Breaking the spell of the tensor product

Adam Bzowski
Universytet Warszawski

UNIWERSYTET WARSZAWSKI

Norway grants

NARODOWE CENTRUM NAUKI

based on [2008.02810]

on work with Alessandra Gnecchi and Thomas Hertog: [1802.02580]

and ongoing research with Ruben Monten and John Gardiner.

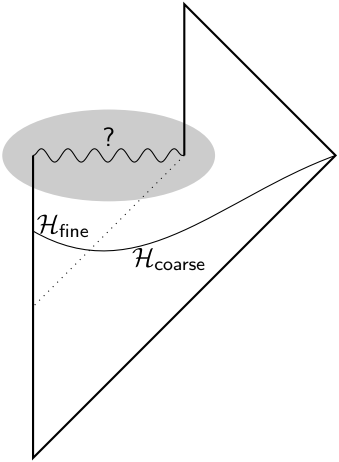
Warszawa, December 17, 2021

Motivation

Problem:

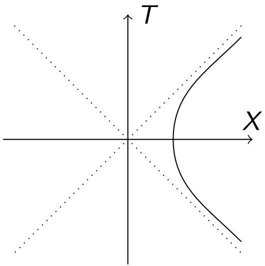
- Black hole evaporates in a finite boundary time.
- Thermal spectrum means that entropy rises and information is lost.
- Can we extract the information from beyond the horizon?
- Is the interior encoded in the exterior?.

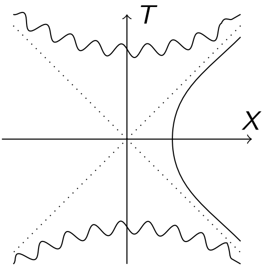
We will explore the possibility that the tensor product factorization fails, $\mathcal{H} \cong \mathcal{H}_{\text{fine}} \otimes \mathcal{H}_{\text{coarse}}$.



Motivation

It's Equivalence principle.





- Minkowski vacuum $|\Omega\rangle$ vs. Rindler vacuum $|0\rangle$.
- Non-intertial frame $\longmapsto$ fictitious force.
- $|\Omega\rangle \sim e^{-a^{-1}\sum_{\omega k} \hat{a}^{L\dagger}_{\omega k} \hat{a}^{R\dagger}_{\omega k}}|0\rangle$.

- Kruskal vacuum $|\Omega\rangle$ vs. Schwarzschild vacuum $|0\rangle$.
- Intertial frame $\longmapsto$ physical force.
- $|\Omega\rangle \sim e^{-\beta\sum_{\omega k} \hat{a}^{L\dagger}_{\omega k} \hat{a}^{R\dagger}_{\omega k}}|0\rangle$.

Motivation

Information paradox = incompatibility among: [AMPS, '13]

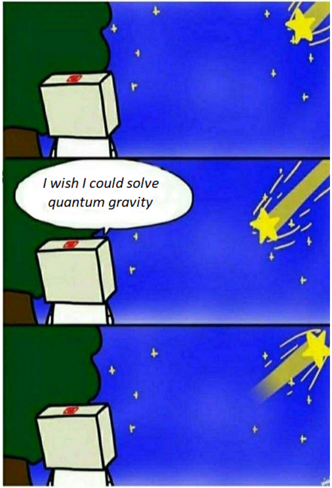
- Unitary black hole formation and evaporation
- Validity of the effective field theory description for the asymptotic observer
- Existence of black hole microstates visible to an asymptotic observer as states with exponentially small energy differences, thus $S_{BH} = \log(\#\text{dof})$
- No drama at the horizon (equivalence principle): an infalling observer in the near horizon region sees nothing out of ordinary.

[AMPS, '13] suggests dropping the equivalence principle.
But then, why have a tensor product?

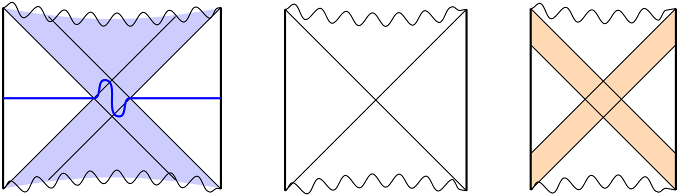
Motivation

Idea: analyze systems that violate the factorization property and have to do something with gravity.

1. Wormholes:
   - BDHM dictionary (AdS/CFT).
   - BTZ black hole (factorizable) and JT wormhole (non-factorizable).
   - Reproduce the known results from the new paradigm.
2. Quantum mechanical model:
   - What can we learn about black holes from double well potential?
   - Approximate factorization, state-dependence and chaos.



Why wormholes?



- Holography: can we reach the left boundary from the right boundary in the two-sided black hole.
- From boundary to bulk: bulk reconstruction: [Hamilton, Kabat, Lifschytz, Lowe, '04; Penington, '20],

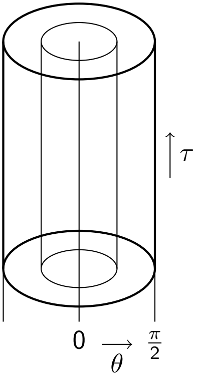
1. Evolution in both boundaries related, $H_L \sim H_R$? [Guica, Ross, '14; Harlow, Jafferis, '18]
2. Wormhole state: close to the thermofield double? [Maldacena, Stanford, Yang, '17]
3. But is it a state in the tensor product $\mathcal{H}_L \otimes \mathcal{H}_R$? [Arias, Botta Cantcheff, Silva, '10; Harlow, Jafferis, '18].

BDHM dictionary

- The BDHM dictionary [Banks, Douglas, Horowitz, Martinec, '98] is the operatorial form of holography.
- The construction here follows [Kaplan, '16].
- Global anti-de Sitter (AdS) metric

$$\mathrm{d}s^2 = \frac{L^2}{\cos^2\theta}\left(-\mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\Omega^2_{d-1}\right).$$

- Let Φ be a Klein-Gordon field satisfying $(-\Box + m^2)\Phi = 0$.
- The mass is parameterized as $m^2 = \Delta(\Delta - d)$.
- Two solutions: $\Phi \sim \mathrm{src}\cos^{d-\Delta}\theta + \mathrm{vev}\cos^{\Delta}\theta$.
- We set $\mathrm{src} = 0$ and quantize Φ.



BDHM dictionary

- The solution is

$$\Phi_{AdS}(\tau,\theta,\Omega)=\sum_{k=0}^{\infty}\sum_{\ell}\left(\phi_{k\ell}\alpha_{k\ell}+\phi^{*}_{k\ell}\alpha^{*}_{k\ell}\right),$$

where

$$\phi_{k\ell}(\tau,\theta,\Omega)=c_{k\ell}e^{-i\omega_{k\ell}\tau}Y_{\ell}(\Omega)\cos^{\Delta}\theta\,\sin^{\ell}\theta\,P_{k}^{(\ell+\frac{d}{2}-1,\Delta-\frac{d}{2})}(\cos(2\theta)).$$

- These are standing waves: the frequencies are quantized:

$$\omega_{k\ell}=\Delta+\ell+2k,\quad k=0,1,2,\ldots$$

- The coefficients are elevated to creation-annihilation operators

$$\left[\hat{\alpha}_{k\ell},\hat{\alpha}^{\dagger}_{k'\ell'}\right]=\delta_{kk'}\delta_{\ell\ell'}.$$

- The vacuum state is $|\Omega\rangle$ defined by the condition $\hat{\alpha}_{k\ell}|\Omega\rangle=0$.
- Hilbert space $\mathcal{H}_{AdS}$ is spanned by $\hat{\alpha}^{\dagger}_{k_1\ell_1}\ldots\hat{\alpha}^{\dagger}_{k_n\ell_n}|\Omega\rangle$.

BDHM dictionary

- At the boundary

$$\textcolor{red}{\mathcal{O}(\tau,\Omega)} = \sum_{k=0}^{\infty}\sum_{\ell}\left(\hat{\alpha}_{k\ell}\varphi_{k\ell} + \hat{\alpha}^{\dagger}_{k\ell}\varphi^{*}_{k\ell}\right),$$

where

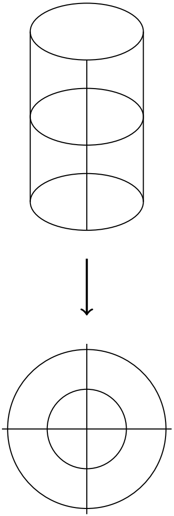
$$\varphi_{k\ell}(\tau,\Omega) = \lim_{\theta\to\frac{\pi}{2}} F^{-\Delta}(\theta)\phi_{k\ell} = \tilde{c}_{k\ell}e^{-\mathrm{i}\omega_{k\ell}\tau}Y_{\ell}(\Omega).$$

- Check: Euclidean operators

$$\mathcal{O}^{Eu}(t,\Omega) = e^{-\Delta t}\mathcal{O}(\tau = -\mathrm{i}t,\Omega),$$

produce the generalized free field correlators, *e.g.*,

$$\langle\Omega|\mathcal{O}^{Eu}(z,\bar{z})\mathcal{O}^{Eu}(0)|\Omega\rangle = \frac{1}{L}\frac{1}{|z|^{2\Delta}}.$$



Two boundaries

$$\hat{\Phi} = \sum_k (\phi_k \hat{a}_k + \phi_k^* \hat{a}_k^\dagger)$$

- Hilbert space: $\mathcal{H}_L$,
- Boundary modes: $\varphi_k^L = \lim_L \phi_k$,
- Operators: $\mathcal{O}_L = \sum_k (\varphi_k^L \hat{\alpha}_k^L + \varphi_k^{L*} \hat{\alpha}_k^{L\dagger})$

- Hilbert space: $\mathcal{H}_R$,
- Boundary modes: $\varphi_k^R = \lim_R \phi_k$,
- Operators: $\mathcal{O}_R = \sum_k (\varphi_k^R \hat{\alpha}_k^R + \varphi_k^{R*} \hat{\alpha}_k^{R\dagger})$

- One can map some combinations of the bulk operators $\hat{a}_k, \hat{a}_k^\dagger$ to $\hat{\alpha}_k^{L,R}, \hat{\alpha}_k^{L,R\dagger}$.
- The constructed boundary Hilbert spaces depend on the bulk region considered.
- Reeh-Schlieder property for AdS holds [Morrison, '14; Banerjee, Bryan, Papadodimas, Raju, '16].

BTZ black hole

- The BTZ black hole

$$\mathrm{d}s^2 = -(\rho^2 - \rho_h^2)\mathrm{d}t^2 + \frac{L^2\mathrm{d}\rho^2}{\rho^2 - \rho_h^2} + \rho^2\mathrm{d}\varphi^2,$$

- General solution:

$$\phi_{\omega n}(t,\rho,\varphi) = c_{\omega n}^{BTZ} e^{-\mathrm{i}\omega t + \mathrm{i}n\varphi} R_{\omega n}(\rho),$$

  - Field decomposition in all wedges:

$$\hat{\Phi}_{BTZ} = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \sum_{n=-\infty}^{\infty} \left(\phi_{\omega n}^{L*}\hat{a}_{\omega n}^{L} + \phi_{\omega n}^{R}\hat{a}_{\omega n}^{R} + h.c.\right)$$

  - In Schwarzschild modes $\phi_{\omega n}^{L,R}$ the split of the Hilbert space is explicit:

$$\mathcal{H} \cong \mathcal{H}_L \otimes \mathcal{H}_R, \qquad |0\rangle = |0\rangle_L \otimes |0\rangle_R,$$

with $\mathcal{H}_L$ spanned by $\hat{a}_{\omega n}^{L\dagger}$ and $\mathcal{H}_R$ spanned by $\hat{a}_{\omega n}^{R\dagger}$.

BTZ black hole

- What if we used Kruskal modes?

$$\chi^R_{\omega n} = \frac{\phi^R_{\omega n} + e^{-\frac{\pi\omega L}{\rho_h}}\phi^L_{\omega n}}{\sqrt{1-e^{-\frac{2\pi\omega L}{\rho_h}}}}, \qquad \chi^L_{\omega n} = \frac{\phi^L_{\omega n} + e^{-\frac{\pi\omega L}{\rho_h}}\phi^R_{\omega n}}{\sqrt{1-e^{-\frac{2\pi\omega L}{\rho_h}}}}.$$

- The action of the corresponding creation-annihilation operators $\hat{b}^{L,R\dagger}_{\omega n}, \hat{b}^{L,R}_{\omega n}$ is not limited to a single boundary,

$$\lim_R \hat{\Phi}_{\omega n}(t,\varphi) = \lim_R \chi^{L*}_{\omega n}\hat{b}^L_{\omega n} + \lim_R \chi^R_{\omega n}\hat{b}^R_{\omega n} + \lim_R \chi^L_{\omega n}\hat{b}^{L\dagger}_{\omega n} + \lim_R \chi^{R*}_{\omega n}\hat{b}^{R\dagger}_{\omega n}$$

- The Hilbert spaces spanned by $\hat{b}^{L\dagger}_{\omega n}$ and $\hat{b}^{R\dagger}_{\omega n}$ are not boundary Hilbert spaces.
- But the full Hilbert space still splits, up to the Bogoliubov transformation, $\mathcal{S}\mathcal{H}_\Omega \cong \mathcal{H} \cong \mathcal{H}_L \otimes \mathcal{H}_R$, where $\mathcal{S}$ implements

$$\hat{a}^R_{\omega n} = \frac{\hat{b}^R_{\omega n} + e^{-\frac{\pi\omega L}{\rho_h}}\hat{b}^{L\dagger}_{\omega n}}{\sqrt{1-e^{-\frac{2\pi\omega L}{\rho_h}}}}, \qquad \hat{a}^L_{\omega n} = \frac{\hat{b}^L_{\omega n} + e^{-\frac{\pi\omega L}{\rho_h}}\hat{b}^{R\dagger}_{\omega n}}{\sqrt{1-e^{-\frac{2\pi\omega L}{\rho_h}}}}.$$

- Kruskal vacuum $|\Omega\rangle$ is the thermofield double. It satisfies $\hat{b}^{L,R}_{\omega n}|\Omega\rangle = 0$.

BTZ black hole

1. Universality: the boundary Hilbert spaces are the same as if the bulk was replaced by empty AdS.
2. Decoupling: the action of the boundary operator is limited to the corresponding boundary.

Conclusions:

- $\mathcal{H}_{L,R}$ spanned by $\hat{a}^{L,R\dagger}_{\omega n}$ are boundary Hilbert spaces, those spanned by $\hat{b}^{L,R\dagger}_{\omega n}$ are not.
- The split does not depend on quantization choice, but it is more explicit is some.
- If the Hilbert space splits, there exist complex bulk modes vanishing on all boundaries but one.

AdS_2 wormhole

- Motivation: Jackiw-Teitelboim gravity [Arias, Botta Cantcheff, Silva, '10; Harlow, Jafferis, '18].
- AdS_2 spacetime

$$\mathrm{d}s^2 = \frac{L^2}{\cos^2\theta}\left(-\mathrm{d}\tau^2 + \mathrm{d}\theta^2\right)$$

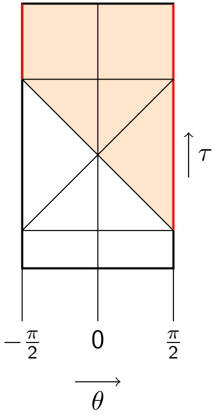
has 2 asymptotic boundaries at $\theta = \pm\frac{\pi}{2}$.

- The bulk field satisfies

$$\left[-\frac{\partial^2}{\partial\tau^2} + \frac{\partial^2}{\partial\theta^2} - \frac{\Delta(\Delta-1)}{\cos^2\theta}\right] X(\tau,\theta) = 0.$$

- Field decomposition reads

$$X(\tau,\theta) = \sum_{n=0}^{\infty}\left[a_n\phi_n + a_n^*\phi_n^*\right].$$



AdS_2 wormhole

- Take $\Delta = 1$. The solutions are

$$\chi^0_m(\tau,\theta) = \frac{1}{\sqrt{\pi m}} e^{-im\tau} \sin\left[m\left(\theta - \tfrac{\pi}{2}\right)\right], \quad n+1 = m = 1,2,3\dots$$

- At the two boundaries:

$$\Pi_R(\tau) = \partial_\theta X(\tau, \tfrac{\pi}{2}) = 2\sum_{m=1}^{\infty}\sqrt{\frac{m}{\pi}}\left[\mathrm{Re}\, a_m \cos(m\tau) + \mathrm{Im}\, a_m \sin(m\tau)\right],$$

$$\Pi_L(\tau) = \partial_\theta X(\tau, -\tfrac{\pi}{2}) = 2\sum_{m=1}^{\infty}(-1)^m\sqrt{\frac{m}{\pi}}\left[\mathrm{Re}\, a_m \cos(m\tau) + \mathrm{Im}\, a_m \sin(m\tau)\right].$$

- At the right boundary $\chi_n \sim \cos\theta\varphi_n$, so we identify $\hat{a}_n = \hat{a}^R_n$.
- At the right boundary $\chi_n \sim \cos\theta(-1)^n\varphi_n$, so we identify $\hat{a}_n = (-1)^n\hat{a}^L_n$.

AdS$_2$ wormhole

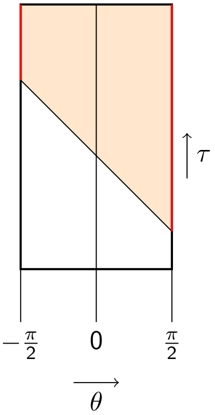
- The Hilbert space $\mathcal{H}$ does not split. In fact $\mathcal{H} = \mathcal{H}_L = \mathcal{H}_R$.
- Left and right operators satisfy

$$\hat{a}_n^L = (-1)^n \hat{a}_n^R = \hat{P} \hat{a}_n^R \hat{P} = e^{\mathrm{i}\pi H} \hat{a}_n^R e^{-\mathrm{i}\pi H}.$$

- Hamiltonians are all equal, $H_L = H_R = H$.
- Correct corrrelators, [Maldacena, Qi, '18]

$$\langle \Omega | \hat{X}_R(\tau) \hat{X}_R(\tau') | \Omega \rangle = \frac{\Gamma(\Delta)}{2^{2\Delta+1}\sqrt{\pi}\Gamma(\Delta + \frac{1}{2})} \frac{e^{-\mathrm{i}\pi\Delta}}{\sin^{2\Delta}\left(\frac{\tau-\tau'-\mathrm{i}\epsilon}{2}\right)},$$

$$\langle \Omega | \hat{X}_L(\tau) \hat{X}_R(\tau') | \Omega \rangle = \frac{\Gamma(\Delta)}{2^{2\Delta+1}\sqrt{\pi}\Gamma(\Delta + \frac{1}{2})} \frac{1}{\cos^{2\Delta}\left(\frac{\tau-\tau'-\mathrm{i}\epsilon}{2}\right)}.$$



- Similar conclusions: [Arias, Botta Cantcheff, Silva, '10].

GJW wormhole

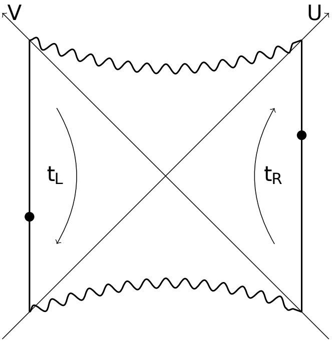
In [Gao, Jafferis, Wall, '17] a wormhole has been opened by coupling CFTs living on two boundaries of a BTZ black hole:

$$S = S_L + S_R + \int d^2x\, h(t,x)\mathcal{O}_L(-t,x)\mathcal{O}_R(t,x).$$

- Violation of the averaged null energy condition (ANEC) is a prerequisite,

$$\int_{-\infty}^{\infty} T_{\mu\nu}k^\mu k^\nu \mathrm{d}\lambda < 0.$$

- Negative energy can be provided by the non-local coupling the boundary theories.



Effective interaction

What if we pretended to work on the tensor product $\mathcal{H}_L \otimes \mathcal{H}_R$?

- Let
$$\hat{\alpha}_n = \hat{\alpha}_n^R + (-1)^n \hat{\alpha}_n^L, \qquad \hat{\beta}_n = \hat{\alpha}_n^R - (-1)^n \hat{\alpha}_n^L.$$
- We need
$$\hat{\beta}_n |\psi\rangle = \hat{\beta}_n^\dagger |\psi\rangle = 0 \quad \text{for all } n = 0, 1, 2, \ldots$$
and the Hamiltonian
$$H_\alpha = \sum_{n=0}^{\infty} (\Delta + n) \hat{\alpha}_n^\dagger \hat{\alpha}_n.$$
- Thus, formally,
$$H_{\text{int}} = \sum_{n=0}^{\infty} (\Delta + n)(-1)^n \left[\hat{\alpha}_n^{L\dagger} \hat{\alpha}_n^R + \hat{\alpha}_n^{R\dagger} \hat{\alpha}_n^L \right]$$
$$= \sum_{n=0}^{\infty} (\Delta + n) \left[e^{-i\Delta\pi} \hat{\alpha}_n^{L\dagger}(\tau + \pi) \hat{\alpha}_n^R(\tau) + e^{i\Delta\pi} \hat{\alpha}_n^{R\dagger}(\tau) \hat{\alpha}_n^L(\tau + \pi) \right].$$

Summary

- The Hilbert space of excitations over the wormhole does not factorize into the tensor product of boundary Hilbert spaces.
- This results in the operatorial relation between boundary operators, *e.g.*, $\hat{a}_n^R = (-1)^n \hat{a}_n^L$ and $H_L = H_R$.
- The interaction is an avatar of the description of the system on the tensor product.
- The interaction Hamiltonian is that of 'infinite squeezing'.
- Thermal partition function does not factorize, $\mathrm{Tr}\, e^{-\beta H} \neq \mathrm{Tr}\, e^{-\beta(H_L+H_R)}$.

Firewalls

Pick a single mode n: $\hat{a}_n^{L,R} \mapsto \hat{a}_{L,R}$.

- Firewalls: Among 3 number operators $N_L = \hat{a}_L^\dagger \hat{a}_L$, $N_R = \hat{a}_R^\dagger \hat{a}_R$, $N_A = \hat{a}^\dagger \hat{a}$ only 2 can be small.
- Give up the Equivalence principle: the near horizon region is not vacuum. In a generic microstate $|\Psi\rangle$ the infalling observer sees the firewall,

$$\langle\Psi|N_A|\Psi\rangle \sim N \gg 1.$$

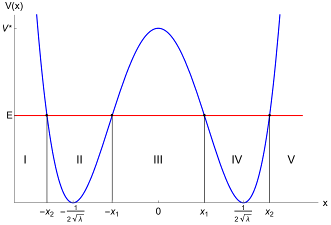
- State Dependence, [Papadodimas, Raju '13]: The notion of particles for the infalling observer is microstate-dependent. Tells us something about the structure of the Hilbert space describing quantum fluctuations.
- According to [Marolf, Polchinski, '15] state dependence violates rules of quantum mechanics (Born rule).

Double well potential

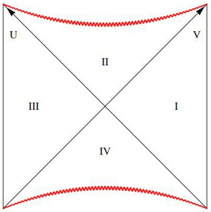
What if we replace $\hat{a}^L = \hat{a}^R$ by $\hat{a}^L = \hat{a}^R + c\mathbf{1}$?

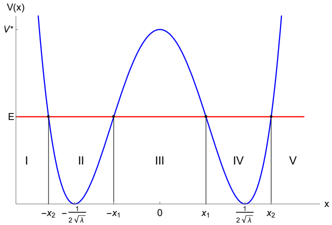
- Minima at $x_{L,R} = \frac{1}{2\omega\sqrt{\lambda}}$.
- One can think $\lambda = \frac{1}{N^2}$,
- At maximum $V_* = \frac{1}{32\lambda}$,
- We set $\omega = 1$.
- Field operators satisfy $\hat{x}_R - \hat{x}_L = N\mathbf{1}$.
- Define a decoupling limit as $\lambda \to 0$. Physically, the system can be thought of two decoupled harmonic oscillators, described by a tensor product Hilbert space.

$$V(x) = \frac{1}{32\lambda} - \frac{1}{4}\omega^2 x^2 + \frac{\lambda}{2}\omega^4 x^4$$



Toy model interpretation



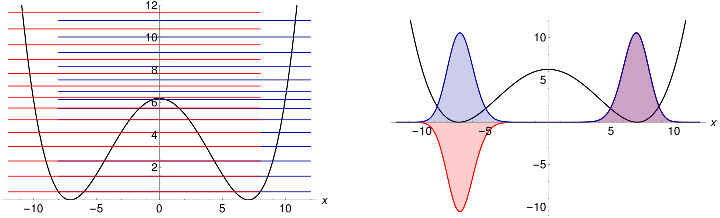


- Asymptotic regions
- BH microstates
- Excitations on top of $|0_k\rangle_R |0_{-k}\rangle_L$
- Decoupling

- Two minima
- Lowest energy states
- Excitations of two HO of frequency k
- $\lambda \to 0$

Energy levels

Energy eigenstates of a given $\pm$ parity, $H\Psi_n^\pm = E_n^\pm \Psi_n^\pm$.



- Energy differences are non-perturbatively small

$$\Delta E_n = E_n^- - E_n^+ = e^{-\frac{1}{6\lambda}} P_n(\lambda^{-1/2}) = o(\lambda^\infty).$$

- When $n \sim 1/\lambda \sim N^2$, $\Delta E_N \sim N^{N^2} e^{-N^2}$: non-perturbative effects become dominant.

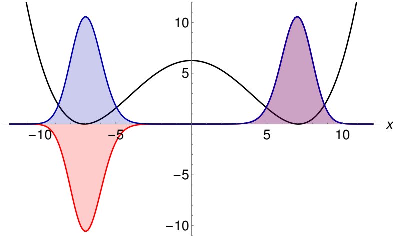
Microstates

- States $\Psi_n^{\pm}$ are indistinguishable to a single asymptotic observer within the perturbation thoery. We have microstates:

$$\mathcal{M} = \{\alpha_+ \Psi_0^+ + \alpha_- \Psi_0^- \ : \ \alpha_\pm \in \mathbb{C}\}.$$

- Each $\mu \in \mathcal{M}$ is a perturbative vacuum.
- We have semi-classical degeneracy and hence entropy,

$$S_B = \log \dim \mathcal{H}_{\text{fine}} = \log 2.$$



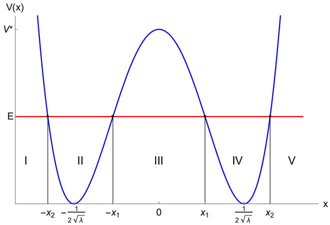
Excitations

- HO normalized eigenfunctions,

$$\varphi_n(x) = \frac{1}{\pi^{1/4}\sqrt{2^n n!}} H_n(x) e^{-\frac{x^2}{2}} ,$$

- Define

$$|n_R\rangle \ : \ \varphi_n^R(x) = \varphi_n(x - x_R),$$
$$|n_L\rangle \ : \ \varphi_n^L(x) = (\Theta\varphi_n^R)(x) = \varphi_n(x - x_L).$$



- Assign creation and annihilation operators a_L, a_L^+, a_R, a_R^+.
- Total Hilbert space $\mathcal{H}$ is isomorphic to each Fock space $\mathcal{F}_L$ and $\mathcal{F}_R$ *separately*,

$$\mathcal{H} \cong \mathcal{F}_R \cong \mathcal{F}_L \ ;$$

There is no tensor product.

Consequences

Naive number operators for left/right asymptotic observers

$$N_L = H_L^{(0)} = a_L^+ a_L, \qquad N_R = H_R^{(0)} = a_R^+ a_R$$

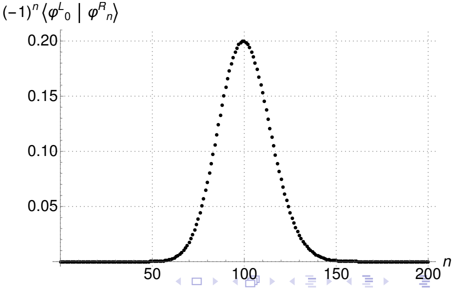
are *weird*

$$\langle \varphi_0^L | N_L | \varphi_0^L \rangle = \langle \varphi_0^R | N_R | \varphi_0^R \rangle = 0 \ ,$$
$$\langle \varphi_0^L | N_R | \varphi_0^L \rangle = \langle \varphi_0^R | N_L | \varphi_0^R \rangle = \frac{1}{2} N^2$$

and diverge in the decoupling limit $N \to \infty$.

For the right observer, semiclassical *left* states are highly excited

$$\langle \varphi_0^L | \varphi_n^R \rangle = \frac{(-1)^n e^{-\frac{1}{4\lambda}}}{\sqrt{2^n \lambda^n n!}}$$



Defining effective theory

- There is no state where N_L, N_R and $N_A = N_L + N_R$ are all small: a firewall?
- No, a_R, a_R^+ are *non-local*, ie., they do something horrible to φ_n^L.
- We cannot define

$$\hat{a}_R \varphi_n^R \stackrel{?}{=} \sqrt{n}\varphi_{n-1}^R, \qquad \hat{a}_R \varphi_n^L \stackrel{?}{=} 0,$$

$$\hat{a}_R^+ \varphi_n^R \stackrel{?}{=} \sqrt{n+1}\varphi_{n+1}^R, \qquad \hat{a}_R^+ \varphi_n^L \stackrel{?}{=} 0$$

  because the set $\{\varphi_n^L, \varphi_n^R\}$ is overcomplete [Jafferis '17].
- A solution: truncate the basis at finite $n \leq N$, [Papadodimas, Raju '13].
- Better solution: orthogonalize $\{\varphi_n^L, \varphi_n^R\}_n$.

Effective theory

- Symmetric and antisymmetric combinations of all energy eigenstates,

$$\Psi_n^L = \frac{1}{\sqrt{2}}(\Psi_n^+ - \Psi_n^-), \qquad \Psi_n^R = \frac{1}{\sqrt{2}}(\Psi_n^+ + \Psi_n^-)$$

  span two Hilbert subspaces (perturbative Hilbert spaces)

$$\mathcal{H}_L = \mathrm{span}\{\Psi_n^L\}_n, \qquad \mathcal{H}_R = \mathrm{span}\{\Psi_n^R\}_n.$$

- $\langle \Psi_m^L | \Psi_n^R \rangle = 0$

$$\mathcal{H} = \mathcal{H}_L \oplus \mathcal{H}_R, \qquad \mathcal{H}_L \perp \mathcal{H}_R, \qquad \Theta\mathcal{H}_L = \mathcal{H}_R, \qquad \Theta\mathcal{H}_R = \mathcal{H}_L.$$

- Projected operators : $\hat{a}_L = P_L a_L P_L, \qquad \hat{a}_R = P_R a_R P_R.$
- Number operators are

$$\hat{N}_L = \hat{a}_L^+ \hat{a}_L, \qquad \hat{N}_R = \hat{a}_R^+ \hat{a}_R, \qquad \hat{N}_A = \hat{N}_L + \hat{N}_R.$$

Consequences

- Every state φ_n^R can be approximated by a perturbative state up to non-perturbative effects, $\varphi_n^R \notin \mathcal{H}_R$ but $\|P_R \varphi_n^R\| = 1 - o(\lambda^\infty)$.
- Hatted operators are non-local up to non-perturbative effects, $[\hat{a}_R, \hat{a}_R^+] \neq 1$, but $[\hat{a}_R, \hat{a}_R^+] = 1 + o(\lambda^\infty)$, [Kabat Lifshitz '14, Raju '17, Anninos, Monten '19]
- No firewall: number operators $\hat{N}_{L,R,A}$ are non-perturbatively close to $N_{L,R,A}$, but are well-behaved:

$$\langle \mu | \hat{N}_{L,R,A} | \mu \rangle = O(\sqrt{\lambda}), \quad \text{for generic } \mu \in \mathcal{M}.$$

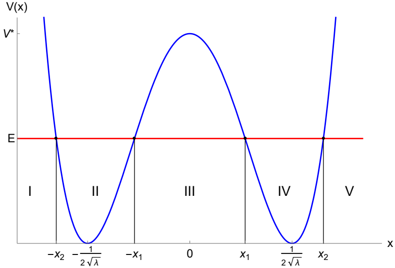
- The Hilbert space factorizes into the tensor product $\mathcal{H} \sim \mathcal{H}_L \otimes \mathcal{H}_R$ only approximately at low energies. Effective operators are microstate-dependent.
- Only some states in $\mathcal{F}_L \otimes \mathcal{F}_R$ are physical.
- Effective theory breaks for energies $\sim V_* \sim N^2$ or times $t \sim 1/E$, [Raju '17]

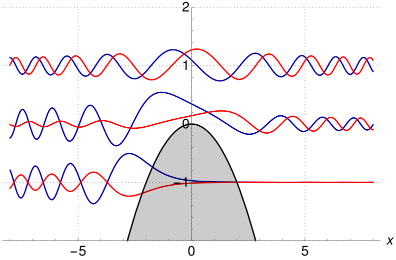
Hawking radiation as tunneling

- BH evaporation as tunneling: [Parhik, Wilczek '99, Gaddam, Papadoulaki, Betzios '16].
- Tunneling rate in WKB: $\Gamma = e^{-2\Lambda}$, $\Lambda = \int_{-x_1}^{x_1} \sqrt{2(V(x)-E)}dx$,
- At $E \sim V_*$ with $\delta = V_* - E$ the potential can be approximated by the inverted harmonic oscillator.
- One finds

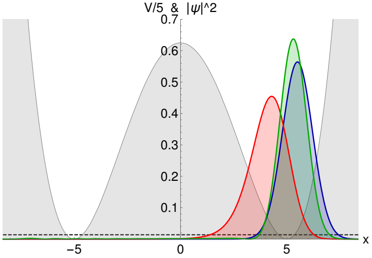
$$\Lambda(\lambda,\delta) = \sqrt{2}\pi\delta + 3\sqrt{2}\pi\lambda\delta^2 + O(\delta^3),$$

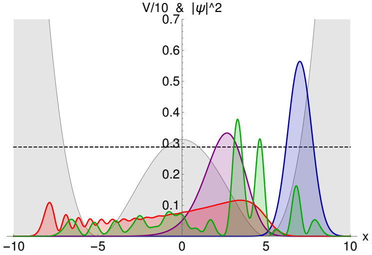
which means that in our model $\omega \sim \sqrt{\lambda}\delta$, $M \sim 1/\sqrt{\lambda} = N$.





Chaotic evolution





- Classical particle with $E \ll V_*$ stays on a closed orbit.
- The period diverges logarithmically when $E \to V_*$:

$$T_{\text{trapped}} = \sqrt{2} \log \left(\frac{2}{\lambda \delta} \right) + O(\lambda),$$

- Close to the tip: $x(t) = x_0 \cosh(\nu t) + v_0/\nu \sinh(\nu t)$. Hence

$$\delta x(t) \sim e^{\nu t} (\delta x_0 + \delta v_0/\nu).$$

This is by definition chaotic behavior with the Lapunov exponent $\nu = \omega/\sqrt{2} = 1/\sqrt{2}$.

Summary

Quantum mechanical system of the double well potential exhibits characteristic behavior associated with a pair of entangled modes in the quantum black holes:

- The Hilbert space does not factorize into the tensor product $\mathcal{F}_L \otimes \mathcal{F}_R$. Instead $\mathcal{H} \cong \mathcal{H}_L \oplus \mathcal{H}_R$.
- One can define natural creation-annihilation and firewall-free number operators, which agree with naive ones up to non-perturbative effects. The new operators remain local, up to non-perturbative terms.
- The factorization into the tensor product is approximate at low energies up to non-perturbative effects.
- A choice of non-perturbative vacuum leads to state-dependence.
- Hawking radiation has a natural interpretation as tunneling.

Summary

Ideas presented:

- Weirdness of black holes can be realized in simple toy models.
- The split of the Hilbert space into the tensor product of the boundary spaces is questionable.
- Interactions between boundaries of a wormhole can be seen as the avatar of the non-trivial tensor structure.
- Possibility of building new models quantum black holes?

Summary

Thank you!