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Report on the Thesis ‘**Topologically non-trivial black hole spacetimes**’ by Maciej Ossowski.

The PhD Thesis ‘Topologically non-trivial black hole spacetimes’ consists of five chapters in a total of 131 pages. The Thesis is based on five papers, [1, 2, 3, 4, 5]. Papers [1, 2, 3, 5] have been written jointly with Ossowski’s PhD supervisor Professor Jerzy Lewandowski. Paper [4] has been written jointly with Dobkowski–Ryłko and Lewandowski. [1] has been published in *Classical and Quantum Gravity*. [2, 3, 4] have been published in *Phys Rev D*. Both journals are regarded as the leading journals in general relativity. Paper [5] has appeared as a preprint on arXiv. The Thesis is written in first person, despite the fact that all results appeared in joint publications.

The subject of the Thesis is a mathematical description of black holes and their horizons in space–times with the so called NUT parameter. These space–times are not asymptotically flat or de–Sitter/anti–de Sitter but instead contain topologically non–trivial regions described by the Hopf fibration or other non–trivial bundles over a two–sphere. There is also a string–type singularity which can be regularised in several ways. According to the famous quote of the equally famous physicist Charles Misner these space–times are *counterexamples to almost everything*. Therefore, although a physical realisation of NUT space–times in the Universe is highly hypothetical, it is very interesting to test various the-

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orems and constructions on NUT examples. One aim of the thesis is adapting the theory of isolated horizons developed by Professor Lewandowski and his co-workers in the last two decades to space-times with a NUT parameter.

I am afraid I found the thesis to be poorly written, full of typos, and errors. It kicks off in the abstract with a statement that Taub–NUT ‘contains two equivalent maximal analytic extensions’. Surely, the author meant to say ‘non-equivalent’? There are also mistakes in calculations, e.g. on page 53 the flat metric (2.45) pulls back, by the coordinate transformation (2.45) to

$$g_E = \frac{1}{2x}dx^2 + 2xa^2d\phi^2$$

and not the metric (2.47). I hope that the author will be given a chance to thoroughly check the manuscript and the calculations before publishing the thesis. In my summary below I shall therefore focus on the correct, and to large extend impressive, results of the Thesis.

Chapter 1 is a review of the Taub–NUT and Plebański–Demiański metrics as well as the theory of isolated horizons. Chapter 2 gives a novel interpretation of the cosmological Kerr-NUT space times, and - together with Chapter 3 - develops a theory of the associated Killing horizons . This differs from the original Misner interpretation, and extends to the Plebański–Demiański class of solutions. Some interesting mathematics is unveiled here in terms of principal $U(1)$ fibrations over a sphere. In Chapter 4 the analysis is extended to horizons with conical singularities. I find this part of the Thesis particularly interesting and ‘topical’, as some recent results of Harvey Reall and his coworkers indicate that development of singularities on horizons is a robust consequence of General Relativity. Chapter 5 addresses the theory of horizons, where the associated $U(1)$ fibration has a Riemann surface of genus higher than zero as a base-space. The so called Type D equation (a set of conditions of a metric on the spatial horizon cross-sections and a one-form in this cross section) is skilfully solved in this case. The black holes admitting such horizons have been named ‘topological’ in some previous studies. The can only exist in the presence of a non-zero cosmological constant, and their analysis in the Kerr-NUT context is an impressive result.

Conclusions. While I have my reservations about the quality of the write up, I find the overall results impressive, original and novel enough to warrant an award of a PhD.

Yours sincerely,



Maciej Dunajski