Review on *Marsden–Meyer–Weinstein Reduction Theories and their Applications to Energy-Momentum Methods*

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1 General theoretical background and independence

The thesis focuses on extending the Marsden-Meyer-Weinstein reduction and the energy momentum method to different geometries. From a theoretical and technical point of view, this dissertation stands out for the extension and complexity of the results.

The field of geometric mechanics took its modern form with the introduction of symplectic geometry. Nonetheless, it is common to use different geometric structures to describe those systems that cannot be describe with symplectic geometry. A lot of current investigations consist in translating the classical results of symplectic geometry to the new geometries. In this regard, the objective of the thesis, as well as the way the results are presented and justified, conforms to what is expected of an independent researcher of the field.

It is common for a PhD student to learn and use several different geometric structures en the development of their thesis. Nonetheless, this thesis stands out for its extensive assortment of geometries. For mechanics, there is Poisson, t-dependent symplectic, cosymplectic and contact geometries. For field theories, the PhD candidate has made contribution to most of the geometries based on the vector-valued approach: k-polysymplectic, k-polycosymplectic and k-contact. The focus on this approach is justified by the fact that the Marsden-Meyer-Weinstein reduction for field theories is a famously difficult problem, for which there has been solid progress only in the vector-valued approach. All these geometric structures are adequately

presented in chapter 1, with extensive references to the relevant literature. Moreover, in other projects, the student has work with the other two major approaches to geometric field theory: the infinite-dimensional and the covariant.

Reduction is considered one of the most relevant and challenging problem in geometric mechanics. Regarding the Marsden-Meyer-Weinstein reduction, its extension to field theories has proven to be challenging. A sign of its technical complexity is the fact that a relatively high proportion of the published papers in the topic contain relevant mistakes. This dissertation proves several new extensions of the Marsden-Meyer-Weinstein reduction, relating them with previous attempts in the literature. The explanation of the relevant technical points of the proofs shows the expertise of the student.

All and all, the student has shown proficiency of an extremely wide assortment of approaches and techniques, some of them particularly challenging.

During the PhD the student has published 9 papers, with 9 collaborators different from their supervisors. The paper "Symmetries and Noether's theorem for action-dependent multicontact field theories" is a result of a research visit outside of Warsaw in a different (albeit related) topic. The paper "Reduction of exact symplectic manifolds and energy hypersurface" is not co-authored with their supervisors nor any senior researchers. In may opinion, this shows the ability of the student to conduct independent research.

2 Originality and innovation

When a symmetry is found in a system, one would like to use it to simplify the system. The geometric version of this classical problem is the Marsden-Meyer-Weinstein reduction. The related concept of momentum map is cornerstone in the study of the integrability and stability of a system. Extending these results to systems described by other geometries is one of the main objectives of geometric mechanics. This dissertation represents a substantial advancement in this long-standing problem.

The advances in this thesis, in regards of the Marsden-Meyer-Weinstein reduction, are based in previous results and concepts: the inclusion of non-Ad*-equivariance momentum maps in symplectic geometry, the reduction of (Ad*-equivariance) k-polysymplectic manifolds and the correspondence between contact distributions and symplectic $\mathbb{R}^{\times}$-principle bundles. In order to proof the Marsden-Meyer-Weinstein reductions, the student has innova-

tively intertwined and extended this results, and supplemented them with a precise description of the level sets of the momentum maps and its orthogonal spaces. As an example, the correction of the Marsden-Meyer-Weinstein reduction for contact systems is derived from the k-contact case (with $k = 1$) which, in its turn, is derived from the k-polysymplectic by extending the correspondence between contact and symplectic geometries to the vector-valued case.

The dissertation also develops energy-momentum methods to study the stability of the system. These results are specially interesting because, even though they are clearly relevant, they are only sparsely considered for non-symplectic systems. In particular, the thesis introduces new methods to study the stability of time-dependent systems, and proposes a new approach to study the stability of field theories. This is very innovative as it is a geometric and finite-dimensional approach, in contrast with the usual infinite-dimensional extension of Lyapunov theory.

Another original contribution of this thesis is the use of geometric structures commonly used in field theories to study mechanical systems. In the work "An energy-momentum method for ordinary differential equations with an underlying k-polysymplectic manifold", the k-polysymplectic structure can be though as a rigidity property that helps characterize the mechanical system.

3 Presentation

The dissertation is superbly written. It contains an introduction which clearly exposes the aim of the thesis and its motivation. The literature review is detailed. The manuscript contains a nice summary of the structures, properties and previous results used. The new results and proofs are clearly presented. This is not an easy achievement in this case, because some proofs are very technical. They are accompanied by helpful explanations, relating the results of different sections, clearly explaining possible confusions and including illustrative examples when necessary. The results are discussed in relation to the previous literature.

4 Justification of distinction

In my opinion, this PhD dissertation deserves a distinction. The quantity and quality of the research the student far exceeds a typical PhD thesis in the field. Their works has received the attention of the community. To sub-

stantiate this claim, I point out that it has been cited by several researchers, from the mathematicians working on the forefront of reduction to physicists. The most valuable results are, in my opinion:

- The clarification of the conditions for the Marsden-Meyer-Weinstein reduction for k-polysymplectic systems. It entails the comparison of half a dozen papers with conflicting results during the last decade. The mistakes are made evident with explicit examples. This stabilizes the foundations of the Marsden-Meyer-Weinstein reduction for field theories, which will have a great impact in future research in the topic.

- The energy momentum method for k-polysymplectic represents a new geometric approach to study the stability of field theories. This is a relevant question in different disciplines. For instance, identifying stable modifications of general relativity is, currently, one of the main obstacle in theoretical cosmology. Current methods use an infinite-dimensional version of Lyapunov theory. The energy-momentum method present in this dissertation represent a new geometric and finite-dimensional approach to stability.

- The Marsden-Meyer-Weinstein reduction for k-contact systems. It represents a new version of the theorem for field theories, which is in itself very relevant. Moreover, it corrected the same theorem for the contact case.

5 Conclusion

My evaluation of this PhD dissertation is positive, and deserves a distinction.

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