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Report on the doctoral thesis of Maciej Kierkla
entitled „Theoretical consistency and phenomenology of supercooled cosmological phase transtions"

The dissertation of mgr. Maciej Kierkla provides an extensive and precise study of a model with radiative symmetry breaking and a first-order phase transition. Assuming it took place in the early Universe, such phase transition would generate a gravitational-wave signal observable at future space interferometers like LISA. By using advanced, state-of-the-art techniques in thermal field theory, the Candidate and collaborators have managed to significantly reduce the theory uncertainties typically marring the computation of the phase-transition parameters.

The dissertation is structured into seven chapters. These include an initial Introduction, followed by two chapters dedicated to an overview of the physics of effective actions, thermal field theory, and phase transitions. The main body of the dissertation is presented in Chapters 4, 5, and 6, which are dedicated, respectively, to the detailed presentation of a conformal extension of the Standard Model (SM); an extensive description of the theoretical uncertainties associated with the computation of the bubble nucleation rate and the proposed techniques to reduce such uncertainties; and, finally, the impact of such uncertainties on the phenomenology of the conformal extension of the SM introduced in chapter 4. A final Summary chapter describes potential venues for improvement and challenges for the future. These chapters are followed by three Appendix sections, dedicated to technical aspects of the computations described in the main body, and by an extensive bibliography.

The dissertation is based on four articles and one Proceedings of Science co-authored by the Candidate, all of which are available on the arXiv repository as preprints. Three of the articles are published in the Journal of High Energy Physics (JHEP), one of the leading journals in the field. The fourth one is presumably currently under review. Together with a fifth article published in JHEP, these publications represent the entirety of the Candidate's scientific output to date.

I have no remarks about chapter 2, which reviews the formalism of the quantum effective action, background field method, and radiative symmetry breaking for massless scalar electrodynamics. This chapter introduces some of the language and notions that will be employed later on.

Chapter 3 offers a review of thermal field theory and first-order phase transitions. The imaginary time formalism and Matsubara modes are introduced. The Candidate then proceeds to review in some detail the issue of the residual scale dependence of finite-temperature effective actions when the „daisy" diagram resummation method is employed to cancel infrared divergences. The proposed solution, which will reinstate a full renormalization-scale independent thermal action, is to match the 4-dimensional theory to a 3-dimensional theory via dimensional reduction. The Candidate spends some time reviewing this concept, and it is already clear at this stage that the use of dimensional reduction will constitute one of the central themes of the dissertation. For this reason, I am a bit surprised by the considerable amount of typos introduced in this chapter:

- A log seems to be missing in eq. 3.13
- One insertion of the background squared seems to be missing in eq. 3.31
- Inequalities seem to be inverted in eq. 3.45
- *T* should be squared in the first line of eq. 3.54
- *T^2* appears to be missing in the r.h.s. of eq. 3.66

and so on. Even if this chapter reviews material that is relatively well known, paying a little bit more attention to the details would have improved the reading experience. The chapter concludes with a review of thermal bubble nucleation.

Chapter 4 is the first of three based on the articles co-authored by the Candidate. It introduces the conformal dark SU(2) extension of the SM and its parameter space. This is the model chosen for the theoretical and phenomenological analysis. A supercooled cosmological phase transition will be studied in later chapters for the model introduced here. My main objection in this chapter regards the motivation for the choice of this -rather than another- model of New Physics. The Candidate refers to the likely presence of vector dark matter, which seems to be really the only phenomenological justification provided in favor of this choice. If so, then, why is there no reference to the parameter space favored by the relic density and/or direct detection? I see that such discussion was made in the original paper (ref. [1]), but it is absent in the dissertation. I see this as a point of conceptual weakness and I will come back to this later.

Chapter 5 features the most substantive part of the dissertation. In it, the full finite-temperature analysis of the conformal SU(2) extension of the SM is performed. The vanilla, "daisy" computation of the thermal bubble nucleation rate is here replaced by a new computation based on dimensional reduction and effective field theory matching, which is performed in two ways and improves significantly on the treatment adopted by and large in the literature. In the first way the calculation is performed at next-to-leading order (NLO) in the derivative expansion method, which is valid under the assumption that the masses of gauge modes are larger than the magnitude of the momenta of the nucleating scalar field. It is explained that this technique is not reliable already at NLO due to errors introduced by the spatial gauge modes. The second way utilizes the state-of-the art technique of functional determinants, which fully restore the renormalization scale invariance of the thermal potential. Figure 5.5 shows in practice the quantitative impact of the theoretical uncertainties associated with the different methods on the nucleation temperature. Note, again, the unfortunate presence of typos in the masses of eqs. 5.20 and 5.58, 5.59 and some minor imprecision (at *n>3* the *c2n* operators in eq. 5.20 are *irrelevant*, not marginal).

Chapter 6 is dedicated to the phenomenology of the model with special attention to the gravitational-wave signal at LISA. The effects of the two methods introduced in chapter 5 for the computation of the parameters of the first-order phase transition are analyzed quantitatively, showing sizable modifications to the percolation temperature, latent heat, and transition speed with respect to the „daisy” treatment. Chapters 5 and 6 contain the core of the dissertation. The main point is that the theoretical uncertainties must be handled with care because they have a non-negligible effect. But this leaves me with some questions I would like the Candidate to address:

1. Figure 5.5 shows that the different methods introduced in chapter 5 can modify the nucleation temperature by a factor of two-to-several (at fixed mass and coupling). Has

the Candidate considered the uncertainties due to the model itself? Specifically, how drastically does the nucleation temperature change in one method (say, NLO det) if we were to add a sizable Yukawa coupling between the dark scalar and a dark fermion?

2. The description of theoretical uncertainties given in section 5.4, while very detailed, is at times somewhat confusing. Can the Candidate spend a few words describing in detail figure 5.4? More specifically, what is the expected quantitative impact of the corrections *beyond* NLO (red squares) in the two methods based on dimensional reduction?
3. It is stated in the dissertation that a detailed treatment of theoretical uncertainties is necessary because the parameters of the phase transition will be reconstructed precisely from LISA data or similar. However, when comparing figures 6.11 and 6.15 I do not see any difference in the signal-to-noise ratio (SNR) computed with daisy vs NLO det. In my opinion this would imply that the LISA SNR is likely not sensitive to the uncertainties of the phase-transition parameters. Could the Candidate explain how it would be possible to reconstruct those parameters faithfully from experimental data? Will the reconstruction not be based on a likelihood function taking as input quantities like the SNR?
4. In the plots of chapter 6, it is clear that the chosen model undergoes the electroweak phase transition only for limited mass and gauge coupling ranges. Are those ranges consistent with the relic density of dark matter and the constraints from direct detection? If they are, how sensitive is the region favored by dark matter to the uncertainties investigated in chapter 5? This circles back to my initial question about the choice of model. Since the SM admits a second-order phase transition, we need a *reason* to look for a first-order transition. Such a phenomenon would certainly be interesting if it were associated with a model improving on some of the weaknesses of the SM (for example by having neutrino masses, DM, improving electroweak naturalness, having no U(1) Landau pole, explaining the baryon asymmetry, etc.) Can the Candidate give me a good reason why we should believe that the analysis done for this model, while detailed and thorough, has anything to do with Nature?

Despite the minor issues noted above and my critical remarks, I assess the thesis as a whole very positively. In particular, the results presented in chapters 4-6 constitute a robust analysis of the theoretical uncertainties associated with the computation of the parameters of a supercooled phase transition and the proposed solutions are effective and innovative. These findings represent original contributions that extend beyond the scope of the existing literature. The Candidate demonstrates both technical competence and a strong command of the theoretical framework, and it is evident that he has acquired the necessary qualifications for conducting research at the doctoral level.

To sum up, I am of the opinion that the dissertation fully meets the standards expected of a PhD thesis, and I recommend that the procedure towards the defence be allowed to proceed.

Conclusion the Polish language:

Podsumowując, w mojej ocenie rozprawa doktorska w pełni spełnia wymagania stawiane pracom doktorskim i wnioskuje o dopuszczenie magistra Kierkli do dalszych etapów przewodu doktorskiego.

Enrico Maria Sessolo