

Bell non-locality is a fundamental concept in quantum mechanics that reveals the non-classical correlations between entangled quantum systems, which cannot be explained by any local hidden variable theory. This phenomenon is not just of theoretical interest but also plays a critical role in the practical certification of quantum devices. Specifically, Bell non-locality provides a powerful tool for certifying the operation of quantum devices in a device-independent manner, meaning that the certification does not rely on trusting the internal workings of the devices but rather on the statistical properties of the measurement outcomes.

This thesis comprises a collection of three papers that explore Bell non-locality and its most direct application, the certification of quantum devices. The first part of the thesis offers a brief introduction to these topics, while the second part presents each of the three studies.

The first of these studies presents a standard approach to both topics by constructing a family of Bell inequalities and characterising their optimal realisation. This includes subjecting the measurements of one of the parties to a device-independent certification protocol and self-testing the optimal shared state.

In the second study, we introduce a variation of the random access code (RAC) protocol, termed biased RACs. We then develop a semi-device-independent certification protocol for the decoding measurements that covers all cases of 2-bit encoding and some cases of 3-bit encoding.

Finally, the last study explores the use of general Machine Learning and data science techniques in the simplest Bell scenario and its variant for the correlation space, framing the problem as a classification task to distinguish non-signalling behaviours between quantum and not quantum. While Machine Learning models do not outperform existing methods, our exploration of behaviour classification techniques has led to the development of new variants of the see-saw and NPA algorithms, allowing us to explore non-exposed points on the boundary of the CHSH quantum set.