

Kinetic theory and hydrodynamics of nearly-integrable gases

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Integrable one-dimensional quantum gases have attracted significant attention. One could argue that this has happened for two primary reasons: analytical access to strongly correlated, non-perturbative physics and the availability of numerous experimental realizations. Recently, the field has shifted toward non-equilibrium dynamics, driven by theoretical breakthroughs such as generalized Gibbs ensemble (GGE) and generalized hydrodynamics (GHD).

The presence of additional integrability-breaking effects in experiments, such as external trapping or additional long-range interactions, calls for the development of perturbative techniques to tackle the dynamics of nearly-integrable gases. The external trapping occurs generically in the experiments, while the long-range interactions are especially prominent in dipolar quantum gases of elements with large magnetic dipole moments, such as dysprosium. The theoretical treatment of such systems is the main topic of this thesis.

The general goal was to provide a framework for understanding of two classes of non-equilibrium phenomena in nearly-integrable gases. The first class of problems which we tackle is related to thermalization or relaxation more broadly. In the context of gases, the description of relaxation is typically provided by kinetic theory, with the cornerstone being the BBGKY (Born-Bogoliubov-Green-Kirkwood-Yvon) hierarchy for correlation functions. The hierarchy is the starting point for further approximations and upon truncation, leads to collision operators such as the Boltzmann or Landau equations. Such collision kernels provide the time scales for relaxation and the fixed points of the dynamics.

In this thesis, a BBGKY-like framework was adapted to nearly-integrable systems. The resulting hierarchy can be efficiently truncated. A kinetic equation describing thermalization was derived and successfully tested against numerical simulations and experimental data for thermalization rates in dipolar gas experiments. The geometry of such experiments, in which 1D gases are placed in a 2D lattice, motivated the study of integrability breaking through interactions between two tubes. In such a scenario, interesting athermal stationary states were found, and an understanding through the kinematics of two-body scattering in 1D Galilean systems was proposed.

The second part of this contribution is related to the hydrodynamics of nearly-integrable gases, that is the effective theory of dynamics emerging at large spatiotemporal scales. The structure of hydrodynamics is intimately tied to the conservation laws present in the system. The set of conserved charges changes drastically upon integrability breaking, and so does the hydrodynamics.

Building on the frameworks of the GGE and GHD, we have shown that at sufficiently large space-time scales, the hydrodynamic universality class of nearly-integrable gases changes from GHD to a one-dimensional version of the Navier-Stokes (NS) equations. Transport coefficients – viscosity and thermal conductivity were computed explicitly. This goal was achieved by a suitable generalization of the classic derivation of NS hydrodynamics from the Boltzmann equation through the Chapman-Enskog (ChE) expansion. The logic of the ChE method was applied also to a two-tube system, where athermal states were found. Surprisingly, the Burgers equation was uncovered there in the large scale dynamics of the density.

Finally, we considered the problem of anomalous transport in 1D non-integrable gases. Using nonlinear fluctuating hydrodynamics, the well-established framework to tackle problems in this field, we found a mechanism leading to the suppression (with the inverse of the integrability breaking parameter) of anomalies in transport. We formulated a general prediction that nearly-integrable gases are characterized by three different hydrodynamic universality classes, which occur in different space-time regimes.

Keywords: hydrodynamics, kinetic theory, integrable systems, one-dimensional quantum gases, thermalization, transport