Adam Gomułka 405984

Zadanie. 1.

Bardzo dobre rozwiązanie! 5 pkt

$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$ Energia kinetyczna

$V = -q \vec{v} \cdot \vec{A} = -q \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ Bx \\ 0 \end{bmatrix} = -q \dot{y} B x$ Potencjał

Czyli: Lagranżjan

$L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q \dot{y} B x$ ✓

Pędy uogólnione $p_i = \frac{\partial L}{\partial \dot{q}_i}$ $q_i = \{x, y, z\}$

$p_x = m\dot{x}$ $p_y = m\dot{y} + qBx$ $p_z = m\dot{z}$ ✓

Hamiltonian:

$H = \sum_j p_j \dot{q}_j - L = m\dot{x}^2 + m\dot{y}^2 + qBx\dot{y} + m\dot{z}^2 - \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - q\dot{y}Bx =$

$= \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \left\{ \begin{array}{l} \dot{x} = \frac{p_x}{m} \\ \dot{y} = \frac{p_y - qBx}{m} \\ \dot{z} = \frac{p_z}{m} \end{array} \right\} =$

$= \frac{1}{2} m \left(\frac{p_x^2}{m^2} + \frac{(p_y - qBx)^2}{m^2} + \frac{p_z^2}{m^2} \right) =$

$= \frac{1}{2m} \left(p_x^2 + (p_y - qBx)^2 + p_z^2 \right)$ ✓

↓

$$\{v_x, v_y\} = \sum_i \left(\frac{\partial \dot{x}}{\partial q_i} \frac{\partial \dot{y}}{\partial p_i} - \frac{\partial \dot{y}}{\partial q_i} \frac{\partial \dot{x}}{\partial p_i} \right) =$$

$$= \left\{ \begin{array}{l} \text{pierwszy} \\ \text{człon zawsze} \\ \text{się zeruje, bo} \\ \frac{\partial \dot{x}}{\partial q_i} = 0 \ \forall_i \end{array} \right\} = -\left[\frac{\partial \dot{y}}{\partial y} \frac{\partial \dot{x}}{\partial p_y} + \frac{\partial \dot{y}}{\partial x} \frac{\partial \dot{x}}{\partial p_x} + \frac{\partial \dot{y}}{\partial z} \frac{\partial \dot{x}}{\partial p_z} \right] =$$

$$= \text{[redacted]} - \frac{\partial}{\partial x}\left(\frac{p_y - qBx}{m} \right) \frac{\partial}{\partial p_x}\left(\frac{p_x}{m} \right) =$$

$$= -\left(-\frac{qB}{m} \right) \cdot \frac{1}{m} = \frac{qB}{m^2}$$ ✓