faugeras, M. Potemski, Excitonic resonances in thin films of WSe₂: from monolayer to bulk material. *Nanoscale* **7**, 10421 (2015). [doi:10.1039/c5nr01536g](https://doi.org/10.1039/c5nr01536g)
- [204] T. Smoleński, M. Goryca, M. Koperski, C. Faugeras, T. Kazimierczuk, A. Bogucki, K. Nogajewski, P. Kossacki, M. Potemski, Tuning valley polarization in a WSe₂ monolayer with a tiny magnetic field. *Phys. Rev. X* **6**, 021024 (2016). [doi:10.1103/PhysRevX.6.021024](https://doi.org/10.1103/PhysRevX.6.021024)
- [205] Z. Ye, L. Waldecker, E. Y. Ma, D. Rhodes, A. Antony, B. Kim, X.-X. Zhang, M. Deng, Y. Jiang, Z. Lu, D. Smirnov, K. Watanabe, T. Taniguchi, J. Hone, T. F. Heinz, Efficient generation of neutral and charged biexcitons in encapsulated WSe₂ monolayers. *Nat. Commun.* **9**, 3718 (2018). [doi:10.1038/s41467-018-05917-8](https://doi.org/10.1038/s41467-018-05917-8)
- [206] B. Radisavljevic, A. Radenovic, J. Brivio, V. Giacometti, A. Kis, Single-layer MoS₂ transistors. *Nat. Nanotechnol.* **6**, 147 (2011). [doi:10.1038/nnano.2010.279](https://doi.org/10.1038/nnano.2010.279)
- [207] S.-Y. Lee, T.-Y. Jeong, S. Jung, K.-J. Yee, Refractive index dispersion of hexagonal boron nitride in the visible and near-infrared. *physica status solidi (b)* **256**, 1800417 (2019). [doi:https://doi.org/10.1002/pssb.201800417](https://doi.org/10.1002/pssb.201800417)
- [208] A. O. Slobodeniuk, Ł. Bala, M. Koperski, M. R. Molas, P. Kossacki, K. Nogajewski, M. Bartos, K. Watanabe, T. Taniguchi, C. Faugeras, M. Potemski, Fine structure of k-excitons in multilayers of transition metal dichalcogenides. *2D Mater.* **6**, 025026 (2019). [doi:10.1088/2053-1583/ab0776](https://doi.org/10.1088/2053-1583/ab0776)

- [209] Y. Li, A. Chernikov, X. Zhang, A. Rigosi, H. M. Hill, A. M. van der Zande, D. A. Chenet, E.-M. Shih, J. Hone, T. F. Heinz, Measurement of the optical dielectric function of monolayer transition-metal dichalcogenides: MoS₂, MoSe₂, WS₂, and WSe₂. *Phys. Rev. B* **90**, 205422 (2014). [doi:10.1103/PhysRevB.90.205422](https://doi.org/10.1103/PhysRevB.90.205422)
- [210] M. Fox, *Quantum optics: an introduction*, Oxford master series in atomic, optical, and laser physics (Oxford Univ. Press, Oxford, 2006).
- [211] V. Savona, L. Andreani, P. Schwendimann, A. Quattropani, Quantum well excitons in semiconductor microcavities: Unified treatment of weak and strong coupling regimes. *Solid State Commun.* **93**, 733 (1995). [doi:10.1016/0038-1098\(94\)00865-5](https://doi.org/10.1016/0038-1098(94)00865-5)
- [212] F. Tassone, F. Bassani, L. Andreani, Resonant and surface polaritons in quantum wells. *Il Nuovo Cimento D* **12**, 1673 (1990). [doi:10.1007/bf02451267](https://doi.org/10.1007/bf02451267)
- [213] V. Walther, R. Johne, T. Pohl, Giant optical nonlinearities from Rydberg excitons in semiconductor microcavities. *Nat. Commun.* **9**, 1309 (2018). [doi:10.1038/s41467-018-03742-7](https://doi.org/10.1038/s41467-018-03742-7)
- [214] B. Piętka, M. R. Molas, N. Bobrovska, M. Król, R. Mirek, K. Lekenta, P. Stepnicki, F. Morier-Genoud, J. Szczytko, B. Deveaud, M. Matuszewski, M. Potemski, 2s exciton-polariton revealed in an external magnetic field. *Phys. Rev. B* **96**, 081402 (2017). [doi:10.1103/PhysRevB.96.081402](https://doi.org/10.1103/PhysRevB.96.081402)
- [215] W. Bao, X. Liu, F. Xue, F. Zheng, R. Tao, S. Wang, Y. Xia, M. Zhao, J. Kim, S. Yang, Q. Li, Y. Wang, Y. Wang, L.-W. Wang, A. H. MacDonald, X. Zhang, Observation of Rydberg exciton polaritons and their condensate in a perovskite cavity. *Proceedings of the National Academy of Sciences* **116**, 20274 (2019). [doi:10.1073/pnas.1909948116](https://doi.org/10.1073/pnas.1909948116)
- [216] J. Gu, V. Walther, L. Waldecker, D. Rhodes, A. Raja, J. C. Hone, T. F. Heinz, S. Kéna-Cohen, T. Pohl, V. M. Menon, Enhanced nonlinear interaction of polaritons via excitonic Rydberg states in monolayer WSe₂. *Nat. Commun.* **12**, 2269 (2021). [doi:10.1038/s41467-021-22537-x](https://doi.org/10.1038/s41467-021-22537-x)
- [217] S.-Y. Chen, C. Zheng, M. S. Fuhrer, J. Yan, Helicity-resolved Raman scattering of MoS₂, MoSe₂, WS₂, and WSe₂ atomic layers. *Nano Lett.* **15**, 2526 (2015). [doi:10.1021/acs.nanolett.5b00092](https://doi.org/10.1021/acs.nanolett.5b00092)

- [218] G. Wang, L. Bouet, D. Lagarde, M. Vidal, A. Balocchi, T. Amand, X. Marie, B. Urbaszek, Valley dynamics probed through charged and neutral exciton emission in monolayer WSe₂. *Phys. Rev. B* **90**, 075413 (2014). [doi:10.1103/PhysRevB.90.075413](https://doi.org/10.1103/PhysRevB.90.075413)
- [219] C. R. Zhu, K. Zhang, M. Glazov, B. Urbaszek, T. Amand, Z. W. Ji, B. L. Liu, X. Marie, Exciton valley dynamics probed by Kerr rotation in WSe₂ monolayers. *Phys. Rev. B* **90**, 161302 (2014). [doi:10.1103/PhysRevB.90.161302](https://doi.org/10.1103/PhysRevB.90.161302)
- [220] A. Singh, K. Tran, M. Kolarczik, J. Seifert, Y. Wang, K. Hao, D. Pleskot, N. M. Gabor, S. Helmrich, N. Owschimikow, U. Woggon, X. Li, Long-lived valley polarization of intravalley trions in monolayer WSe₂. *Phys. Rev. Lett.* **117**, 257402 (2016). [doi:10.1103/PhysRevLett.117.257402](https://doi.org/10.1103/PhysRevLett.117.257402)
- [221] G. Wang, C. Robert, M. M. Glazov, F. Cadiz, E. Courtade, T. Amand, D. Lagarde, T. Taniguchi, K. Watanabe, B. Urbaszek, X. Marie, In-plane propagation of light in transition metal dichalcogenide monolayers: Optical selection rules. *Phys. Rev. Lett.* **119**, 047401 (2017). [doi:10.1103/PhysRevLett.119.047401](https://doi.org/10.1103/PhysRevLett.119.047401)
- [222] Y. Zhou, G. Scuri, D. S. Wild, A. A. High, A. Dibos, L. A. Jauregui, C. Shu, K. D. Greve, K. Pistunova, A. Y. Joe, T. Taniguchi, K. Watanabe, P. Kim, M. D. Lukin, H. Park, Probing dark excitons in atomically thin semiconductors via near-field coupling to surface plasmon polaritons. *Nat. Nanotechnol.* **12**, 856 (2017). [doi:10.1038/nnano.2017.106](https://doi.org/10.1038/nnano.2017.106)
- [223] X.-X. Zhang, T. Cao, Z. Lu, Y.-C. Lin, F. Zhang, Y. Wang, Z. Li, J. C. Hone, J. A. Robinson, D. Smirnov, S. G. Louie, T. F. Heinz, Magnetic brightening and control of dark excitons in monolayer WSe₂. *Nat. Nanotechnol.* **12**, 883 (2017). [doi:10.1038/nnano.2017.105](https://doi.org/10.1038/nnano.2017.105)
- [224] M. R. Molas, C. Faugeras, A. O. Slobodeniuk, K. Nogajewski, M. Bartos, D. M. Basko, M. Potemski, Brightening of dark excitons in monolayers of semiconducting transition metal dichalcogenides. *2D Mater.* **4**, 021003 (2017). [doi:10.1088/2053-1583/aa5521](https://doi.org/10.1088/2053-1583/aa5521)
- [225] K. Łempicka-Mirek, M. Król, H. Sigurdsson, A. Wincukiewicz, P. Morawiak, R. Mazur, M. Muszyński, W. Piecek, P. Kula, T. Stefaniuk, M. Kamińska, L. De Marco, P. G. Lagoudakis, D. Ballarini, D. Sanvitto, J. Szczytko, B. Piętko, Electrically tunable Berry curvature and

- strong light-matter coupling in birefringent perovskite microcavities at room temperature. [arXiv:2203.05289](https://arxiv.org/abs/2203.05289) (2022).
- [226] Y. Fu, H. Zhu, J. Chen, M. P. Hautzinger, X.-Y. Zhu, S. Jin, Metal halide perovskite nanostructures for optoelectronic applications and the study of physical properties. *Nat. Rev. Mater.* **4**, 169 (2019). [doi:10.1038/s41578-019-0080-9](https://doi.org/10.1038/s41578-019-0080-9)
- [227] R. Su, A. Fieramosca, Q. Zhang, H. S. Nguyen, E. Deleporte, Z. Chen, D. Sanvitto, T. C. H. Liew, Q. Xiong, Perovskite semiconductors for room-temperature exciton-polaritonics. *Nat. Mater.* **20**, 1315 (2021). [doi:10.1038/s41563-021-01035-x](https://doi.org/10.1038/s41563-021-01035-x)
- [228] M. Baranowski, P. Plochocka, Excitons in metal-halide perovskites. *Adv. Energy Mater.* **10**, 1903659 (2020). [doi:10.1002/aenm.201903659](https://doi.org/10.1002/aenm.201903659)
- [229] J. Huang, Y. Yuan, Y. Shao, Y. Yan, Understanding the physical properties of hybrid perovskites for photovoltaic applications. *Nat. Rev. Mater.* **2**, 17042 (2017). [doi:10.1038/natrevmats.2017.42](https://doi.org/10.1038/natrevmats.2017.42)
- [230] S. D. Stranks, G. E. Eperon, G. Grancini, C. Menelaou, M. J. P. Alcocer, T. Leijtens, L. M. Herz, A. Petrozza, H. J. Snaith, Electron-hole diffusion lengths exceeding 1 micrometer in an organometal trihalide perovskite absorber. *Science* **342**, 341 (2013). [doi:10.1126/science.1243982](https://doi.org/10.1126/science.1243982)
- [231] D. R. Ceratti, Y. Rakita, L. Cremonesi, R. Tenne, V. Kalchenko, M. Elbaum, D. Oron, M. A. C. Potenza, G. Hodes, D. Cahen, Self-healing inside APbBr₃ halide perovskite crystals. *Adv. Mater.* **30**, 1706273 (2018). [doi:10.1002/adma.201706273](https://doi.org/10.1002/adma.201706273)
- [232] K. Miyata, D. Meggiolaro, M. T. Trinh, P. P. Joshi, E. Mosconi, S. C. Jones, F. D. Angelis, X.-Y. Zhu, Large polarons in lead halide perovskites. *Sci. Adv.* **3**, e1701217 (2017). [doi:10.1126/sciadv.1701217](https://doi.org/10.1126/sciadv.1701217)
- [233] X.-K. Liu, W. Xu, S. Bai, Y. Jin, J. Wang, R. H. Friend, F. Gao, Metal halide perovskites for light-emitting diodes. *Nat. Mater.* **20**, 10 (2021). [doi:10.1038/s41563-020-0784-7](https://doi.org/10.1038/s41563-020-0784-7)
- [234] L. Protesescu, S. Yakunin, M. I. Bodnarchuk, F. Krieg, R. Caputo, C. H. Hendon, R. X. Yang, A. Walsh, M. V. Kovalenko, Nanocrystals of cesium lead halide perovskites (CsPbX₃, X = Cl, Br, and I): Novel optoelectronic materials showing bright emission with wide color gamut. *Nano Lett.* **15**, 3692 (2015). [doi:10.1021/nl5048779](https://doi.org/10.1021/nl5048779)

- [235] S. G. R. Bade, J. Li, X. Shan, Y. Ling, Y. Tian, T. Dilbeck, T. Besara, T. Geske, H. Gao, B. Ma, K. Hanson, T. Siegrist, C. Xu, Z. Yu, Fully printed halide perovskite light-emitting diodes with silver nanowire electrodes. *ACS Nano* **10**, 1795 (2016). doi:10.1021/acsnano.5b07506
- [236] L. Zhao, N. Rolston, K. M. Lee, X. Zhao, M. A. Reyes-Martinez, N. L. Tran, Y.-W. Yeh, N. Yao, G. D. Scholes, Y.-L. Loo, A. Selloni, R. H. Dauskardt, B. P. Rand, Influence of bulky organo-ammonium halide additive choice on the flexibility and efficiency of perovskite light-emitting devices. *Adv. Funct. Mater.* **28**, 1802060 (2018). doi:10.1002/adfm.201802060
- [237] W. Xu, Q. Hu, S. Bai, C. Bao, Y. Miao, Z. Yuan, T. Borzda, A. J. Barker, E. Tyukalova, Z. Hu, M. Kawecki, H. Wang, Z. Yan, X. Liu, X. Shi, K. Uvdal, M. Fahlman, W. Zhang, M. Duchamp, J.-M. Liu, A. Petrozza, J. Wang, L.-M. Liu, W. Huang, F. Gao, Rational molecular passivation for high-performance perovskite light-emitting diodes. *Nat. Photonics* **13**, 418 (2019). doi:10.1038/s41566-019-0390-x
- [238] Y. Rong, Y. Hu, A. Mei, H. Tan, M. I. Saidaminov, S. I. Seok, M. D. McGehee, E. H. Sargent, H. Han, Challenges for commercializing perovskite solar cells. *Science* **361**, eaat8235 (2018). doi:10.1126/science.aat8235
- [239] Y. Liang, Q. Shang, Q. Wei, L. Zhao, Z. Liu, J. Shi, Y. Zhong, J. Chen, Y. Gao, M. Li, X. Liu, G. Xing, Q. Zhang, Lasing from mechanically exfoliated 2D homologous Ruddlesden–Popper perovskite engineered by inorganic layer thickness. *Adv. Mater.* **31**, 1903030 (2019). doi:10.1002/adma.201903030
- [240] Q. A. Akkerman, S. G. Motti, A. R. Srimath Kandada, E. Mosconi, V. D’Innocenzo, G. Bertoni, S. Marras, B. A. Kamino, L. Miranda, F. De Angelis, A. Petrozza, M. Prato, L. Manna, Solution synthesis approach to colloidal cesium lead halide perovskite nanoplatelets with monolayer-level thickness control. *J. Am. Chem. Soc.* **138**, 1010 (2016). doi:10.1021/jacs.5b12124
- [241] S. T. Ha, X. Liu, Q. Zhang, D. Giovanni, T. C. Sum, Q. Xiong, Synthesis of organic–inorganic lead halide perovskite nanoplatelets: Towards high-performance perovskite solar cells and optoelectronic devices. *Adv. Opt. Mater.* **2**, 838 (2014). doi:10.1002/adom.201400106

- [242] W. Niu, A. Eiden, G. Vijaya Prakash, J. J. Baumberg, Exfoliation of self-assembled 2D organic-inorganic perovskite semiconductors. *Appl. Phys. Lett.* **104**, 171111 (2014). doi:10.1063/1.4874846
- [243] J.-C. Blancon, A. V. Stier, H. Tsai, W. Nie, C. C. Stoumpos, B. Traoré, L. Pedesseau, M. Kepenekian, F. Katsutani, G. T. Noe, J. Kono, S. Tretiak, S. A. Crooker, C. Katan, M. G. Kanatzidis, J. J. Crochet, J. Even, A. D. Mohite, Scaling law for excitons in 2D perovskite quantum wells. *Nat. Commun.* **9**, 2254 (2018). doi:10.1038/s41467-018-04659-x
- [244] T. Fujita, Y. Sato, T. Kuitani, T. Ishihara, Tunable polariton absorption of distributed feedback microcavities at room temperature. *Phys. Rev. B* **57**, 12428 (1998). doi:10.1103/PhysRevB.57.12428
- [245] G. Lanty, A. Bréhier, R. Parashkov, J. S. Lauret, E. Deleporte, Strong exciton–photon coupling at room temperature in microcavities containing two-dimensional layered perovskite compounds. *New J. Phys.* **10**, 065007 (2008). doi:10.1088/1367-2630/10/6/065007
- [246] R. Su, C. Diederichs, J. Wang, T. C. H. Liew, J. Zhao, S. Liu, W. Xu, Z. Chen, Q. Xiong, Room-temperature polariton lasing in all-inorganic perovskite nanoplatelets. *Nano Lett.* **17**, 3982 (2017). doi:10.1021/acs.nanolett.7b01956
- [247] R. Su, S. Ghosh, J. Wang, S. Liu, C. Diederichs, T. C. H. Liew, Q. Xiong, Observation of exciton polariton condensation in a perovskite lattice at room temperature. *Nat. Phys.* **16**, 301 (2020). doi:10.1038/s41567-019-0764-5
- [248] J. Wang, H. Xu, R. Su, Y. Peng, J. Wu, T. C. H. Liew, Q. Xiong, Spontaneously coherent orbital coupling of counterrotating exciton polaritons in annular perovskite microcavities. *Light Sci. Appl.* **10**, 45 (2021). doi:10.1038/s41377-021-00478-w
- [249] R. Su, J. Wang, J. Zhao, J. Xing, W. Zhao, C. Diederichs, T. C. H. Liew, Q. Xiong, Room temperature long-range coherent exciton polariton condensate flow in lead halide perovskites. *Sci. Adv.* **4**, eaau0244 (2018). doi:10.1126/sciadv.aau0244
- [250] J. Feng, J. Wang, A. Fieramosca, R. Bao, J. Zhao, R. Su, Y. Peng, T. C. H. Liew, D. Sanvitto, Q. Xiong, All-optical switching based on interacting exciton polaritons in self-assembled perovskite microwires. *Sci. Adv.* **7**, eabj6627 (2021). doi:10.1126/sciadv.abj6627

- [251] J. Wu, S. Ghosh, R. Su, A. Fieramosca, T. C. H. Liew, Q. Xiong, Nonlinear parametric scattering of exciton polaritons in perovskite microcavities. *Nano Lett.* **21**, 3120 (2021). [doi:10.1021/acs.nanolett.1c00283](https://doi.org/10.1021/acs.nanolett.1c00283)
- [252] J. Wu, R. Su, A. Fieramosca, S. Ghosh, J. Zhao, T. C. H. Liew, Q. Xiong, Perovskite polariton parametric oscillator. *Adv. Photonics* **3**, 055003 (2021). [doi:10.1117/1.ap.3.5.055003](https://doi.org/10.1117/1.ap.3.5.055003)
- [253] T. Wang, Z. Zang, Y. Gao, C. Lyu, K. Peng, K. Watanabe, T. Taniguchi, X. Liu, W. Bao, Y. Ye, Electrically pumped polarized exciton-polaritons in a halide perovskite microcavity. [arXiv:2203.01144](https://arxiv.org/abs/2203.01144) (2022).
- [254] J. Wang, R. Su, J. Xing, D. Bao, C. Diederichs, S. Liu, T. C. Liew, Z. Chen, Q. Xiong, Room temperature coherently coupled exciton-polaritons in two-dimensional organic-inorganic perovskite. *ACS Nano* **12**, 8382 (2018). [doi:10.1021/acsnano.8b03737](https://doi.org/10.1021/acsnano.8b03737)
- [255] A. Fieramosca, L. D. Marco, M. Passoni, L. Polimeno, A. Rizzo, B. L. T. Rosa, G. Cruciani, L. Dominici, M. D. Giorgi, G. Gigli, L. C. Andreani, D. Gerace, D. Ballarini, D. Sanvitto, Tunable out-of-plane excitons in 2D single-crystal perovskites. *ACS Photonics* **5**, 4179 (2018). [doi:10.1021/acsp Photonics.8b00984](https://doi.org/10.1021/acsp Photonics.8b00984)
- [256] A. Fieramosca, L. Polimeno, V. Ardizzone, L. D. Marco, M. Pugliese, V. Maiorano, M. D. Giorgi, L. Dominici, G. Gigli, D. Gerace, D. Ballarini, D. Sanvitto, Two-dimensional hybrid perovskites sustaining strong polariton interactions at room temperature. *Sci. Adv.* **5**, eaav9967 (2019). [doi:10.1126/sciadv.aav9967](https://doi.org/10.1126/sciadv.aav9967)
- [257] L. Polimeno, A. Fieramosca, G. Lerario, M. Cinquino, M. D. Giorgi, D. Ballarini, F. Todisco, L. Dominici, V. Ardizzone, M. Pugliese, C. T. Prontera, V. Maiorano, G. Gigli, L. D. Marco, D. Sanvitto, Observation of two thresholds leading to polariton condensation in 2D hybrid perovskites. *Adv. Opt. Mater.* **8**, 2000176 (2020). [doi:10.1002/adom.202000176](https://doi.org/10.1002/adom.202000176)
- [258] J. Anandan, The geometric phase. *Nature* **360**, 307 (1992). [doi:10.1038/360307a0](https://doi.org/10.1038/360307a0)
- [259] E. Cohen, H. Larocque, F. Bouchard, F. Nejdassattari, Y. Gefen, E. Karimi, Geometric phase from Aharonov-Bohm to Pancharatnam-Berry and beyond. *Nat. Rev. Phys.* **1**, 437 (2019). [doi:10.1038/s42254-019-0071-1](https://doi.org/10.1038/s42254-019-0071-1)

- [260] Quantal phase factors accompanying adiabatic changes. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* **392**, 45 (1984). [doi:10.1098/rspa.1984.0023](https://doi.org/10.1098/rspa.1984.0023)
- [261] Y. Aharonov, J. Anandan, Phase change during a cyclic quantum evolution. *Phys. Rev. Lett.* **58**, 1593 (1987). [doi:10.1103/PhysRevLett.58.1593](https://doi.org/10.1103/PhysRevLett.58.1593)
- [262] D. Xiao, M.-C. Chang, Q. Niu, Berry phase effects on electronic properties. *Rev. Mod. Phys.* **82**, 1959 (2010). [doi:10.1103/RevModPhys.82.1959](https://doi.org/10.1103/RevModPhys.82.1959)
- [263] M. Z. Hasan, C. L. Kane, Colloquium: Topological insulators. *Rev. Mod. Phys.* **82**, 3045 (2010). [doi:10.1103/RevModPhys.82.3045](https://doi.org/10.1103/RevModPhys.82.3045)
- [264] A. V. Nalitov, D. D. Solnyshkov, G. Malpuech, Polariton $\mathbb{Z}$ topological insulator. *Phys. Rev. Lett.* **114**, 116401 (2015). [doi:10.1103/PhysRevLett.114.116401](https://doi.org/10.1103/PhysRevLett.114.116401)
- [265] P. Kokhanchik, H. Sigurdsson, B. Piętko, J. Szczytko, P. G. Lagoudakis, Photonic Berry curvature in double liquid crystal microcavities with broken inversion symmetry. *Phys. Rev. B* **103**, L081406 (2021). [doi:10.1103/PhysRevB.103.L081406](https://doi.org/10.1103/PhysRevB.103.L081406)
- [266] M. Matuszewski, A. Opala, R. Mirek, M. Furman, M. Król, K. Tyszka, T. Liew, D. Ballarini, D. Sanvitto, J. Szczytko, B. Piętko, Energy-efficient neural network inference with microcavity exciton polaritons. *Phys. Rev. Applied* **16**, 024045 (2021). [doi:10.1103/PhysRevApplied.16.024045](https://doi.org/10.1103/PhysRevApplied.16.024045)
- [267] R. Mirek, A. Opala, P. Comaron, M. Furman, M. Król, K. Tyszka, B. Seredyński, D. Ballarini, D. Sanvitto, T. C. H. Liew, W. Pacuski, J. Suffczyński, J. Szczytko, M. Matuszewski, B. Piętko, Neuro-morphic binarized polariton networks. *Nano Lett.* **21**, 3715 (2021). [doi:10.1021/acs.nanolett.0c04696](https://doi.org/10.1021/acs.nanolett.0c04696)
- [268] M. Król, R. Mirek, D. Stephan, K. Lekenta, J.-G. Rousset, W. Pacuski, A. V. Kavokin, M. Matuszewski, J. Szczytko, B. Piętko, Giant spin Meissner effect in a nonequilibrium exciton-polariton gas. *Phys. Rev. B* **99**, 115318 (2019). [doi:10.1103/PhysRevB.99.115318](https://doi.org/10.1103/PhysRevB.99.115318)
- [269] K. Tyszka, M. Furman, R. Mirek, M. Król, A. Opala, B. Seredyński, J. Suffczyński, W. Pacuski, M. Matuszewski, J. Szczytko, B. Piętko, Le-

- aky Integrate-and-Fire Mechanism in Exciton-Polariton Condensates for Photonic Spiking Neurons. [arXiv:2111.13123](https://arxiv.org/abs/2111.13123) (2021).
- [270] K. Łempicka, M. Furman, M. Muszyński, M. Król, A. Wincukiewicz, K. Rechcińska, R. Mazur, W. Piecek, M. Kamińska, J. Szczytko, B. Piętka, *International Photonics and OptoElectronics Meeting 2019 (OFDA, OEDI, ISST, PE, LST, TSA)* (Optical Society of America, 2019), p. JW4A.66. [doi:10.1364/ISST.2019.JW4A.66](https://doi.org/10.1364/ISST.2019.JW4A.66).
- [271] M. Król, R. Mirek, K. Lekenta, J.-G. Rousset, D. Stephan, M. Nawrocki, M. Matuszewski, J. Szczytko, W. Pacuski, B. Piętka, Spin polarized semimagnetic exciton-polariton condensate in magnetic field. *Sci. Rep.* **8**, 6694 (2018). [doi:10.1038/s41598-018-25018-2](https://doi.org/10.1038/s41598-018-25018-2)
- [272] B. Seredyński, M. Król, P. Starzyk, R. Mirek, M. Ściesiek, K. Sobczak, J. Borysiuk, D. Stephan, J.-G. Rousset, J. Szczytko, B. Piętka, W. Pacuski, (Cd,Zn,Mg)Te-based microcavity on MgTe sacrificial buffer: Growth, lift-off, and transmission studies of polaritons. *Phys. Rev. Materials* **2**, 043406 (2018). [doi:10.1103/PhysRevMaterials.2.043406](https://doi.org/10.1103/PhysRevMaterials.2.043406)
- [273] J.-G. Rousset, B. Piętka, M. Król, R. Mirek, K. Lekenta, J. Szczytko, W. Pacuski, M. Nawrocki, Magnetic field effect on the lasing threshold of a semimagnetic polariton condensate. *Phys. Rev. B* **96**, 125403 (2017). [doi:10.1103/PhysRevB.96.125403](https://doi.org/10.1103/PhysRevB.96.125403)
- [274] R. Mirek, M. Król, K. Lekenta, J.-G. Rousset, M. Nawrocki, M. Kulczykowski, M. Matuszewski, J. Szczytko, W. Pacuski, B. Piętka, Angular dependence of giant Zeeman effect for semimagnetic cavity polaritons. *Phys. Rev. B* **95**, 085429 (2017). [doi:10.1103/PhysRevB.95.085429](https://doi.org/10.1103/PhysRevB.95.085429)
- [275] B. Piętka, N. Bobrowska, D. Stephan, M. Teich, M. Król, S. Winnerl, A. Pashkin, R. Mirek, K. Lekenta, F. Morier-Genoud, H. Schneider, B. Deveaud, M. Helm, M. Matuszewski, J. Szczytko, Doubly dressed bosons: Exciton polaritons in a strong terahertz field. *Phys. Rev. Lett.* **119**, 077403 (2017). [doi:10.1103/PhysRevLett.119.077403](https://doi.org/10.1103/PhysRevLett.119.077403)
- [276] K. Łempicka, K. Norowski, M. Grzeszczyk, M. Król, K. Lekenta, A. Babiński, B. Piętka, J. Szczytko, Relative reflection difference as a method for measuring the thickness of the exfoliated MoSe₂ layers. *Acta Phys. Pol. A* **132**, 316 (2017). [doi:10.12693/aphyspola.132.316](https://doi.org/10.12693/aphyspola.132.316)
- [277] J.-G. Rousset, B. Piętka, M. Król, R. Mirek, K. Lekenta, J. Szczytko, J. Borysiuk, J. Suffczyński, T. Kazimierczuk, M. Goryca, T. Smoleński,

- P. Kossacki, M. Nawrocki, W. Pacuski, Strong coupling and polariton lasing in Te based microcavities embedding (Cd,Zn)Te quantum wells. *Appl. Phys. Lett.* **107**, 201109 (2015). [doi:10.1063/1.4935791](https://doi.org/10.1063/1.4935791)
- [278] B. Piętka, D. Zygmunt, M. Król, M. R. Molas, A. A. L. Nicolet, F. Morier-Genoud, J. Szczytko, J. Łusakowski, P. Zięba, I. Tralle, P. Stępnicki, M. Matuszewski, M. Potemski, B. Deveaud, Magnetic field tuning of exciton-polaritons in a semiconductor microcavity. *Phys. Rev. B* **91**, 075309 (2015). [doi:10.1103/PhysRevB.91.075309](https://doi.org/10.1103/PhysRevB.91.075309)
- [279] R. Mirek, A. Opala, M. Furman, M. Król, K. Tyszka, B. Seredyński, W. Pacuski, J. Suffczyński, J. Szczytko, M. Matuszewski, B. Piętka, Neural networks based on ultrafast time-delayed effects in exciton-polaritons. [arXiv:2201.00225](https://arxiv.org/abs/2201.00225) (2022).