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From classical hydrodynamics to quantum hydrodynamics and back again – how the Navier-Stokes equations describe quantum systems

Although the Navier-Stokes equations are the foundation of modern hydrodynamics, adopting them to quantum systems has so far been a major challenge. Researchers from the Faculty of Physics at the University of Warsaw, Maciej Łebek, M.Sc. and Miłosz Panfil, Ph.D., Prof. UW, have shown that these equations can be generalised to quantum systems, specifically quantum liquids in which the motion of particles is restricted to one dimension. This discovery opens up new avenues for research into transport in one-dimensional quantum systems. The paper, published in the prestigious Physical Review Letters, was awarded an 'editors' suggestion'.

Liquids are among the basic states of matter and play a key role in nature and technology. The equations of hydrodynamics, known as the Navier-Stokes equations, describe their motion and interactions with the environment. Solutions to these equations allow us to predict the behaviour of fluids under various conditions, from the ocean currents and the blood flow in blood vessels, to the dynamics of quark-gluon plasma on subatomic scales. The Navier-Stokes equations, formulated in the 19th century based on the principles of conservation of mass, momentum and energy, belong to classical physics. However, the motion of particles is governed by the laws of quantum mechanics, which raises the question of whether these equations can be derived from the principles of quantum physics.

The relationship between hydrodynamics and the microscopic description of the movement of the particles forming a liquid is not only of theoretical, but also of practical importance. The Navier-Stokes equations contain certain quantities, known as transport coefficients, which determine how quickly disturbances in the fluid dissipate, meaning how quickly the system returns to equilibrium. Their values cannot be deduced without knowledge of the microscopic interactions between the molecules. Deriving these equations from the microscopic laws makes it possible to determine the relationship of the transport coefficients to the interactions between the molecules.

Navier-Stokes equations in quantum systems

The application of the Navier-Stokes equations to quantum systems has so far been a major challenge. Researchers from the Faculty of Physics at the University of Warsaw, Maciej Łebek, M.Sc. and Miłosz Panfil, Ph.D., Prof. UW, have addressed this issue in the context of quantum liquids, in which the motion of particles is restricted to one dimension. Under specific conditions, such systems exhibit quantum integrability, i.e. the presence of multiple conservation laws. This feature has important consequences - it makes it possible to accurately describe the state of the fluid (using the wave function) also in the case of strong interactions between particles. 'In combination with numerous conservation laws, this has allowed the formulation of equations describing the hydrodynamics of these systems, called generalised hydrodynamics. The generalised hydrodynamics equations are much more complex than the Navier-Stokes equations. Despite their complexity, they have been confirmed in experiments with ultracold quantum gases and were the starting point of our work,' explains Maciej Łebek, M.Sc., first author of the paper, published in the prestigious journal Physical Review Letters

Another difference between the Navier-Stokes equations and the generalised hydrodynamics equations is the range of applicability. The Navier-Stokes equations hold for most liquids, whereas the generalised hydrodynamics equations apply only to integrable systems. - In our study, we have taken into account the influence of additional interactions between particles that break the integrability. If they are sufficiently weak, the dynamics of the system can still be described by the generalised hydrodynamics equations, supplemented with an additional term describing non-integrable interactions. As a result, the equations take a structure reminiscent of the Boltzmann kinetic equation' explains Dr Miłosz Panfil, Prof. UW.

In a paper published in "Physical Review Letters", the UW researchers showed that the Navier-Stokes equations are derived from generalised hydrodynamics with an additional Boltzmann term, and derived formulae for transport coefficients such as viscosity and thermal conductivity. 'Interestingly, the derived values of these coefficients have two contributions - one from integrable interactions and the other from interactions that break integrability. Classical kinetic theory for weakly interacting liquids predicts zero viscosity, which contradicts experimental results. Our method, on the other hand, provides a viscosity value different from zero, which is due to the subtle interplay between the two types of interactions, the researcher explains.

Transport in quantum systems

The UW researchers' results show that the ideas of hydrodynamics are also applicable in quantum conditions. They are an example of the microscopic derivation of transport coefficients in strongly interacting systems. They also have practical relevance for contemporary experiments on ultracold atoms conducted in laboratories around the world. The discovery opens up new possibilities for research on transport in one-dimensional quantum systems. In the future, the researchers plan to extend the theory to more complex systems and to experimentally test the model's predictions.

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Faculty of Physics of the University of Warsaw

Physics and astronomy at the University of Warsaw appeared in 1816 as part of the then Faculty of Philosophy. In 1825, the Astronomical Observatory was established. Currently, the Faculty of Physics at the University of Warsaw consists of the following institutes: Experimental Physics, Theoretical Physics, Geophysics, the Department of Mathematical Methods in Physics, and the Astronomical Observatory. The research covers almost all areas of modern physics on scales from quantum to cosmological. The Faculty's research and teaching staff consists of over 250 academic teachers. About 1,100 students and over 170 doctoral students study at the Faculty of Physics UW. The

University of Warsaw is among the 300 best universities in the world, educating in the field of physics according to Shanghai's Global Ranking of Academic Subjects.

SCIENTIFIC PUBLICATION:

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Pictured: water whirls. Solutions to the Navier-Stokes equations make it possible to predict the behaviour of liquids under a variety of conditions, from the motion of the oceans, to the flow of blood in blood vessels, to the dynamics of quark-gluon plasma on subatomic scales (photo by Maciej Łebek, Faculty of Physics, Warsaw University).