Classical and quantum Kneser graphs

Abstract

We present a construction of quantum Kneser graphs, which yields classical Kneser graphs in the commutative case. We also generalize this construction to obtain quantum versions of generalized Kneser graphs, such as Johnson graphs. We discuss several basic properties of the constructed quantum graphs, such as their reflexivity and regularity. Then, we investigate their quantum chromatic numbers and establish upper bounds on them by constructing explicit colorings. We also investigate spectral lower bounds on quantum chromatic numbers for several examples and formulate a conjecture about their asymptotic behavior. Lastly, we present an alternate approach to quantization of Kneser graphs and a few results obtained for the quantum Mycielski construction.