Wormholes, geons, and the illusion of the tensor product

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based on [2212.10652]

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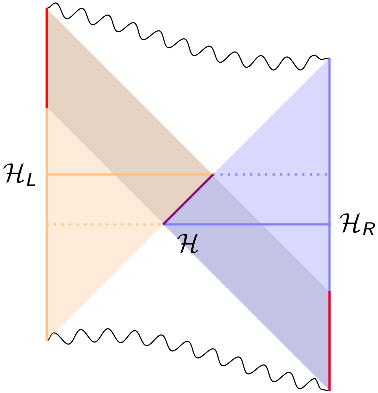
Motivation



- The most persistent assumption regarding holographic, traversable wormholes: $\mathcal{H} \cong \mathcal{H}_L \otimes \mathcal{H}_R$.
- $|0\rangle_L \otimes |0\rangle_R$ dual to AdS^2
- $|TFD\rangle$ dual to BH
- highly entangled state dual to WH?

I want to argue that $\mathcal{H}_L \otimes \mathcal{H}_R \cong \mathcal{H} \otimes \mathcal{H}_{wh}$.

Illusions

What if we keep pretending that $\mathcal{H} \cong \mathcal{H}_L \otimes \mathcal{H}_R$?

1. **Illusion of null states.** Some states in $\mathcal{H}_L$ and $\mathcal{H}_R$ must be identified [Baez, Vicary, '14], [Harlow, Jafferis, '18]. This leads to physical and null states.
2. **Illusion of null operators.** With the physical Hilbert space $\mathcal{H}$ 'smaller' than the tensor product $\mathcal{H}_L \otimes \mathcal{H}_R$ the algebra of observables does not factorize either [Leutheusser, Liu, '22], [Witten, '22].
3. **Illusion of entanglement.** States dual to wormholes are believed to be highly entangled, [Kourkoulou, Maldacena, '17], [Maldacena, Qi, '18], [Su, '20], [Lin, '22].

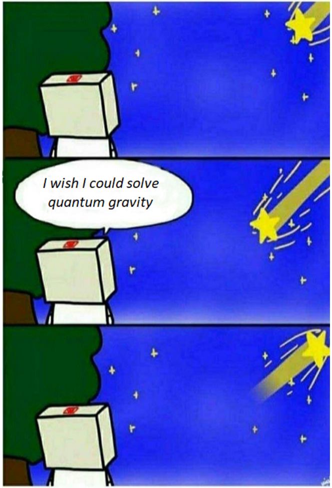
$$(\hat{a}^R_{wh} - \hat{a}^{L\dagger}_{wh})|\psi\rangle = 0, \qquad (\hat{a}^L_{wh} - \hat{a}^{R\dagger}_{wh})|\psi\rangle = 0,$$

4. **Illusion of interactions.** Effectively, the interactions act as the projectors onto the states satisfying the above identities, [Kourkoulou, Maldacena, '17].

Motivation

Gravity has something to do with null states.

1. General argument for non-factorization.
2. Geon as an example of a wormhole.
3. Illusions.
4. What can we learn about black holes from double well potential?
5. Approximate factorization, state-dependence and chaos.



Quantization

Quantize the bulk:

- Consider a scalar field on a fixed background,

$$S = -\frac{1}{2}\int \mathrm{d}^{d+1}x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + m^2\Phi^2\right].$$

- Impose enough boundary conditions for hyperbolicity.
- Let $\mathcal{M}^{\mathbb{C}}$ denote the space of complexified solutions to the Klein-Gordon equation $(-\Box_g + m^2)\Phi = 0$.
- Klein-Gordon scalar product

$$(\Phi,\Psi) = -\mathrm{i}\int_{\Sigma_t}\mathrm{d}^d x\sqrt{\gamma}\, n^\mu\left[\Phi\partial_\mu\Psi^* - \partial_\mu\Phi\,\Psi^*\right],$$

- Pick the polarization

$$\mathcal{M}^{\mathbb{C}} \cong \mathcal{H}^{(1)} \oplus (\mathcal{H}^{(1)})^*,$$

Quantization

- Hilbert space

$$\mathcal{H} = Sym(\mathcal{H}^{(1)}) = \mathbb{C} \oplus \mathcal{H}^{(1)} \oplus (\mathcal{H}^{(1)} \circledS \mathcal{H}^{(1)}) \oplus \dots$$

- Select complete basis of $\mathcal{M}^{\mathbb{C}}$: $\{\phi_n\}_n$ such that, $(\phi_m, \phi_n) = \delta_{mn}$. Denote ϕ_n by $|1\rangle_n$.
- Creation-annihilation operators

$$\hat{a}_n^\dagger |j\rangle_n = \sqrt{j+1}|j+1\rangle_n, \qquad \hat{a}_n |j\rangle_n = \sqrt{j}|j-1\rangle_n, \qquad \hat{a}_n|0\rangle = 0.$$

- Canonical commutation relations

$$\left[\hat{a}_\phi, \hat{a}_\psi^\dagger\right] = (\phi, \psi)\,\mathbf{1}, \qquad \langle 1_\phi | 1_\psi \rangle = (\phi, \psi).$$

Quantization

- Choose a foliation $\{\Sigma_t\}_t$.
- In the vicinity of, say, Σ_1 choose coordinates such that

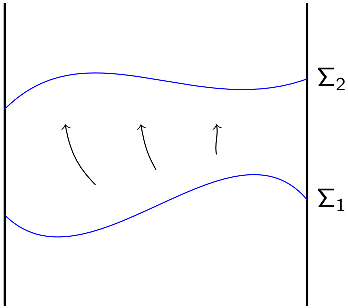
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \gamma_{ij}(x,t)\mathrm{d}x^i\mathrm{d}x^j$$

- Negative frequency modes on Σ_1 are

$$\phi_{\omega\ell}(t,x) = \frac{e^{-\mathrm{i}\omega t}}{\sqrt{2\omega}} f_{\omega\ell}(x), \quad \omega > 0,$$

where $f_{\omega\ell}$ is a time-independent wave function satisfying

$$\Delta_x f_{\omega\ell} = (m^2 - \omega^2) f_{\omega\ell}.$$



Quantization

- Wave functions are square-integrable and serve as initial data

$$\mathcal{H}_t^{(1)} = \{\phi_{\omega\ell}^{\Sigma_t}\}_{\omega\ell} \cong \{f_{\omega\ell}^{\Sigma_t}\}_{\omega\ell} \cong \mathcal{I}_t^- .$$

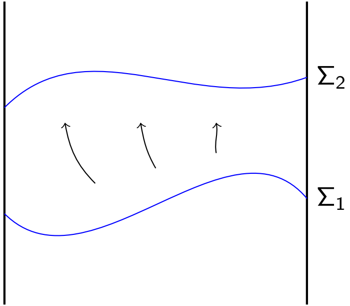
- The foliation determines the Hamiltonian and (instantaneous) vacuum $|0\rangle$

$$\hat{H} = \sum_\ell \int \frac{\mathrm{d}\omega}{2\pi} \omega \hat{a}^\dagger_{\omega\ell} \hat{a}_{\omega\ell}.$$

- We have

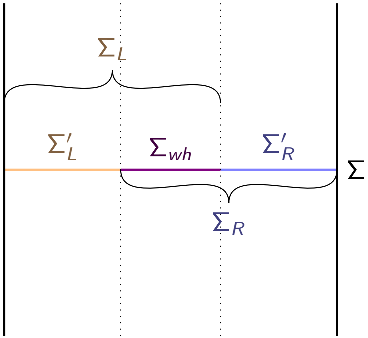
$$\mathcal{I}_t^{\mathbb{C}} \cong \mathcal{I}_t^- \oplus \mathcal{I}_t^+, \qquad \mathcal{I}_t^+ = (\mathcal{I}_t^-)^*.$$

- Evolution $U : \mathcal{H}_1 \to \mathcal{H}_2$ given by the Bogoliubov transformation.



Factorization

If two regions, Σ_L, Σ_R do not overlap, $\mathcal{I}^- \cong \mathcal{I}_L^- \oplus \mathcal{I}_R^-$ and the Hilbert space splits, $\mathcal{H} \cong \mathcal{H}_L \otimes \mathcal{H}_R$.



- Wave functions $f_L \oplus f_R \in \mathcal{I}_L^- \oplus \mathcal{I}_R^-$ must agree on the overlap, $f_R - f_L = 0$.
- We have

$$\mathcal{I}_L^- \oplus \mathcal{I}_R^- \xrightarrow{\cong} \mathcal{I}^- \oplus \mathcal{I}_{wh}^-$$

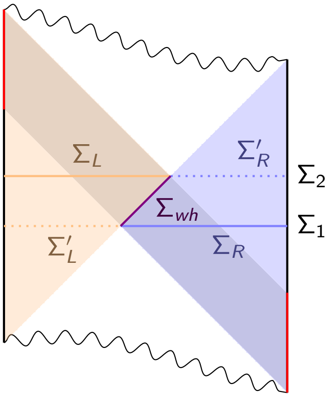
- And thus non-factorization property

$$\mathcal{H}_L \otimes \mathcal{H}_R \cong \mathcal{H} \otimes \mathcal{H}_{wh}.$$

- Furthermore

$$\mathcal{H}_L \cong \mathcal{H}_{L'} \otimes \mathcal{H}_{wh}, \quad \mathcal{H}_R \cong \mathcal{H}_{R'} \otimes \mathcal{H}_{wh}.$$

Factorization



- The isomorphisms in the non-factorization property

$$\mathcal{H}_L \otimes \mathcal{H}_R \cong \mathcal{H} \otimes \mathcal{H}_{wh}$$

  are now up to Bogoliubov transformations.

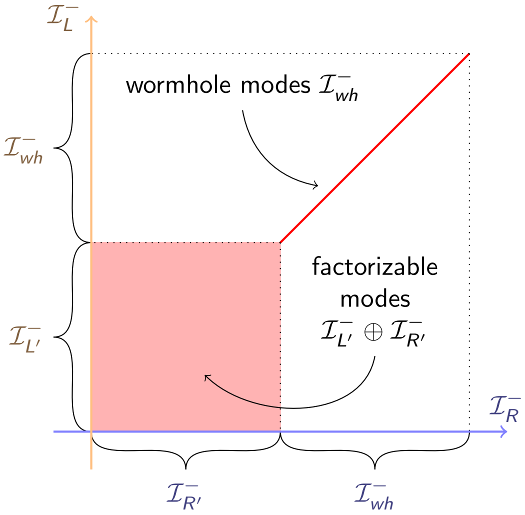
- Furthermore

$$\mathcal{H}_L \cong \mathcal{H}_{L'} \otimes \mathcal{H}_{wh},$$
$$\mathcal{H}_R \cong \mathcal{H}_{R'} \otimes \mathcal{H}_{wh}$$

  from which

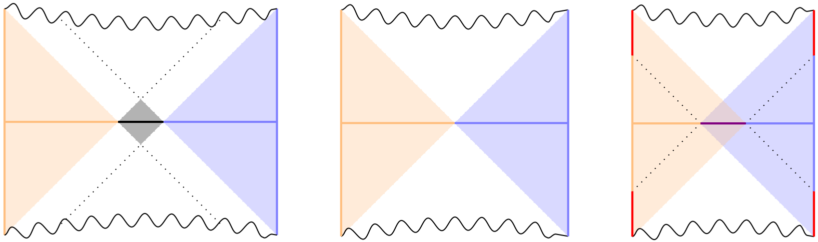
$$\mathcal{H} \cong \mathcal{H}_{L'} \otimes \mathcal{H}_{wh} \otimes \mathcal{H}_{R'}.$$

Factorization



- There are 2 copies of $\mathcal{H}_{wh}$ in $\mathcal{H}_L \otimes \mathcal{H}_R$, but only one in $\mathcal{H}$.

Holography



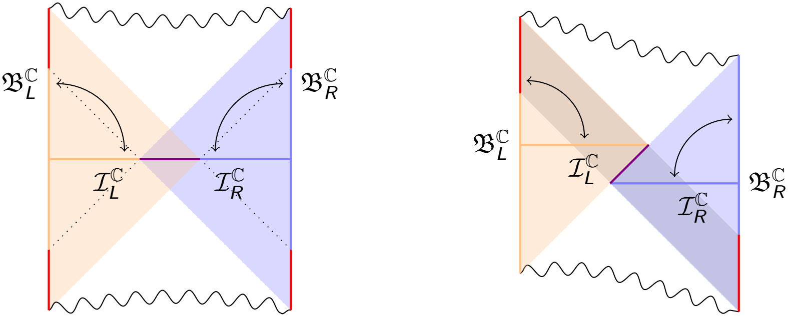
- Given a bulk field Φ we can take its boundary values

$$\varphi_I = \lim_{z\to 0} z^{-\Delta_I}\Phi, \quad I = L, R.$$

- By $\mathfrak{B}_L^{\mathbb{C}}$ and $\mathfrak{B}_R^{\mathbb{C}}$ denote the set of all complex boundary values on the left and right boundary component.
- Define

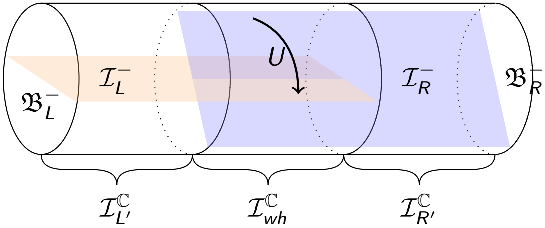
$$\mathfrak{D}^{\mathbb{C}} = \{(\varphi_L, \varphi_R) \,:\, \Phi_{\mathbb{C}} \in \mathcal{M}^{\mathbb{C}}\} \subseteq \mathfrak{B}_L^{\mathbb{C}} \oplus \mathfrak{B}_R^{\mathbb{C}}.$$

Holography



- Holography provides a relation between the Cauchy and boundary data.
- Operatorial form: BDHM dictionary, [Banks, Douglas, Horowitz, Martinec, '98].
- Real-time holography: [Skenderis, van Rees, '09], [Botta-Cantcheff, Martínez, Silva, '15], [Christodoulou, Skenderis, '16].

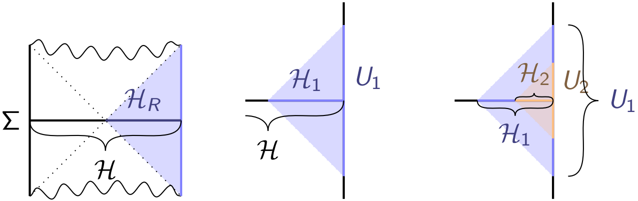
Holography



- Use boundary modes $\varphi_I(t, x_j) \sim e^{\pm i\omega t} Y_{\omega\ell}(x_j)$ to construct bulk subspaces $\mathcal{I}_L^-$ and $\mathcal{I}_R^-$ of $\mathcal{I}^{\mathbb{C}}$.
- Glue together along $\mathcal{I}_{wh}^{\mathbb{C}}$,

$$\mathbf{1} \otimes U \ : \ \mathcal{H}_L = \mathcal{H}_{L'} \otimes \mathcal{H}_{wh} \xrightarrow{\cong} \mathcal{H}_L^\partial = \mathcal{H}_{L'}^\partial \otimes \mathcal{H}_{wh}^\partial .$$

Summary



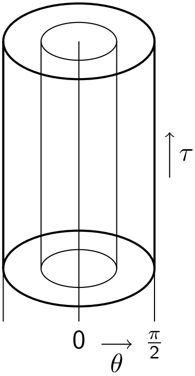
- Wormhole modes propagating between the boundaries imply $\mathfrak{D}^{\mathbb{C}} \subset \mathfrak{B}_L^{\mathbb{C}} \oplus \mathfrak{B}_R^{\mathbb{C}}$.
- Holography maps boundary conditions for wormhole modes to initial conditions for wormhole modes.
- $\mathcal{I}_L^{\mathbb{C}} \oplus \mathcal{I}_R^{\mathbb{C}} \cong \mathcal{I}^{\mathbb{C}} \oplus \mathcal{I}_{wh}^{\mathbb{C}}$ implies $\mathcal{H}_L \otimes \mathcal{H}_R \cong \mathcal{H} \otimes \mathcal{H}_{wh}$.
- Non-factorization is a statement about Hilbert spaces, but in some representation it is more visible than in others.

AdS

- The construction here follows [Kaplan, '16].
- Global anti-de Sitter (AdS) metric

$$\mathrm{d}s^2 = \frac{L^2}{\cos^2\theta}\left(-\mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\Omega_{d-1}^2\right).$$

- Let Φ be a Klein-Gordon field satisfying $(-\Box + m^2)\Phi = 0$.
- The mass is parameterized as $m^2 = \Delta(\Delta - d)$.
- Two solutions: $\Phi \sim \text{src}\cos^{d-\Delta}\theta + \text{vev}\cos^{\Delta}\theta$.
- We set $\text{src} = 0$ and quantize Φ.



AdS

- The solution is

$$\Phi_{AdS}(\tau,\theta,\Omega) = \sum_{k=0}^{\infty}\sum_{\ell}\left(\phi_{k\ell}\alpha_{k\ell} + \phi^*_{k\ell}\alpha^*_{k\ell}\right),$$

where

$$\phi_{k\ell}(\tau,\theta,\Omega) = c_{k\ell}e^{-i\omega_{k\ell}\tau}Y_\ell(\Omega)\cos^\Delta\theta\sin^\ell\theta\, P_k^{(\ell+\frac{d}{2}-1,\Delta-\frac{d}{2})}(\cos(2\theta)).$$

- These are standing waves: the frequencies are quantized:

$$\omega_{k\ell} = \Delta + \ell + 2k, \quad k = 0, 1, 2, \ldots$$

- The coefficients are elevated to creation-annihilation operators

$$\left[\hat{\alpha}_{k\ell}, \hat{\alpha}^\dagger_{k'\ell'}\right] = \delta_{kk'}\delta_{\ell\ell'}.$$

- The vacuum state is $|\Omega\rangle$ defined by the condition $\hat{\alpha}_{k\ell}|\Omega\rangle = 0$.
- Hilbert space $\mathcal{H}_{AdS}$ is spanned by $\hat{\alpha}^\dagger_{k_1\ell_1}\ldots\hat{\alpha}^\dagger_{k_n\ell_n}|\Omega\rangle$.

AdS

- At the boundary

$$\color{red}{\mathcal{O}(\tau,\Omega)} = \sum_{k=0}^{\infty}\sum_{\ell}\left(\hat{\alpha}_{k\ell}\varphi_{k\ell} + \hat{\alpha}^{\dagger}_{k\ell}\varphi^{*}_{k\ell}\right),$$

where

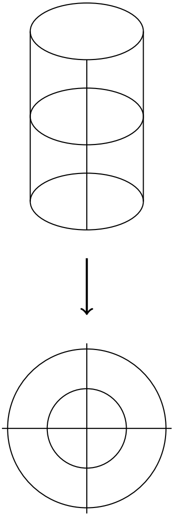
$$\varphi_{k\ell}(\tau,\Omega) = \lim_{\theta\to\frac{\pi}{2}} F^{-\Delta}(\theta)\phi_{k\ell} = \tilde{c}_{k\ell}e^{-\mathrm{i}\omega_{k\ell}\tau}Y_{\ell}(\Omega).$$

- Check: Euclidean operators

$$\mathcal{O}^{Eu}(t,\Omega) = e^{-\Delta t}\mathcal{O}(\tau=-\mathrm{i}t,\Omega),$$

produce the generalized free field correlators, *e.g.*,

$$\langle\Omega|\mathcal{O}^{Eu}(z,\bar{z})\mathcal{O}^{Eu}(0)|\Omega\rangle = \frac{1}{L}\frac{1}{|z|^{2\Delta}}.$$



BTZ black hole

- The BTZ black hole

$$ds^2 = -(\rho^2 - \rho_h^2)\mathrm{d}t^2 + \frac{L^2 \mathrm{d}\rho^2}{\rho^2 - \rho_h^2} + \rho^2 \mathrm{d}\varphi^2,$$

- General solution:

$$\phi_{\omega n}(t, \rho, \varphi) = c_{\omega n}^{BTZ} e^{-\mathrm{i}\omega t + \mathrm{i}n\varphi} R_{\omega n}(\rho),$$

  - Field decomposition in all wedges:

$$\hat{\Phi}_{BTZ} = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{n=-\infty}^{\infty} \left(\phi_{\omega n}^{L*} \hat{\alpha}_{\omega n}^{L} + \phi_{\omega n}^{R} \hat{\alpha}_{\omega n}^{R} + h.c.\right)$$

  - In Schwarzschild modes $\phi_{\omega n}^{L,R}$ the split of the Hilbert space is explicit:

$$\mathcal{H} \cong \mathcal{H}_L \otimes \mathcal{H}_R, \qquad |0\rangle = |0\rangle_L \otimes |0\rangle_R,$$

with $\mathcal{H}_L$ spanned by $\hat{\alpha}_{\omega n}^{L\dagger}$ and $\mathcal{H}_R$ spanned by $\hat{\alpha}_{\omega n}^{R\dagger}$.

BTZ black hole

- What if we used Kruskal modes?

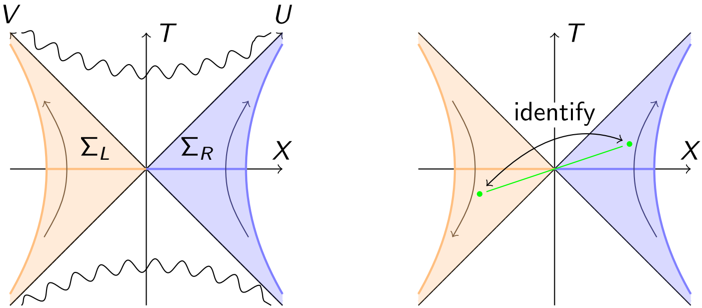
$$\chi^R_{\omega n} = \frac{\phi^R_{\omega n} + e^{-\frac{\pi\omega L}{\rho_h}}\phi^L_{\omega n}}{\sqrt{1 - e^{-\frac{2\pi\omega L}{\rho_h}}}}, \qquad \chi^L_{\omega n} = \frac{\phi^L_{\omega n} + e^{-\frac{\pi\omega L}{\rho_h}}\phi^R_{\omega n}}{\sqrt{1 - e^{-\frac{2\pi\omega L}{\rho_h}}}}.$$

- The action of the corresponding creation-annihilation operators $\hat{\beta}^{L,R\dagger}_{\omega n}$, $\hat{\beta}^{L,R}_{\omega n}$ is not limited to a single boundary.
- The Hilbert spaces spanned by $\hat{\beta}^{L\dagger}_{\omega n}$ and $\hat{\beta}^{R\dagger}_{\omega n}$ are not boundary Hilbert spaces.
- But the full Hilbert space still splits, up to the Bogoliubov transformation, $\mathcal{S}\mathcal{H}_\Omega \cong \mathcal{H} \cong \mathcal{H}_L \otimes \mathcal{H}_R$, where $\mathcal{S}$ implements

$$\hat{\beta}^R_{\omega n} = \frac{\hat{\alpha}^R_{\omega n} - e^{-\frac{\pi\omega L}{\rho_h}}\hat{\alpha}^{L\dagger}_{\omega n}}{\sqrt{1 - e^{-\frac{2\pi\omega L}{\rho_h}}}}, \qquad \hat{\beta}^L_{\omega n} = \frac{\hat{\alpha}^L_{\omega n} - e^{-\frac{\pi\omega L}{\rho_h}}\hat{\alpha}^{R\dagger}_{\omega n}}{\sqrt{1 - e^{-\frac{2\pi\omega L}{\rho_h}}}}.$$

- Kruskal vacuum $|\Omega\rangle$ is the thermofield double. It satisfies $\hat{\beta}^{L,R}_{\omega n}|\Omega\rangle = 0$.

Geon



By the term geon we will refer to the BTZ black hole with the antipodal $\mathbb{Z}_2$ identification

$$\theta(T, X, \varphi) = (-T, -X, \varphi + \pi),$$

Geon

- Geons studied as toy models of black holes, [Louko, Marolf, '98], ['t Hooft, '16], [Betzios, Gaddam, Papadoulaki, '16].
- Instead of folding, we can think of geon as the BTZ geometry with the scalar field obeying antiodal identification,

$$\Phi^{(\pm)} \circ \theta = \pm\Phi^{(\pm)}.$$

- Introduce the geon modes

$$\psi^{(\pm)}_{\omega n} = \frac{1}{\sqrt{2}} \left[\phi^R_{\omega n} \pm (-1)^n \phi^{L*}_{\omega,-n}\right]$$

and the corresponding operators,

$$\hat{\alpha}^{(\pm)}_{\omega n} = \frac{1}{\sqrt{2}} \left[\hat{\alpha}^R_{\omega n} \pm (-1)^n \hat{\alpha}^{L\dagger}_{\omega,-n}\right].$$

- These modes and operators have the specified parity under θ,

$$\psi^{(\pm)}_{\omega n} \circ \theta = \pm\psi^{(\pm)}_{\omega n}, \qquad \hat{\Theta}\, \hat{\alpha}^{(\pm)}_{\omega n}\, \hat{\Theta} = \pm\hat{\alpha}^{(\pm)\dagger}_{\omega n}.$$

- We split $\hat{\Phi}_{BTZ} = \hat{\Phi}^{(+)} + \hat{\Phi}^{(-)}$ into two operators of fixed parity under $\hat{\Theta}$.

Geon

- Hilbert space structure of geon studied in [Sanchez, '86], [Louko, Marolf, '98], [Ross, Guica, '14].
- In [Sanchez, '86] the quantization of the field

$$\hat{\Phi}^{(+)} = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{n=-\infty}^{\infty} \left(\psi^{(+)}_{\omega n} \hat{\alpha}^{(+)}_{\omega n} + \psi^{(+)*}_{\omega n} \hat{\alpha}^{(+)\dagger}_{\omega n} \right)$$

  was considered.
- Problem: $\hat{\alpha}^{(+)}_{\omega n}, \hat{\alpha}^{(+)\dagger}_{\omega n}$ commute,

$$\left[\hat{\alpha}^{(\pm)}_{\omega n}, \hat{\alpha}^{(\pm)\dagger}_{\omega' n'} \right] = 0, \qquad \left[\hat{\alpha}^{(\pm)}_{\omega n}, \hat{\alpha}^{(\mp)\dagger}_{\omega' n'} \right] = 2\pi\delta(\omega - \omega')\delta_{nn'} .$$

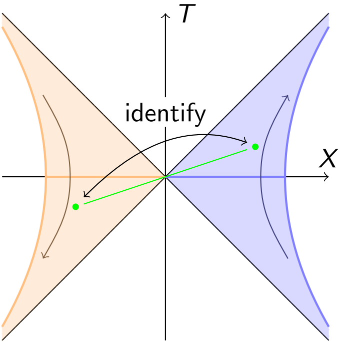
- Proposal of [Sanchez, '86]: take $\mathcal{H} \cong \mathcal{H}_L \otimes \mathcal{H}_R$, but act only with parity-even operators.
- For any state $|\psi\rangle$ the 2-point function is

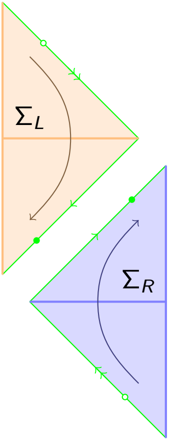
$$\langle \psi | \hat{\Phi}^{(+)}(x) \hat{\Phi}^{(+)}(y) | \psi \rangle = \frac{1}{4} \left[G(x,y) + G(\theta x, y) + G(x, \theta y) + G(\theta x, \theta y) \right] .$$

- In particular $[\hat{\Phi}^{(+)}(x), \hat{\Phi}^{(+)}(y)] \neq 0$, even if x and y are spacelike-separated.

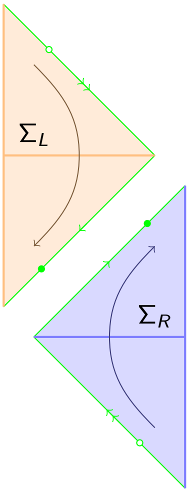
Geon as a wormhole

It is enough to introduce identifications on the horizons.





Geon as a wormhole



- Σ_L and Σ_R are Cauchy slices separately.
- The geon modes are

$$\psi_{\omega n} = \sqrt{2}\psi^{(+)}_{\omega n} = \phi^{R}_{\omega n} + (-1)^n \phi^{L*}_{\omega,-n}.$$

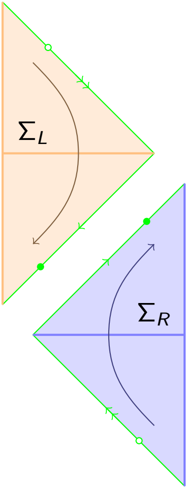
- Its restrictions are $\psi_{\omega n}|_R = \phi^R_{\omega n}$ and $\psi_{\omega n}|_L = (-1)^n \phi^{L*}_{\omega,-n}$.
- There is only one set of creation-annihilation operators,

$$\left[\hat{a}_{\omega n}, \hat{a}^{\dagger}_{\omega' n'}\right] = 2\pi\delta(\omega-\omega')\delta_{nn'}$$

- and the field operator takes form

$$\hat{\Phi}_g = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{n=-\infty}^{\infty} \left(\psi_{\omega n}\hat{a}_{\omega n} + \psi^*_{\omega n}\hat{a}^{\dagger}_{\omega n}\right).$$

Geon as a wormhole



- The Klein-Gordon scalar product

$$(\psi_{\omega n}, \psi_{\omega' n'})_g = (\psi_{\omega n}|_R, \psi_{\omega' n'}|_R)_{BTZ} = -(\psi_{\omega n}|_L, \psi_{\omega' n'}|_L)_{BTZ}$$

forces $\phi^{L*}_{\omega n}$ to be negative frequency in the left wedge.

- Norms are off by a factor of $\sqrt{2}$ when comparing to the BTZ case, $\|\psi_{\omega n}\|^2_{BTZ} = 2\|\psi_{\omega n}\|^2_g$.
- By comparing to the BTZ modes,

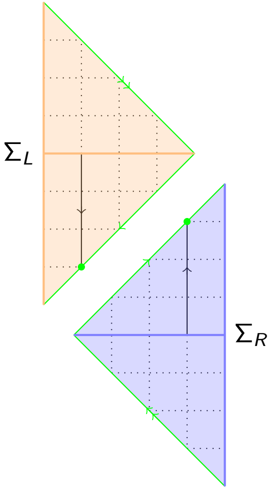
$$\hat{a}_{\omega n} = \hat{a}^R_{\omega n} = (-1)^n \hat{a}^L_{\omega,-n}, \quad \hat{a}^\dagger_{\omega n} = \hat{a}^{R\dagger}_{\omega n} = (-1)^n \hat{a}^{L\dagger}_{\omega,-n}$$

- In particular

$$\mathcal{H}_g \cong \mathcal{H}_R \cong \hat{\Theta}\mathcal{H}_L.$$

- For AdS$_2$ non-factorization pointed out

Geon state



- Left and right Schwarzschild Hamiltonians are related,

$$\hat{H}_R - \hat{H}_L = 0, \quad \hat{H} = \frac{1}{2}(\hat{H}_L + \hat{H}_R),$$

  relations advocated in [Harlow, Jafferis, '18], [Maldacena, Qi, '18].
- Use $\hat{H}_K$ in the Kruskal coordinates to evolve between the wedges

$$U_{RL} = e^{\mathrm{i}(X-T)\hat{H}_K}\hat{\Theta}e^{-\mathrm{i}(X-T)\hat{H}_K} = \hat{\Theta},$$

- Geon state $|G\rangle$ = 'Kruskal vacuum'.

Geon state

What is the dual geon state $|G\rangle$?

- We look for $|G\rangle$ in $\mathcal{H}_g \cong \mathcal{H}_R \cong \hat{\Theta}\mathcal{H}_L$, not $\mathcal{H}_L \otimes \mathcal{H}_R$.
- It must be annihilated by $\hat{b}_{\omega n}$ with left and right creation-annihilation operators related.
- This gives

$$\hat{b}_{\omega n} = \frac{\hat{a}_{\omega n} - e^{-\frac{\beta\omega}{2}}(-1)^n \hat{a}^\dagger_{\omega n}}{\sqrt{1 - e^{-\beta\omega}}}, \qquad \beta = \frac{2\pi L}{\rho_h},$$

- $|G\rangle$ is the squeezed state

$$|G\rangle_{\omega n} = \left(1 - e^{-\beta\omega}\right)^{1/4} \exp\left[\frac{1}{2} e^{-\frac{\beta\omega}{2}} (-1)^n \hat{a}^\dagger_{\omega n} \hat{a}^\dagger_{\omega n}\right] |0\rangle$$
$$= \left(1 - e^{-\beta\omega}\right)^{1/4} \sum_{j=0}^{\infty} (-1)^{nj} e^{-\frac{\beta\omega j}{2}} \sqrt{\frac{(2j-1)!!}{(2j)!!}} |2j\rangle_{\omega n}.$$

Correlation functions

- In Schwarzschild vacuum $|0\rangle$,

$$\langle 0|\hat{\Phi}_g(x)\hat{\Phi}_g(y)|0\rangle_g = \langle 0|\hat{\Phi}_{BTZ}(x)\hat{\Phi}_{BTZ}(y)|0\rangle_{BTZ}$$

  with x and y in the same wedge.

- With x and y in the opposite wedges

$$\langle 0|\hat{\Phi}_g(x)\hat{\Phi}_g(y)|0\rangle_g = \langle 0|\hat{\Phi}_{BTZ}(x)\hat{\Phi}_{BTZ}(\theta y)|0\rangle_{BTZ},$$

- The geon state sees everything,

$$\langle G|\hat{\Phi}_g(x)\hat{\Phi}_g(y)|G\rangle_g = \frac{1}{2}\left[G(x,y) + G(\theta x, y) + G(x, \theta y) + G(\theta x, \theta y)\right],$$

  where

$$G(x,y) = \langle \Omega|\hat{\Phi}_{BTZ}(x)\hat{\Phi}_{BTZ}(y)|\Omega\rangle_{BTZ}$$

Semiclassical approximation a la [Louko, Marolf, '98]

- In the semiclassical approach one treats $\mathcal{H}_L$ and $\mathcal{H}_R$ as two independent Hilbert spaces spanned by their own sets of independent left and right creation-annihilation operators $\hat{\alpha}^{L\dagger}_{\omega n}, \hat{\alpha}^{L}_{\omega n}$ and $\hat{\alpha}^{R\dagger}_{\omega n}, \hat{\alpha}^{R}_{\omega n}$.
- The semiclassical Hilbert space is assumed to be the tensor product, $\mathcal{H}_{\text{semi}} = \mathcal{H}_{BTZ} = \mathcal{H}_L \otimes \mathcal{H}_R$.
- The authors observe the time reversal in the left wedge and designate $\phi^{L*}_{\omega n}$ as negative frequency modes and $\hat{\alpha}^{L\dagger}_{\omega n}$ as the annihilation operators.
- Now

$$\left[\hat{\alpha}^{(\sigma)}_{\omega n}, \hat{\alpha}^{(\sigma')\dagger}_{\omega' n'}\right] = 2\pi\delta(\omega - \omega')\delta_{nn'}\delta^{\sigma\sigma'}$$

are genuine creation-annihilation operators.

Semiclassical approximation a la [Louko, Marolf, '98]

- Semiclassical Hilbert space splits as $\mathcal{H}_{\text{semi}} \cong \mathcal{H}_+ \otimes \mathcal{H}_-$.
- The geon state $|\Omega_g\rangle$ is the usual thermofield double state, but entangling particles between $\mathcal{H}_+$ and $\mathcal{H}_-$,

$$|\Omega_g\rangle_{\omega n} = \sqrt{1 - e^{-\beta\omega}} \exp\left[e^{-\frac{\beta\omega}{2}} \hat{\alpha}^{(-)\dagger}_{\omega n} \hat{\alpha}^{(+)\dagger}_{\omega n} \right] |0\rangle$$

$$= \sqrt{1 - e^{-\beta\omega}} \sum_{j=0}^{\infty} e^{-\frac{\beta\omega j}{2}} |j\rangle^{(+)}_{\omega n} \otimes |j\rangle^{(-)}_{\omega n}.$$

- The geon Hilbert space $\mathcal{H}_g$ can be identified with $\mathcal{H}_+$ up to rescaling.
- But there is no projection from the tensor product $\mathcal{H}_+ \otimes \mathcal{H}_-$ on one of its factors.

Null states from [Louko, Marolf, '98]

- Strong physical states

$$\hat{\alpha}^{(-)}_{\omega n}|\psi\rangle = \hat{\alpha}^{(-)\dagger}_{\omega n}|\psi\rangle = 0$$

  for all ω, n on all physical states $|\psi\rangle \in \mathcal{H}_{\text{semi}}$.
- Doable in constrained quantization.
- Weak physical states

$$\hat{\alpha}^{(-)}_{\omega n}|\psi\rangle = 0$$

  for all ω, n. States that obey this condition are physical. All other states are null.
- $\hat{\alpha}^{(-)\dagger}_{\omega n}|\psi\rangle$ is in general non-vanishing.
- If $|\psi\rangle$ is physical, then $\hat{\alpha}^{(-)\dagger}_{\omega n}|\psi\rangle$ is null due to commutation relations.

Null states from [Louko, Marolf, '98]

- States in $\mathcal{H}_+$ are physical, all other states are null.
- The semiclassical geon state is

$$|G_{\rm semi}\rangle = \sqrt{1-e^{-\beta\omega}}\sum_{j=0}^{\infty}(-1)^{nj}e^{-\frac{\beta\omega j}{2}}\sqrt{\frac{(2j-1)!!}{(2j)!!}}|2j_{\omega n}\rangle_+ \otimes |0\rangle_- + \text{null}$$

$$= \left(1-e^{-\beta\omega}\right)^{1/4}|G\rangle_+ \otimes |0\rangle_- .$$

Unphysical operators from [Louko, Marolf, '98]

- An operator $\mathcal{O}$ is physical if it maps $\mathcal{H}_+$ into itself. This is equivalent to

$$\left[\mathcal{O}, \hat{\alpha}^{(-)}_{\omega n}\right] |\psi\rangle = 0, \qquad \left[\mathcal{O}, \hat{\alpha}^{(-)\dagger}_{\omega n}\right] |\psi\rangle = 0$$

for all ω, n on all physical states $|\psi\rangle \in \mathcal{H}_+$.

- This means that an operator $\mathcal{O}$ is physical if, when presented in terms of the creation-annihilation operators, it contains only $\hat{\alpha}^{(+)}_{\omega n}$ and $\hat{\alpha}^{(+)\dagger}_{\omega n}$, while the operators $\hat{\alpha}^{(-)}_{\omega n}$ and $\hat{\alpha}^{(-)\dagger}_{\omega n}$ are absent.
- The left and right boundary creation-annihilation operators are on their own unphysical.
- Left and right Hamiltonians $\hat{H}_L$ and $\hat{H}_R$ are unphysical. Only combinations such as

$$\hat{H}_R - \hat{H}_L = 0, \qquad \hat{H} = \frac{1}{2}(\hat{H}_L + \hat{H}_R)$$

are physical.

Entanglement from [Guica, Ross, '14]

- In [Guica, Ross, '14] no time reversal was taken.
- Physical states should satisfy

$$\left[\hat{\alpha}^{R}_{\omega n} - (-1)^n \hat{\alpha}^{L\dagger}_{\omega,-n}\right] |\psi\rangle = \left[\hat{\alpha}^{L}_{\omega n} - (-1)^n \hat{\alpha}^{R\dagger}_{\omega,-n}\right] |\psi\rangle = 0.$$

- These are formally satisfied by infinite temperature states

$$|I\rangle = \bigotimes_{\omega n} |I\rangle_{\omega n}, \qquad |I\rangle_{\omega n} = \sum_{j=0}^{\infty} (-1)^{jn} |j\rangle_{\omega,-n} |j\rangle_{\omega n}$$

known also as Kourkoulou-Maldacena state in the context of AdS_2 wormhole.

- Formally non-existent, infinite norm, maximally entangled states.

Entanglement from [Guica, Ross, '14]

- The geon dual state is supposed to be

$$|\Psi_g\rangle_{\omega n} = e^{-\frac{\beta}{4}(\hat{H}_L+\hat{H}_R)}|I\rangle_{\omega n}.$$

- This is TFD with the factor $(-1)^{jn}$,

$$|\Psi_g\rangle_{\omega n} = \sum_{j=0}^{\infty}(-1)^{jn} e^{-\frac{\beta\omega j}{2}}|j\rangle_{\omega,-n}|j\rangle_{\omega n}.$$

- No excited states in $\mathcal{H}_L \otimes \mathcal{H}_R$.

Entanglement from [Guica, Ross, '14]

- Effective description

$$\left[\hat{\alpha}^{R}_{\omega n} - \nu_{\omega n}\hat{\alpha}^{L\dagger}_{\omega,-n}\right]|\psi_{\nu_n}\rangle = 0, \qquad \left[\hat{\alpha}^{L}_{\omega n} - \nu_{\omega n}\hat{\alpha}^{R\dagger}_{\omega,-n}\right]|\psi_{\nu_n}\rangle = 0.$$

- Consider squeezed states and take $\nu_{\omega n} \to (-1)^n$.
-

$$\hat{\gamma}^{L}_{\omega n} = \cosh\lambda_{\omega n}\hat{\alpha}^{L}_{\omega n} - \sinh\lambda_{\omega n}\hat{\alpha}^{R\dagger}_{\omega,-n},$$
$$\hat{\gamma}^{R}_{\omega n} = \cosh\lambda_{\omega n}\hat{\alpha}^{R}_{\omega n} - \sinh\lambda_{\omega n}\hat{\alpha}^{L\dagger}_{\omega,-n},$$

with

$$\nu_{\omega n} = \tanh\lambda_{\omega n}$$

Entanglement from [Guica, Ross, '14]

$$\hat{H}_\nu = \frac{2}{\delta}\left[V_0 + \hat{H}_0 + \hat{H}_{\text{int}}\right],$$

where $V_0 = \omega$ is a constant that can be discarded, $\hat{H}_0$ is the free Hamiltonian in $\mathcal{H}_L \otimes \mathcal{H}_R$ and $\hat{H}_{\text{int}}$ is the interaction Hamiltonian,

$$\hat{H}_0 = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{n=-\infty}^{\infty} \omega \left(\hat{\alpha}^{L\dagger}_{\omega n}\hat{\alpha}^{L}_{\omega n} + \hat{\alpha}^{R\dagger}_{\omega n}\hat{\alpha}^{R}_{\omega n}\right),$$

$$\hat{H}_{\text{int}} = -\int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{n=-\infty}^{\infty} \omega(-1)^n \left(\hat{\alpha}^{L\dagger}_{\omega,-n}\hat{\alpha}^{R\dagger}_{\omega n} + \hat{\alpha}^{L}_{\omega,-n}\hat{\alpha}^{R}_{\omega n}\right)$$

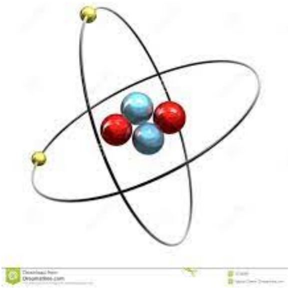
and

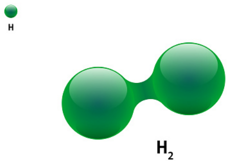
$$\nu_{\omega n} = (-1)^n\sqrt{1-\delta}$$

Summary

- The Hilbert space of excitations over the wormhole does not factorize into the tensor product of boundary Hilbert spaces.
- This results in the operatorial relation between boundary operators, *e.g.*, $\hat{a}^R_{\omega n} = (-1)^n \hat{a}^L_{\omega,-n}$ and $\hat{H}_L = \hat{H}_R$.
- Physical and null states are avatars of these relations.
- The interaction is an avatar of the description of the system on the tensor product.
- The interaction Hamiltonian is that of 'infinite squeezing'.
- Thermal partition function does not factorize, $\mathrm{Tr}\, e^{-\beta \hat{H}} \neq \mathrm{Tr}\, e^{-\beta(\hat{H}_L + \hat{H}_R)}$.

Double well potential





- Atom He: two electrons in a single potential well.
- Factorized Hilbert space: $\mathcal{H} \cong \mathcal{H}_1 \circledS \mathcal{H}_2$.
- Electrostatic interaction.
- Decoupling limit as $e \to 0$.

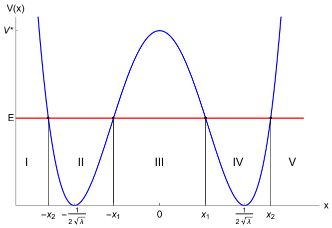
- Molecule H_2^+: single electrons in a double well.
- Un-factorized Hilbert space: $\mathcal{H}_1$.
- No additional interactions.
- Decoupling limit as distance $\to \infty$.

Double well potential

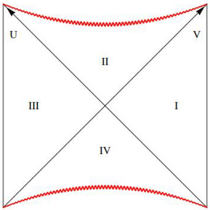
What if we replace $\hat{a}^L = \hat{a}^R$ by $\hat{a}^L = \hat{a}^R + c\mathbf{1}$?

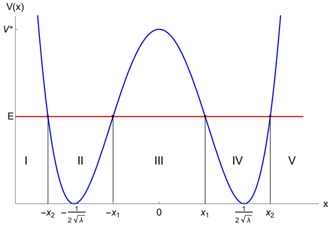
- Minima at $x_{L,R} = \frac{1}{2\omega\sqrt{\lambda}}$.
- One can think $\lambda = \frac{1}{N^2}$,
- At maximum $V_* = \frac{1}{32\lambda}$,
- We set $\omega = 1$.
- Field operators satisfy $\hat{x}_R - \hat{x}_L = N\mathbf{1}$.
- Define a decoupling limit as $\lambda \to 0$. Physically, the system can be thought of two decoupled harmonic oscillators, described by a tensor product Hilbert space.

$$V(x) = \frac{1}{32\lambda} - \frac{1}{4}\omega^2 x^2 + \frac{\lambda}{2}\omega^4 x^4$$



Toy model interpretation



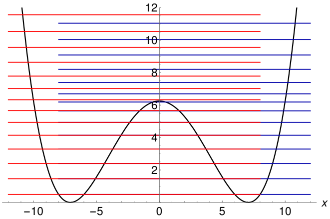


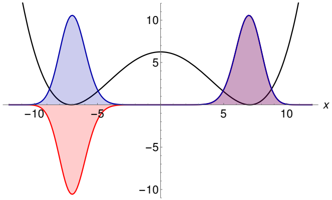
- Asymptotic regions
- BH microstates
- Excitations on top of $|0_k\rangle_R |0_{-k}\rangle_L$
- Decoupling

- Two minima
- Lowest energy states
- Excitations of two HO of frequency k
- $\lambda \to 0$

Energy levels

Energy eigenstates of a given $\pm$ parity, $H\Psi_n^\pm = E_n^\pm \Psi_n^\pm$.





- Energy differences are non-perturbatively small

$$\Delta E_n = E_n^- - E_n^+ = e^{-\frac{1}{6\lambda}} P_n(\lambda^{-1/2}) = o(\lambda^\infty).$$

- When $n \sim 1/\lambda \sim N^2$, $\Delta E_N \sim N^{N^2} e^{-N^2}$: non-perturbative effects become dominant.

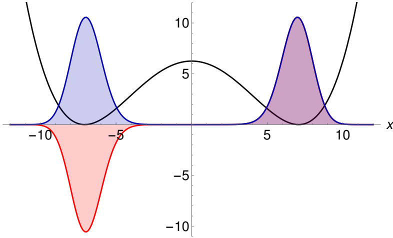
Microstates

- States $\Psi_n^{\pm}$ are indistinguishable to a single asymptotic observer within the perturbation thoery. We have microstates:

$$\mathcal{M} = \{\alpha_+ \Psi_0^+ + \alpha_- \Psi_0^- \ : \ \alpha_\pm \in \mathbb{C}\}.$$

- Each $\mu \in \mathcal{M}$ is a perturbative vacuum.
- We have semi-classical degeneracy and hence entropy,

$$S_B = \log \dim \mathcal{H}_{\text{fine}} = \log 2.$$



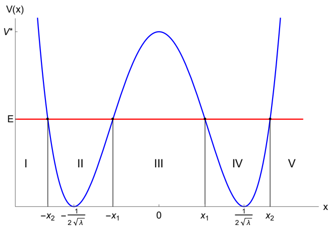
Excitations

- HO normalized eigenfunctions,

$$\varphi_n(x) = \frac{1}{\pi^{1/4}\sqrt{2^n n!}} H_n(x) e^{-\frac{x^2}{2}} ,$$

- Define

$$|n_R\rangle \ : \ \varphi_n^R(x) = \varphi_n(x - x_R),$$
$$|n_L\rangle \ : \ \varphi_n^L(x) = (\Theta\varphi_n^R)(x) = \varphi_n(x - x_L).$$



- Assign creation and annihilation operators a_L, a_L^+, a_R, a_R^+.
- Total Hilbert space $\mathcal{H}$ is isomorphic to each Fock space $\mathcal{F}_L$ and $\mathcal{F}_R$ *separately*,

$$\mathcal{H} \cong \mathcal{F}_R \cong \mathcal{F}_L \ ;$$

There is no tensor product.

Consequences

Naive number operators for left/right asymptotic observers

$$N_L = H_L^{(0)} = a_L^+ a_L, \qquad N_R = H_R^{(0)} = a_R^+ a_R$$

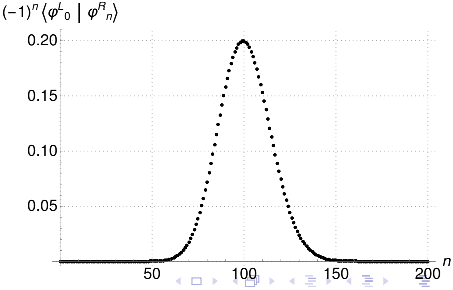
are *weird*

$$\langle \varphi_0^L | N_L | \varphi_0^L \rangle = \langle \varphi_0^R | N_R | \varphi_0^R \rangle = 0 \ ,$$
$$\langle \varphi_0^L | N_R | \varphi_0^L \rangle = \langle \varphi_0^R | N_L | \varphi_0^R \rangle = \frac{1}{2} N^2$$

and diverge in the decoupling limit $N \to \infty$.

For the right observer, semiclassical *left* states are highly excited

$$\langle \varphi_0^L | \varphi_n^R \rangle = \frac{(-1)^n e^{-\frac{1}{4\lambda}}}{\sqrt{2^n \lambda^n n!}}$$



Defining effective theory

- There is no state where N_L, N_R and $N_A = N_L + N_R$ are all small: a firewall?
- No, a_R, a_R^+ are *non-local*, ie., they do something horrible to φ_n^L.
- We cannot define

$$\hat{a}_R \varphi_n^R \stackrel{?}{=} \sqrt{n}\varphi_{n-1}^R, \qquad \hat{a}_R \varphi_n^L \stackrel{?}{=} 0,$$
$$\hat{a}_R^+ \varphi_n^R \stackrel{?}{=} \sqrt{n+1}\varphi_{n+1}^R, \qquad \hat{a}_R^+ \varphi_n^L \stackrel{?}{=} 0$$

because the set $\{\varphi_n^L, \varphi_n^R\}$ is overcomplete [Jafferis '17].

- A solution: truncate the basis at finite $n \leq N$, [Papadodimas, Raju '13].
- Better solution: orthogonalize $\{\varphi_n^L, \varphi_n^R\}_n$.

Effective theory

- Symmetric and antisymmetric combinations of all energy eigenstates,

$$\Psi_n^L = \frac{1}{\sqrt{2}}(\Psi_n^+ - \Psi_n^-), \qquad \Psi_n^R = \frac{1}{\sqrt{2}}(\Psi_n^+ + \Psi_n^-)$$

span two Hilbert subspaces (perturbative Hilbert spaces)

$$\mathcal{H}_L = \mathrm{span}\{\Psi_n^L\}_n, \qquad \mathcal{H}_R = \mathrm{span}\{\Psi_n^R\}_n.$$

- $\langle \Psi_m^L | \Psi_n^R \rangle = 0$

$$\mathcal{H} = \mathcal{H}_L \oplus \mathcal{H}_R, \qquad \mathcal{H}_L \perp \mathcal{H}_R, \qquad \Theta\mathcal{H}_L = \mathcal{H}_R, \qquad \Theta\mathcal{H}_R = \mathcal{H}_L.$$

- Projected operators : $\hat{a}_L = P_L a_L P_L, \qquad \hat{a}_R = P_R a_R P_R.$
- Number operators are

$$\hat{N}_L = \hat{a}_L^+ \hat{a}_L, \qquad \hat{N}_R = \hat{a}_R^+ \hat{a}_R, \qquad \hat{N}_A = \hat{N}_L + \hat{N}_R.$$

Consequences

- Every state φ_n^R can be approximated by a perturbative state up to non-perturbative effects, $\varphi_n^R \notin \mathcal{H}_R$ but $\|P_R \varphi_n^R\| = 1 - o(\lambda^\infty)$.
- Hatted operators are non-local up to non-perturbative effects, $[\hat{a}_R, \hat{a}_R^+] \neq 1$, but $[\hat{a}_R, \hat{a}_R^+] = 1 + o(\lambda^\infty)$, [Kabat Lifshitz '14, Raju '17, Anninos, Monten '19]
- No firewall: number operators $\hat{N}_{L,R,A}$ are non-perturbatively close to $N_{L,R,A}$, but are well-behaved:

$$\langle \mu | \hat{N}_{L,R,A} | \mu \rangle = O(\sqrt{\lambda}), \quad \text{for generic } \mu \in \mathcal{M}.$$

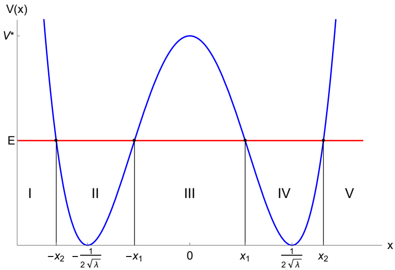
- The Hilbert space factorizes into the tensor product $\mathcal{H} \sim \mathcal{H}_L \otimes \mathcal{H}_R$ only approximately at low energies. Effective operators are microstate-dependent.
- Only some states in $\mathcal{F}_L \otimes \mathcal{F}_R$ are physical.
- Effective theory breaks for energies $\sim V_* \sim N^2$ or times $t \sim 1/E$, [Raju '17]

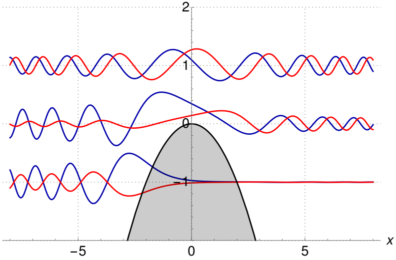
Hawking radiation as tunneling

- BH evaporation as tunneling: [Parhik, Wilczek ‘99, Gaddam, Papadoulaki, Betzios ‘16].
- Tunneling rate in WKB: $\Gamma = e^{-2\Lambda}$, $\Lambda = \int_{-x_1}^{x_1} \sqrt{2(V(x) - E)}dx$,
- At $E \sim V_*$ with $\delta = V_* - E$ the potential can be approximated by the inverted harmonic oscillator.
- One finds

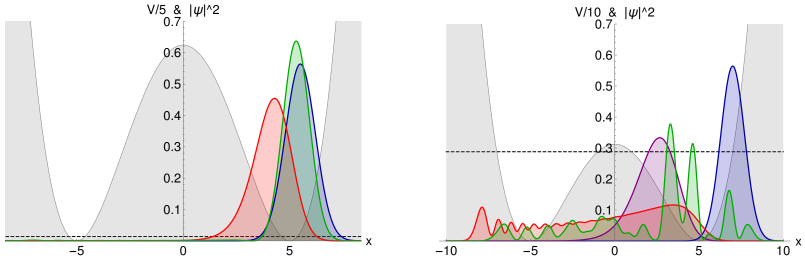
$$\Lambda(\lambda, \delta) = \sqrt{2}\pi\delta + 3\sqrt{2}\pi\lambda\delta^2 + O(\delta^3),$$

which means that in our model $\omega \sim \sqrt{\lambda}\delta$, $M \sim 1/\sqrt{\lambda} = N$.





Chaotic evolution



- Classical particle with $E \ll V_*$ stays on a closed orbit.
- The period diverges logarithmically when $E \to V_*$:

$$T_{\text{trapped}} = \sqrt{2} \log \left(\frac{2}{\lambda \delta} \right) + O(\lambda),$$

- Close to the tip: $x(t) = x_0 \cosh(\nu t) + v_0/\nu \sinh(\nu t)$. Hence

$$\delta x(t) \sim e^{\nu t} (\delta x_0 + \delta v_0/\nu).$$

This is by definition chaotic behavior with the Lapunov exponent $\nu = \omega/\sqrt{2} = 1/\sqrt{2}$.

Summary

Quantum mechanical system of the double well potential exhibits characteristic behavior associated with a pair of entangled modes in the quantum black holes:

- The Hilbert space does not factorize into the tensor product $\mathcal{F}_L \otimes \mathcal{F}_R$. Instead $\mathcal{H} \cong \mathcal{H}_L \oplus \mathcal{H}_R$.
- One can define natural creation-annihilation and firewall-free number operators, which agree with naive ones up to non-perturbative effects. The new operators remain local, up to non-perturbative terms.
- The factorization into the tensor product is approximate at low energies up to non-perturbative effects.
- A choice of non-perturbative vacuum leads to state-dependence.
- Hawking radiation has a natural interpretation as tunneling.

Summary

Ideas presented:

- Weirdness of black holes and wormholes can be realized in simple toy models.
- The split of the Hilbert space into the tensor product of the boundary spaces is questionable.
- Physical states, null states, strong entanglement and interactions between boundaries of a wormhole can be seen as the avatar of the non-trivial tensor structure.
- Possibility of building new models quantum black holes?

Summary

Thank you!