

Heisenberg limit beyond quantum Fisher information

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The Heisenberg limit provides a fundamental bound on the achievable estimation precision with a limited number of N resources used (e.g., atoms, photons, etc.). Using entangled quantum states makes it possible to scale the precision with N better than when resources would be used independently. Consequently, the optimal use of all resources involves accumulating them in a single execution of the experiment. Unfortunately, that implies that the most common theoretical tool used to analyze metrological protocols - quantum Fisher information (QFI) -- does not allow for a reliable description of this problem, as it becomes operationally meaningful only with multiple repetitions of the experiment.

In this thesis, using the formalism of Bayesian estimation and the minimax estimator, I derive asymptotically saturable bounds on the precision of the estimation for the case of noiseless unitary evolution. For the case where the number of resources N is strictly constrained, I show that the final measurement uncertainty is π times larger than would be implied by a naive use of QFI. I also analyze the case where a constraint is imposed only on the average amount of resources, the exact value of which may fluctuate (in which case QFI does not provide any universal bound for precision). In both cases, I study the asymptotic saturability and the rate of convergence of these bounds.

In the following part, I analyze the problem of the Heisenberg limit when multiple parameters are measured simultaneously on the same physical system. In particular, I investigate the existence of a gain from measuring all parameters simultaneously compared to distributing the same amount of resources to measure them independently. Using two examples -- the measurement of multiple phase shifts in a multi-arm interferometer and the measurement of three magnetic field components -- I show the existence of qualitative differences between the results obtained using Bayesian estimation/minimax estimator and those resulting from the use of QFI. I also derive a lower bound on the achievable precision of the measurement in the general case (not always saturable).