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Report on the dissertation of MSc. Bartosz M. Zawora: The Marsden-Meyer-Weinstein reduction theorem and its applications in energy-momentum methods

In this dissertation, the Marsden–Meyer–Weinstein (MMW) theorem for symplectic structures and the energy–momentum method for the stability of solutions of symmetric symplectic Hamiltonian systems are extended to geometric settings close to symplectic geometry. The thesis consists of an introduction and three chapters.

In the introduction, the motivation for the general scientific objectives pursued is presented, with particular emphasis on MMW reduction theory. In particular, the relevance of this theory is highlighted in the study of symmetric symplectic Hamiltonian systems, as well as the need to clarify certain errors and limitations in extensions of this theory to other geometric contexts of interest in Mechanics and Classical Field Theories. These contexts include cosymplectic, k-polysymplectic, and k-contact geometry. The introduction also presents a summary of five scientific articles published by the author of the dissertation (in collaboration with other researchers) that are directly related to the results of the thesis. The essential contributions of these articles are discussed in the final part of this report.

Chapter 1 reviews the basic constructions and known theories in the literature that are used throughout the development of the thesis. This includes the basic foundations of Lyapunov stability for non-autonomous systems, symplectic, cosymplectic, and Poisson geometry, as well as the geometric formulation of Classical Field Theories in their k-polysymplectic, k-polycosymplectic, and k-contact versions. This makes the thesis fully self-contained and facilitates its reading without difficulty.

Chapter 2 constitutes the core of the thesis. In its first part, the original MMW reduction theory is reviewed: (i) its motivation from symplectic reduction by a submanifold; (ii) the importance of the presence of a (not necessarily equivariant) momentum map in reduction and as a generator of first integrals of the symmetric system; (iii) the description of the reduced symplectic space and the reduced dynamics on it; and (iv) its connection with prior work by important researchers such as Smale, Sternberg, Kostant, and others. The theory is then extended to the cosymplectic, k-polysymplectic, k-polycosymplectic, and k-contact settings. Although parts of some of these theories had already been developed, an important contribution of the author is the study of reduction (in all the aforementioned contexts) in the presence of a momentum map that is not necessarily equivariant. Another relevant contribution is the identification of unnecessary technical conditions previously assumed but not required for the proofs, as well as the detection of errors in earlier demonstrations and their correction. Finally, all the developed theories are applied to interesting examples from a geometric, mechanical, and classical field theory perspective.

Chapter 3 is devoted to the development of extensions of the classical energy–momentum method, for symmetric symplectic Hamiltonian systems, to other geometric contexts. This method describes the behavior of solutions in a neighborhood of trajectories induced by Lie symmetries of the system and is based on MMW reduction theory and stability analysis techniques. The aim is to study the stability of relative equilibria, that is, equilibrium points of the reduced system. Using the reduction methods developed in Chapter 2, Chapter 3 discusses energy–momentum methods for analyzing Lyapunov stability on reduced manifolds. Special emphasis is placed on applications to time-dependent and non-autonomous systems. To this end, the symplectic context is considered first, followed by the cosymplectic one, with applications to interesting examples such as almost rigid bodies, quantum systems, and the reduced circular three-body problem. In the final part of the chapter, the theory is extended to the k-polysymplectic context with the development of an energy–momentum method, and further interesting applications of all the previous results are presented, including complex Schwarz equations, k-polysymplectic affine Lie systems, quantum quadratic Hamiltonian operators, and others.

The dissertation concludes with a section summarizing the achievements, discussing ongoing work related to the thesis, and proposing interesting future research directions connected to its topics.

Apart from the interest, relevance, and importance of the results discussed above, the thesis is supported by the publication of five research papers authored by the candidate (in collaboration with other researchers), which contain the main contributions presented in the dissertation. These articles are:

- J. de Lucas, B.M. Zawora, *"A time-dependent energy–momentum method"*. Journal of Geometry and Physics 170, 104364, 2021, 21 pp.

In this paper, an extension of the energy–momentum method is presented to study the stability of non-autonomous Hamiltonian systems invariant under the action of a Lie symmetry group. The systems are reduced via the corresponding MMW reduction theorem, and applications to almost rigid bodies are proposed.

- J. de Lucas, X. Rivas, S. Vilariño, B.M. Zawora, *"On k-polycosymplectic Marsden–Weinstein reductions"*. Journal of Geometry and Physics 191, 104899, 2023, 36 pp.

In this article, certain technical conditions previously assumed in proofs of results related to polysymplectic MMW reduction theory are identified as unnecessary. At the same time, as an application, a k-polycosymplectic MMW reduction theory is proposed and applied to interesting physical systems in classical field theory, such as coupled vibrating strings.

- J. de Lucas, A. Maskalaniec, B.M. Zawora, *"Cosymplectic geometry, reductions, and energy–momentum methods with applications"*. Journal of Nonlinear Mathematical Physics 31, 64, 2024, 58 pp.

This paper proposes a new cosymplectic energy–momentum method to study time-dependent Hamiltonian systems and gradient systems on cosymplectic manifolds. The method is developed so that it can be applied in the presence of a momentum map that is not necessarily equivariant (an assumption made in previous contributions). Applications to classical and quantum systems are also provided.

- L. Colombo, J. de Lucas, X. Rivas, and B.M. Zawora, *"An energy–momentum method for ordinary differential equations with an underlying k-polysymplectic manifold"*. Journal of Nonlinear Science 35, 42, 2025, 54 pp.

In this article, using k-polysymplectic MMW reduction theory, a k-polysymplectic energy–momentum method is proposed, introducing new techniques for stability analysis. This theory is applied to several classical and quantum interesting examples from both mathematical and physical perspectives.

- J. de Lucas, X. Rivas, S. Vilariño, B.M. Zawora, *"Marsden–Meyer–Weinstein reduction for k-contact field theories"*. Preprint: arXiv:2503.03463, 2025, 50 pp.

In this paper, an MMW reduction theory for k-contact geometry is proposed, and at the same time, certain errors in the previous literature on contact reduction are identified and corrected. The results are applied to various interesting examples from both mathematical and physical viewpoints.

To all of the above, one should add four additional research articles authored by B. M. Zawora (in collaboration with other researchers) related to the topics of the dissertation:

- C. Gonera, J. de Lucas, W. Szczesek, B.M. Zawora, *"More on superintegrable models on spaces of constant curvature"*. Regular and Chaotic Dynamics 27, 561–571 (2022).
- A. M. Grundland, J. de Lucas, B.M. Zawora, *"Stability analysis of the (1 + 1)-dimensional Nambu–Goto action gas models"*. J. Phys. A 58, 50LT01 (2025).
- X. Rivas, N. Román-Roy, B.M. Zawora, *"Symmetries and Noether's theorem for action-dependent multicontact field theories"*. Lett. Math. Phys. 115, 108 (2025).
- J. Lange, B.M. Zawora, *"Reduction of exact symplectic manifolds and energy hypersurfaces"*. Proceedings of the 7th International Conference on Geometric Science of Information, LNCS, vol. 16035: 328–336, Springer.

The main results contained in the dissertation are of great interest from both a geometric and a physical point of view, particularly in the field of geometric mechanics and the geometric formulation of classical field theories. This is further supported by their

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publication in several high-level journals in the area. Moreover, the dissertation is very well written and includes an appropriate list of references.

In addition, I would like to highlight the strong work ethic demonstrated by B. M. Zawora. This is evident not only from the results presented in the dissertation, but also from the production and publication of other related contributions, as well as from the ability to collaborate with researchers from different institutions and universities, despite his youth.

For all the above reasons, my assessment of the work carried out by B. M. Zawora is highly positive and, consequently, I fully support the submission of the thesis.

Signed: Juan Carlos Marrero González
Full professor of Geometry and Topology