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Review of Doctoral Dissertation of Maciej Ossowski

The thesis "Topologically non-trivial black hole spacetimes" by Maciej Ossowski presents a study of spacetimes containing topologically non-trivial horizons in general relativity. The thesis is based on five published papers all focused on this topic, written in collaboration with his doctoral advisor Jerzy Lewandowski and one of the papers is also written in collaboration with a former doctoral student Denis Dobkowski-Rylko. This is very good productivity for a PhD student. The thesis includes an appropriate introduction to the topic including, isolated horizons (IH), horizons where the null generators are fibered non-trivially over a two-dimensional spacelike surface, and Taub-NUT spacetimes and their generalisations which provide explicit examples of such horizons.

The main result of the thesis is a new non-singular interpretation of the Kerr-NUT metrics (and the more general Plebanski-Demianski metrics) with a cosmological constant. This is achieved by a careful global analysis of the orbits of the Killing fields and their fixed points, that generalises Misner's interpretation of Taub-NUT spacetime. The resulting spacetimes are $U(1)$-bundles over $R \times S^2$ with total space $R \times S^3$ (or a lens space quotient). Given the local metrics have been known for many decades, it is remarkable that this global analysis had not been previously performed.

The thesis also includes a detailed analysis of the Killing horizons contained in these non-singular solutions. Two qualitatively different cases are identified, depending on whether the Killing field tangent to the $U(1)$-fibre generates a horizon or not, which are referred to as projectively non-singular or projectively singular horizons, respectively. In both cases the horizon is a smooth null hypersurface with topology of a $U(1)$-bundle over S^2, however, the space of null generators is non-singular only in the case of a projectively non-singular horizon. It turns out the the projectively non-singular case occurs only if the horizon is non-extremal and in the generic case the cosmological constant is fixed to some positive value in terms of the other parameters (there are non-generic cases where one of the rotation, NUT or cosmological constant vanish, but these all reduce to Taub-NUT-(A)dS or Kerr-(A)dS). In the projectively singular horizon case, presumably there are no constraints on the sign (or value) of the cosmological constant, although this does not seem to be clearly discussed (at least, I could not easily determine this from the thesis). Indeed, it would be helpful to clarify what the precise parameter constraints are in each case, perhaps by summarising this in a theorem.

The thesis also includes a general analysis of isolated horizons with such non-trivial topology. It is known that Type D IH with topology $R \times S^2$ and axial symmetry must be in the Kerr family of metrics. This was recently generalised to the case where the IH is a $U(1)$-bundle over S^2 where the null generators are tangent to the $U(1)$-fibers by Dobkowski-Rylko, Lewandowski and Racz (this is the projectively non-singular case). The present thesis generalises this work to include the projectively singular case. The general solution is given in terms of several parameters and an investigation of which solutions correspond to the known examples from the (accelerated) Kerr-NUT-(A)dS spacetimes is performed. Due to the complicated nature of the map between the parameters of the general IH solution and of the (accelerated) Kerr-NUT-(A)dS solution, the question of whether this is a bijection is left open. However, it is shown this map is locally invertible if and only if the horizon is non-extremal. Again, it would be helpful to state the precise parameter ranges for the general IH and the (accelerated) Kerr-NUT-(A)dS solutions, in order that this result about local invertibility of the parameter change map could be stated more precisely. The pertinent question here is: could there be Type D IH solutions that are not realised in the Plebanski-Demianski metrics, or does one expect that this map is in fact a bijection?

The final chapter considers IH with a more exotic topology, namely, that of a $U(1)$-bundle over a higher genus Riemann surface. It is assumed that the $U(1)$ is tangent to the generators of the horizon. The classical case of a Type D IH with topology of a higher genus Riemann surface times a line has been ruled out previously by Dobkowski-Rylko, Kaminski, Lewandowski and Szerezewski (employing complex analysis on the surface). In this thesis, their analysis is generalised to cover the aforementioned $U(1)$-bundle topology and all such solutions are derived. The horizons are then also explicitly embedded in static spacetimes (essentially the well known topological black holes with toroidal or hyperbolic sections).

Several times in the thesis these non-singular solutions are referred to as "black hole" solutions. In Misner's interpretation of Taub-NUT spacetime, the region exterior to the outermost horizon possesses closed timelike curves through every point (the $U(1)$-fibre is timelike) and therefore the interpretation as the exterior region to a black hole in not possible. Presumably the same can be said for the projectively non-singular horizons studied in this thesis (since the null generators are periodic), although it is perhaps less clear for the projectively singular case. A natural question that does not appear to have been addressed in the thesis is the following: do any of the new non-singular solutions actually admit a black hole interpretation? Indeed, more generally, it would be interesting to understand the maximal extensions and causal structure of the non-singular solutions. On the other hand, it shown that for $\Lambda > 0$ there are parameter ranges where the non-singular solutions admit a good cosmological interpretation (in these cases the $U(1)$-fibre is always spacelike). Therefore, interestingly, these solutions do appear to give completely smooth and non-pathological cosmological spacetimes, so perhaps cosmology rather than black hole theory is the context in which they are of most physical interest.

A few minors comments and typos:

- Page 49, in the third bullet point the equation $b = 1/(2b)$ appears, which presumably should be $b = 1/(2l)$.
- Equation 2.47 is not correct given the coordinate change eq 2.46. The discussion down to 2.52 may also need to be re-checked (there also appears to be an issue of P^2 versus P in that discussion).
- In equation 2.64 the "r" appearing should be "τ'"
- The formatting of equation 2.81 has cut off some of the equation for $\tilde{\omega}$.
- In several places (e.g. above eq 3.40) it is said "the flat metric on the two-sphere". Presumably what it is meant the "round" metric, or the constant curvature metric.
- Equation 3.43 is not correct, the r.h.s is not a vector field.
- After equation 5.59 presumably one needs $\tilde{\Omega}^2 c_1 < 0$.

- Start of sec 5.2.1 it is said that on a torus one has "global coordinates". Of course, strictly speaking this is not true.

To summarise, overall the thesis is of good quality and contains several new results concerning horizons with non-trivial topology as described above. Therefore, I am happy to recommend it for the award of PhD.

Your sincerely,

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