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Report on the Ph. D. thesis of Mr. Mateusz Czaja

To whom it may concern:

The properties of the B mesons, which constitute of a bottom or anti-bottom quark and a light (anti)quark, contribute in an essential way to our understanding of the fundamental interactions of the elementary particles. Due to a number of dedicated experiments, there is a vast amount of precise experimental data which can be confronted with precision predictions obtained on the theory side. Mateusz Czaja has provided a thesis which is centered around this topic. It has the title "Multiloop Corrections to Semileptonic Decays of the B Meson" and deals with higher order quantum corrections to the decay of a bottom quark to a charm quark and a lepton-neutrino pair.

Mr. Czaja has structured his thesis into three main chapters, which are subdivided into several sections. They are accompanied by an introduction and conclusions.

The Introduction extends over almost 15 pages and puts the thesis in to the general context. It covers a multitude of topics ranging from the experimental situation of some of the B physics observables to the general structure of multi-loop calculations. At the end of the Introduction Mr. Czaja outlines the structure of the thesis.

Chapter 1 covers fundamental aspects of quantum field theory. The author starts by introducing the path integral formalism which he uses to derive Feynman rules. In a next step the Lehmann-Symanzik-Zimmermann reduction formula is introduced and the connection to inclusive decay rates is worked out. It is a nice feature of the thesis that the general discussions are interrupted by "Sample Calculations" where Mr. Czaja shows how the formalism is applied to the main topic of the thesis, i.e. to the semileptonic decay rate of B mesons. In fact, he provides many details to the next-to-leading order calculation where the intermediate expressions are still quite compact. The first "Sample Calculation" is performed in section 3 of chapter 1, where the relevant Feynman rules are provided and the integrand for one of the two-loop Feynman diagrams contributing to the next-to-leading order predictions is shown.

Chapter 1 continues with a discussion of divergences which occur in loop calculations. This leads to the introduction of dimensional regularization as an important tool for the calculation of quantum corrections. The thesis continues with a discussion of the Dirac and colour algebra. The discussion of these two sections is quite elementary and it would have probably been a better idea to move this part to an appendix. Chapter 1 finishes with a further "Sample Calculation" where the calculation of the Feynman diagram already considered before is continued.

In chapter 2 Mr. Czaja discusses the technical aspects which are relevant for his calculation. In the first section he starts with introducing the so-called "integration-by-parts" method which allows to establish relations among scalar integrals. The latter are needed for the computation of the amplitude. After the "Sample Calculation" in section 2 the author discusses how differential equations can be used to compute master integrals. The thesis contains an extensive discussion about the construction of the so-called canonical basis of master integrals. In case it is possible to arrive at such a basis the computation of the master integrals is much simpler than in other bases. This is in particular true for the computation of higher orders is the dimensional parameter ϵ. Mr. Czaja provides instructive examples for the construction of a canonical basis. Unfortunately, for the next-to-next-to-leading order calculation, which is presented in chapter 3, it is not possible to construct a canonical basis for all contributions.

The third technical aspect which is important for the thesis is discussed in section 5: the auxiliary mass flow. Mr. Czaja summaries the ideas and findings of the literature in a competent way and explains the subtleties and challenges. Some parts are illustrated by explicit examples. For the calculation of the loop integrals which Mr. Czaja needs for his calculation he uses the publicly available program `AMFlow`.

Chapter 3 contains the new results obtained in the framework of this thesis. However, also here the first three sections are dedicated to more general results which can also be found in many text books. Mr. Czaja first describes the various contributions to the Standard Model Lagrange density. Afterwards he identifies the relevant terms for his process. The second section in chapter 3 is dedicated to Quantum Chromodynamics (QCD). He provides the Feynman rules which are relevant for his calculation and discusses the running of the strong coupling constant. Afterwards, in section 3, the heavy quark expansion is considered. Section 4 of chapter 3 is dedicated to the leading and next-to-leading order results for the q^2 spectrum of the semileptonic $b \to c$ transition. In this section he can profit from the "sample calculations" of chapters 1 and 2. Finally, the next-to-next-to leading order corrections are discussed in section 5. Among others, he discusses the workflow used for the calculation of the three-loop diagrams and the computation of the counterterm contributions. He puts special emphasis on the contributions with three charm quarks in the final state — the contribution which up to now only Mr. Czaja has computed.

The final section in chapter 3 discusses the numerical results for the q^2 spectrum of the semileptonic B decay. He starts with the contribution with three charm quarks in the final state. For this contribution the master integrals are only available numerically. For this reason he constructs an interpolation which provides an approximation at the percent level. Afterwards the contribution with a single charm quark in the final state is obtained after subtracting the triple-charm contribution from the full result. This brings Mr. Czaja into the position to compare with the results in the literature where he finds good agreement. The final part of section 5 is dedicated to the momentum spectrum which leads to the main results of the thesis shown in Figures 28 and 29.

Mr. Czaja ends his thesis with a summary of his achievements and provides a brief outlook on possible next steps.

Including the list of references the thesis has about 170 pages and is written in good English. For his calculations Mr. Czaja has used modern techniques like differential equations in canonical form, the auxiliary mass flow and semi-automated software to generate and process Feynman amplitudes. He has obtained highly non-trivial results which will be important for future phenomenological studies. Parts of his results were cross-checked against a competing group which has published only shortly before Mr. Czaja and his collaborators have published their results. In my opinion already this part is sufficient for a PhD thesis. Mr. Czaja has in addition computed a contribution which the competing group did not consider. Although this contribution is numerically small it is needed in order to have a complete result. On the positive side I also want to mention the "sample calculations". They are particularly useful for future master or PhD students who want to study similar processes. Furthermore, I also would like to mention that the thesis contains an extensive and detailed list of references.

It should also be stressed that about one year ago Mr. Czaja (together with Prof. Mikolaj Misiak) wrote a very comprehensive mini-review about the process $B_s \to \mu^+\mu^-$ summarizing the various results which have been obtained in recent years. Mr. Czaja also provided important contributions to the paper "The $Q_{1,2}$–Q_7 interference contributions to $b \to s\gamma$ at $\mathcal{O}(\alpha_s^2)$ for the physical value of m_c" where he is one out of eight coauthors. In this paper, which is published in the *European Journal for Physics C*, important next-to-next-to-leading order contributions to the decay of a bottom quark into a charm quark and a photon have been computed. Both topics are not included in the thesis.

My main criticism of the thesis is related to the elementary material contained in particular in chapter 1 and the beginning of chapter 3 such as the path integral formalism and the QCD Lagrange density. It is contained in many text books about quantum field theory and thus should either be part of an appendix or should not be present at all.

As a further criticism on the structure of thesis I want to remark that some of the sections are quite long and contain many different topics. For example, section 2 of chapter 3 contains a discussion of the QCD Lagrange density, the Feynman rules, the running of α_s and the renormalization in terms of counterterms. A subdivision into smaller parts would have increased the readability of the thesis.

In the last two sections of chapter 3 a more detailed discussion of the results would have been helpful. Unfortunately some of the figures are only very briefly described but there is no detailed interpretation of the results shown in the plots.

Despite my criticism I am of the opinion that the thesis is very good and contains important new results which will be used for future phenomenological analyses. Mr. Czaja has demonstrated a deep knowledge of quantum field theory and the ability to apply it to highly non-trivial problems. He has obtained original results. In particular, for the first time he has computed the q^2 spectrum of B meson decays to second order in the perturbative expansion including three charm quarks in the final state. I recommend to proceed with the further steps for the PhD defense procedure.

Yours sincerely,

Matthias Steinhauser