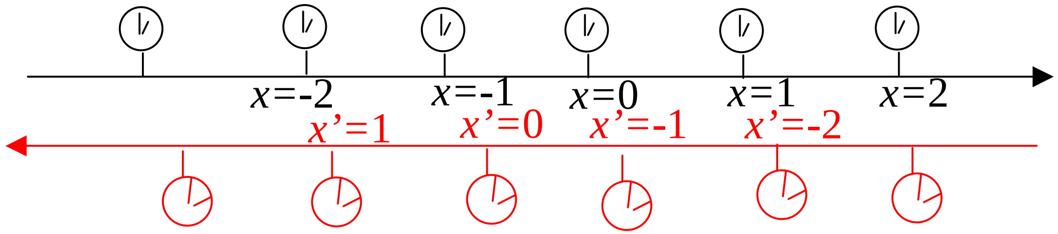
STW 100 lat potem

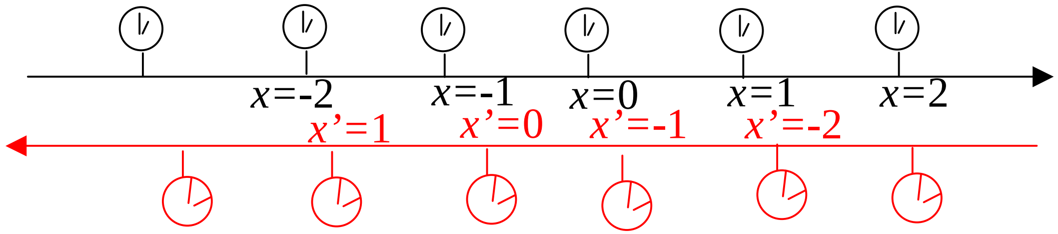
O symetrii i o ciałach w ruchu, ale
bez elektrodynamiki.

Dla $x' = 0$

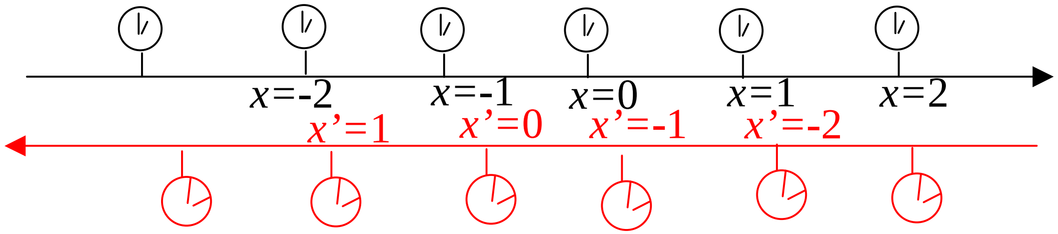


$$x = Vt$$

Dla $x' \neq 0$



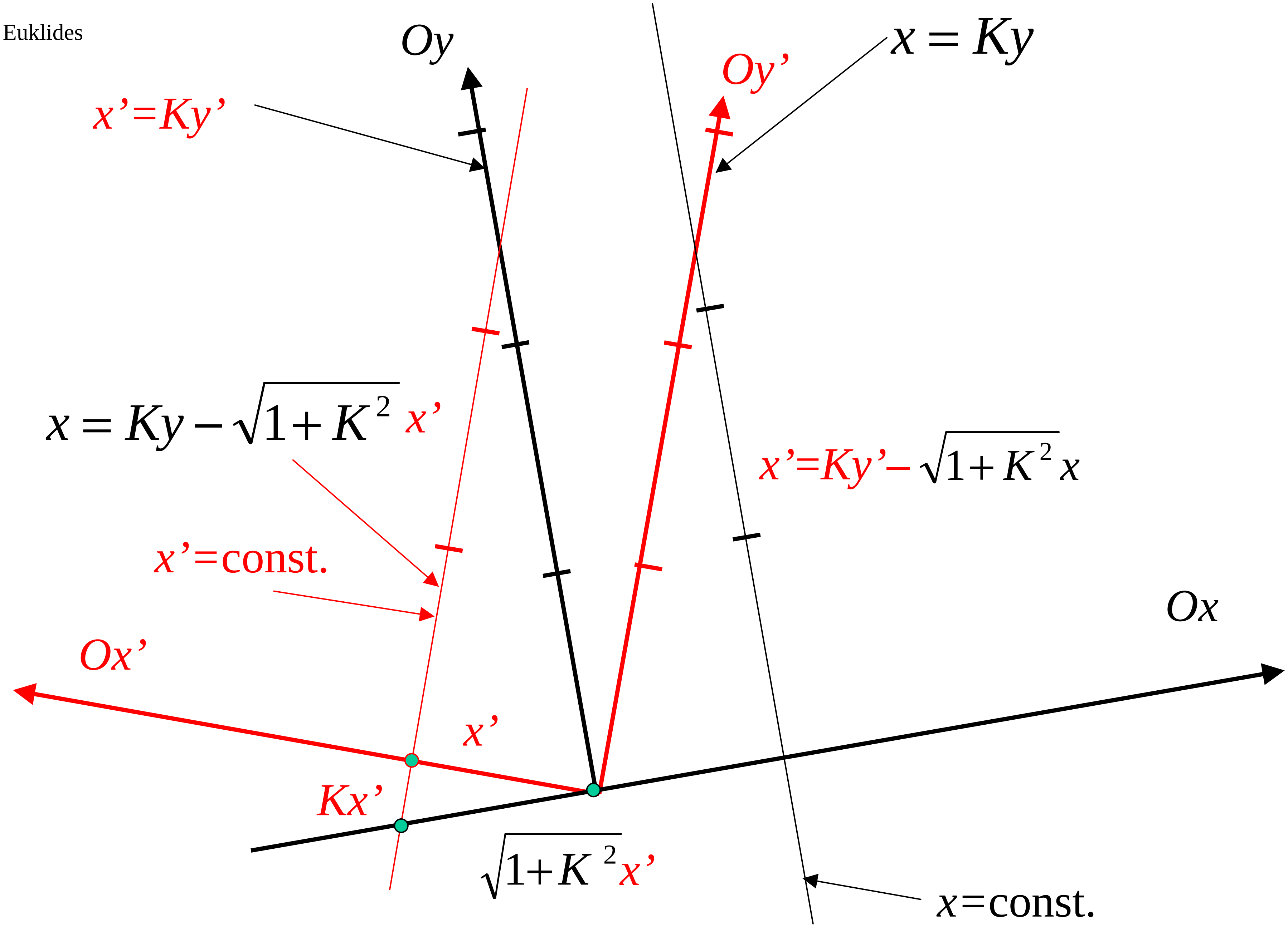
$$x = Vt - a(V) \cdot x'$$



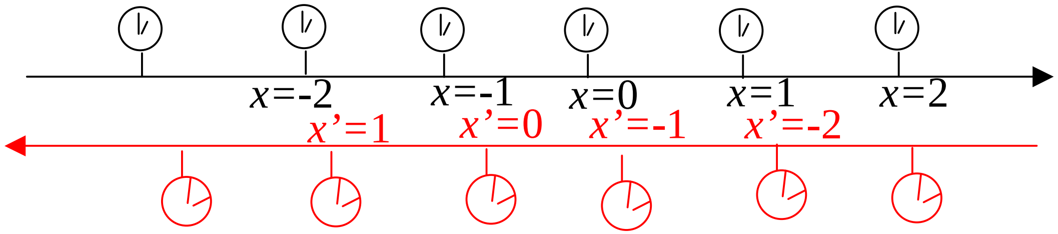
$$x = Vt - a(V)\,x'$$

$$x' = Vt' - a(V)\,x$$

Euklides

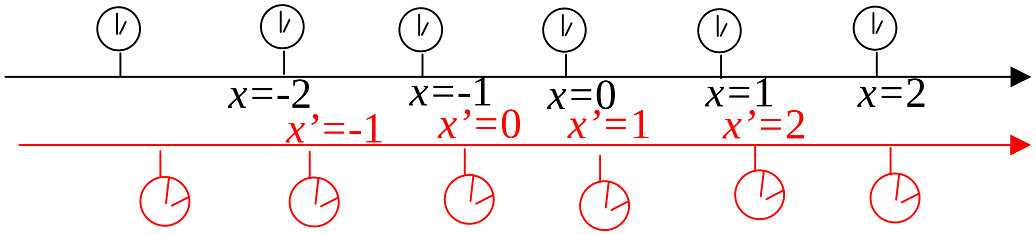


Zegary



$$x = Vt - a(V)\,x'$$

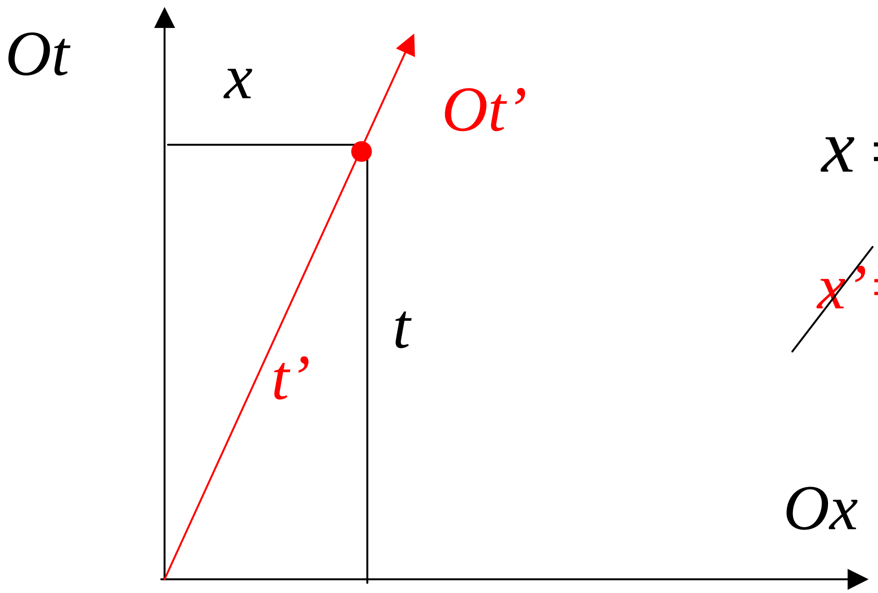
$$x' = Vt' - a(V)\,x$$



$$x = Vt + a(V)\,x'$$

$$x' = -Vt' + a(V)\,x$$

$$(x + x')(1 - a) = V(t - t')$$



$$x = Vt + a(V)\,x'$$

$$x' = -Vt' + a(V)x = 0$$

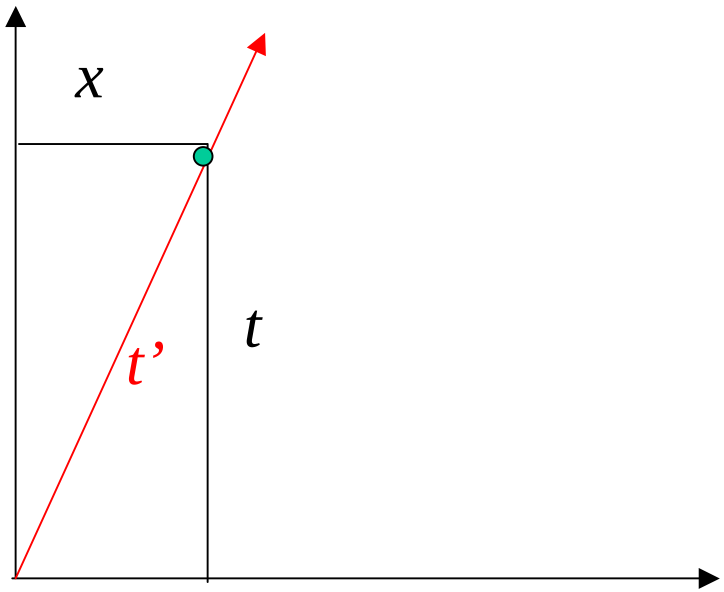
$$t' = a\frac{x}{V} = at$$

$$x = Vt - ax' \qquad x = Ky - ax'$$

$$x' = Vt' - ax \qquad x' = Ky' - ax$$

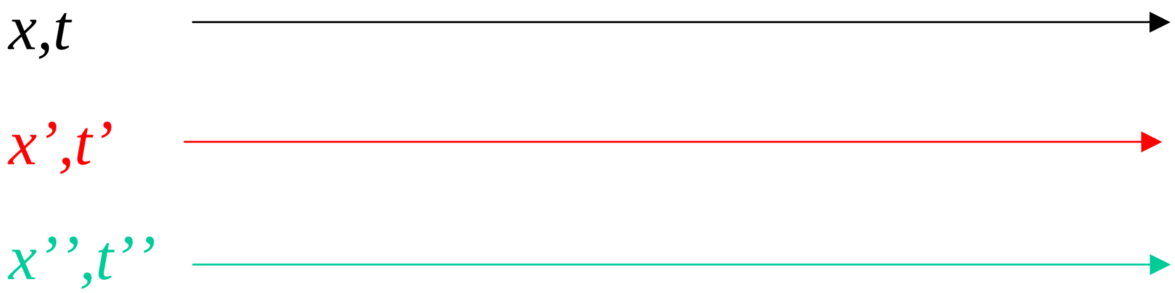
Galileusz: a=1, Euklides: $a = \sqrt{1 + K^2}$

Jakie jeszcze *a* jest możliwe??



Przekątna to wartość t' dla x'=0,
czyli $t'=ax/V=at$

3Układy



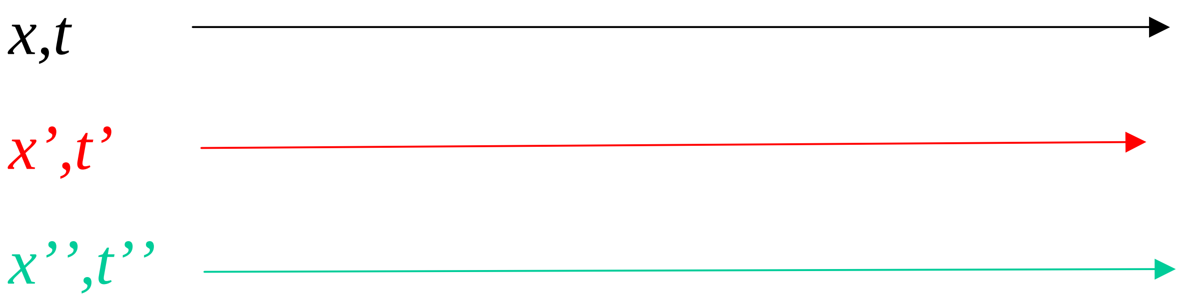
$$x = Vt + a(V)x'$$

$$x' = -Vt' + a(V)x$$

$$x' = v't' + a(v')x''$$

$$x'' = -v't'' + a(v')x'$$

Cd 3 układy



$$x = Vt + a(V)x'$$

$$x' = -Vt' + a(V)x \qquad / \otimes v'$$

$$x' = v't' + a(v')x'' \qquad / \otimes V$$

$$+$$

$$x'' = -v't'' + a(v')x'$$

$$x'(V+v') = v'a(V)x + Va(v')x''$$

$$x' = \frac{a(V)v'x + a(v')V\,x''}{v'+V}$$

1-szy i trzeci

$$x = Vt + a(V)\,\frac{a(V)v'x + a(v')V\,x''}{v'+V}$$

$$x'' = -v't'' + a(v')\,\frac{a(V)v'x + a(v')V\,x''}{v'+V}$$

Stała C

uzyskujemy	**musi być**
$$x=\frac{V+v'}{1+\frac{1-a(V)^2}{V^2}Vv'}t+\frac{a(V)a(v')}{1+\frac{1-a(V)^2}{V^2}Vv'}x''$$	$$x=v\,t+a(v)\,x''$$
$$x''=-\frac{V+v'}{1+\frac{1-a(v')^2}{v'^2}Vv'}t''+\frac{a(V)a(v')}{1+\frac{1-a(v')^2}{v'^2}Vv'}x$$	$$x''=-v\,t''+a(v)\,x$$

musi być:

$$\frac{1-a(V)^2}{V^2}=\frac{1-a(v')^2}{v'^2}\equiv \mathrm{const}=C$$

wyniki

$$\frac{1-a^{2}(V)}{V^{2}}=C \quad \Rightarrow \quad a(V)=\sqrt{1-CV^{2}}$$

$$x=Vt+\sqrt{1-CV^{2}}\,x'$$

$$x'=-Vt'+\sqrt{1-CV^{2}}\,x$$

$$v="v'+V"=\frac{V+v'}{1+CVv'}$$

Zestawienie z Eukl.

$$\frac{1-a^2(V)}{V^2}=C \quad \Rightarrow \quad a(V)=\sqrt{1-CV^2}$$

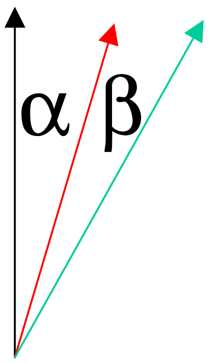
$$x=Vt+\sqrt{1-CV^2}\,x' \qquad x=Ky+\sqrt{1+K^2}\,\boldsymbol{x'}$$

$$x'=-Vt'+\sqrt{1-CV^2}\,x \qquad \boldsymbol{x'=-Ky'+}\sqrt{1+K^2}\,x$$

$$v="v'+V"=\frac{V+v'}{1+CVv'}$$

$$k=\tan(\alpha+\beta)=\frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\cdot\tan(\beta)}=$$

$$=\frac{K+k'}{1-Kk'}$$



Tr. Lorentza1

$$\frac{1-a^2(V)}{V^2}=C \quad \Rightarrow \quad a(V)=\sqrt{1-CV^2}$$

$$x=Vt+\sqrt{1-CV^2}\,x'$$

$$x'=-Vt'+\sqrt{1-CV^2}\,x$$

$$x=\frac{x'+Vt'}{\sqrt{1-CV^2}}$$

$$v="v'+V"=\frac{V+v'}{1+CVv'}$$

Tr Lorentza2

$$\frac{1-a^{2}(V)}{V^{2}}=C \quad \Rightarrow \quad a(V)=\sqrt{1-CV^{2}}$$

$$x=Vt+\sqrt{1-CV^{2}}\,x'$$

$$t=\frac{t'+CVx'}{\sqrt{1-CV^{2}}}$$

$$x'=-Vt'+\sqrt{1-CV^{2}}\,x$$

$$x=\frac{x'+Vt'}{\sqrt{1-CV^{2}}}$$

$$v="v'+V"=\frac{V+v'}{1+CVv'}$$

c

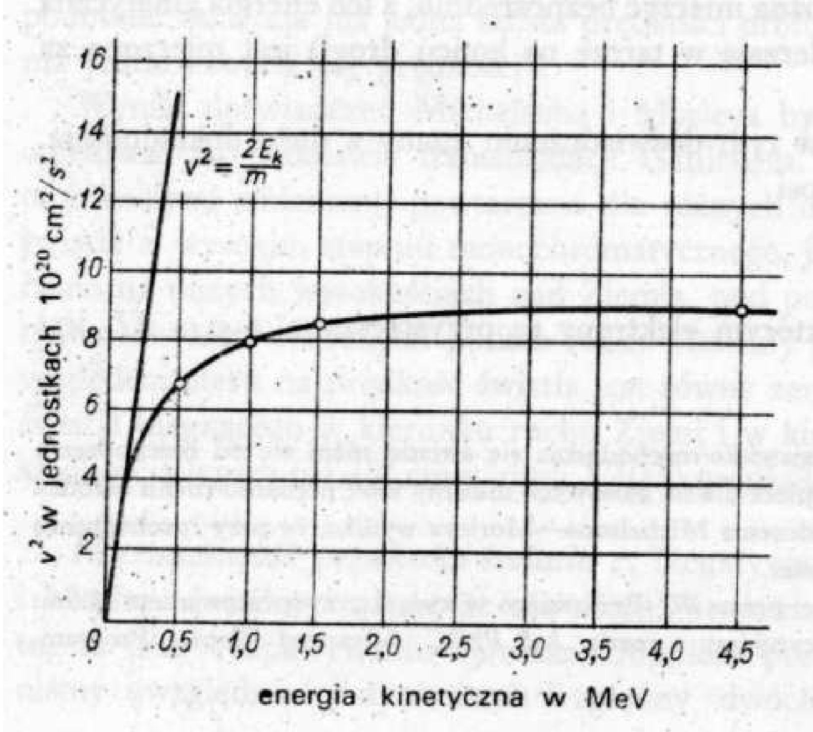
Prędkość *V* nie może być dowolna (minus pod pierwiastkiem):

$$1 - CV^2 \geq 0 \quad \Rightarrow V \leq \frac{1}{\sqrt{C}} = c$$

$$C = 1{,}1 \cdot 10^{-17}\ \frac{s^2}{m^2} = \frac{1}{(300000 km/s)^2} \equiv \frac{1}{c^2}$$

Jest to prędkość graniczna i zarazem absolutna:

$$"V + c" = \frac{c + V}{1 + CVc} = \frac{c + V}{1 + Vc/c^2} = c\,\frac{1 + V/c}{1 + V/c} = c$$



v² w jednostkach 10²⁰ cm²/s²
16
14
12
10
8
6
4
2
0
0,5
1,0
1,5
2,0
2,5
3,0
3,5
4,0
4,5
energia kinetyczna w MeV
v² = 2Eₖ/m

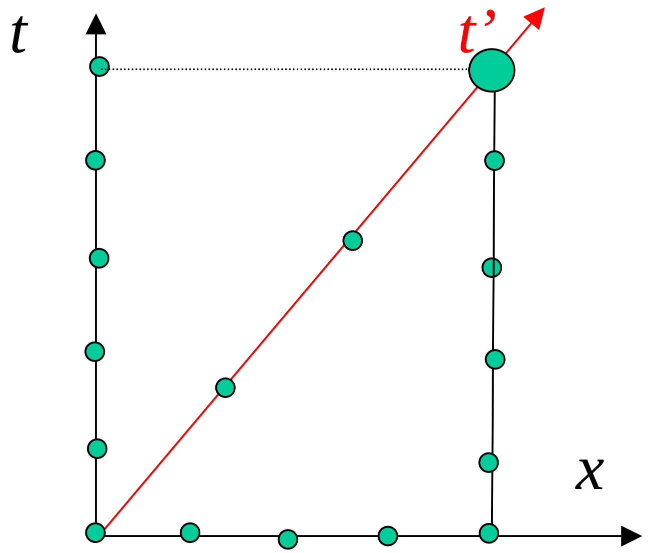
trójkąty

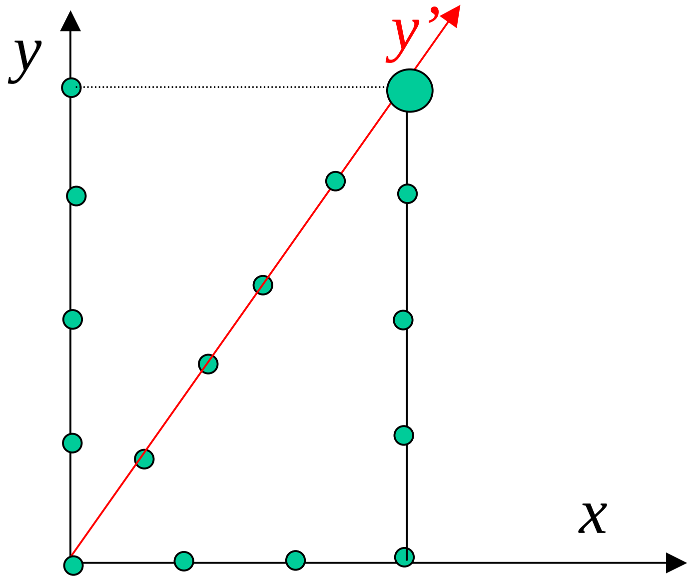
W czasoprzestrzeni

$$x'=0 \Rightarrow t'=\sqrt{1-CV^2}\,x/V=\sqrt{t^2-Cx^2}$$

U Euklidesa

$$x'=0 \Rightarrow y'=\sqrt{1+K^2}\,x/K=\sqrt{y^2+x^2}$$

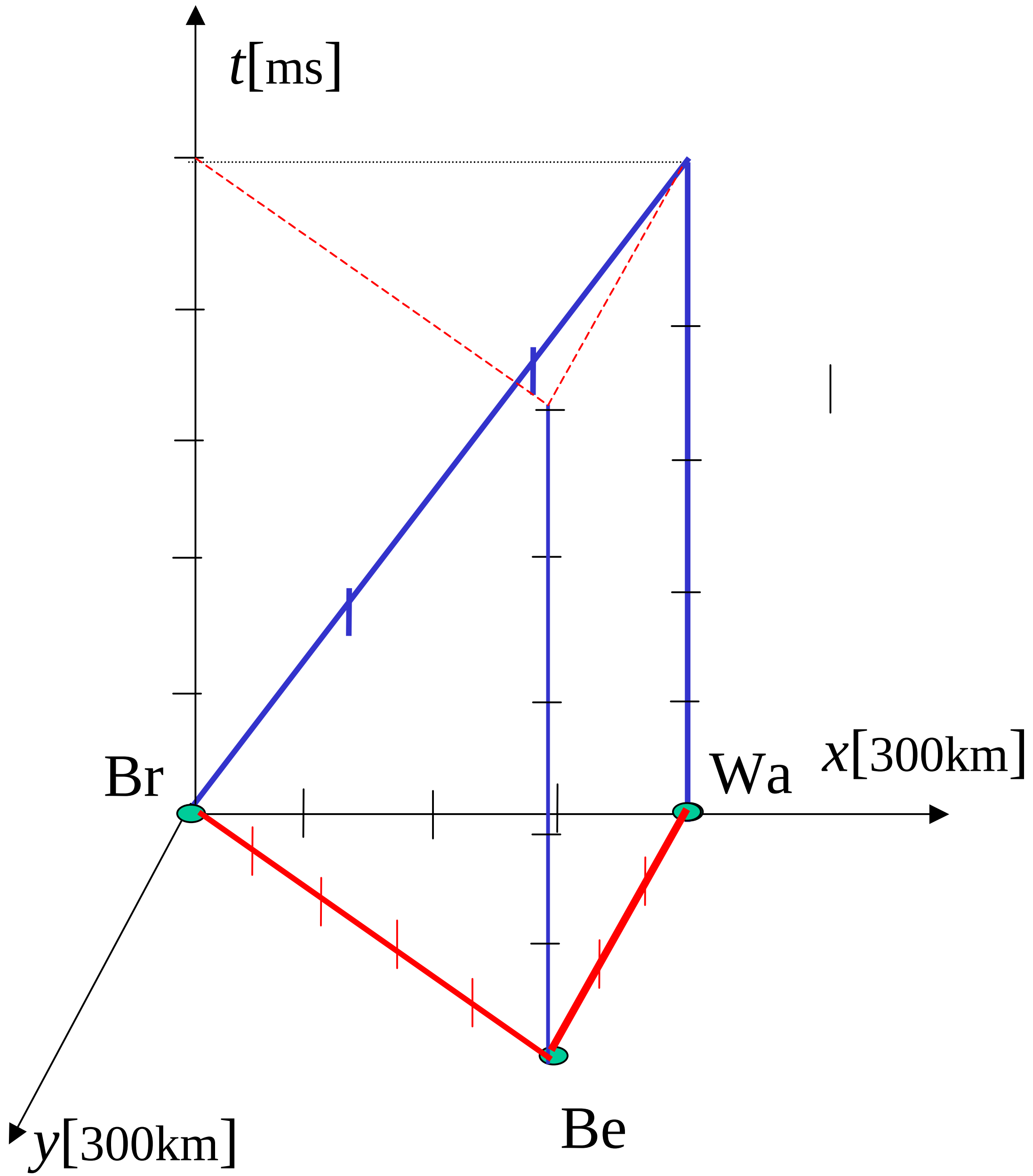


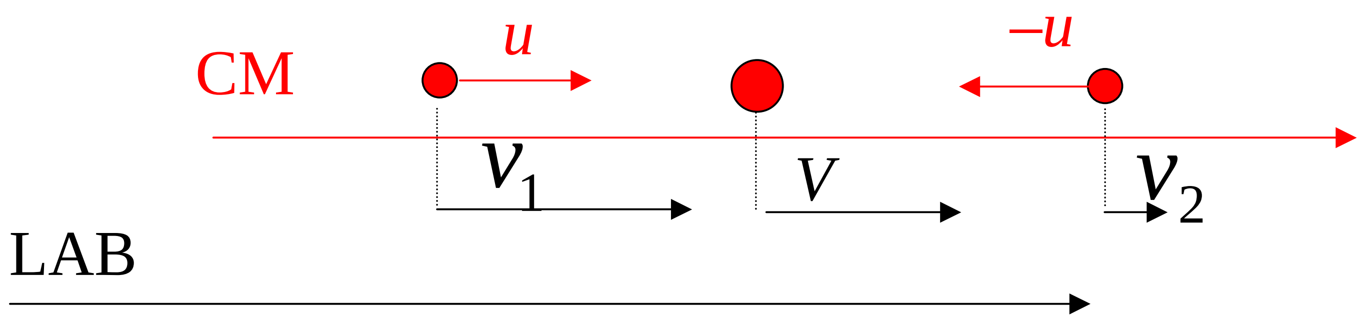


$x=4\cdot300$km, $t=5\cdot1$ms, $t'=3$ms

$x=3$, $y=4$, $y'=5$

Europa"





$$v_1 = \frac{V + u}{1 + Vu / c^2}, v_2 = \frac{V - u}{1 - Vu / c^2}, \quad v_3 = V$$

Typowy problem: Znam prędkości początkowe v_1 i v_2 chcę przewidzieć prędkość końcową v_3.

Dygresja:

Zapomnijmy o STW

$$
\begin{aligned}
+\quad v_1 &= V + u \\
v_2 &= V - u
\end{aligned}
\qquad v_3 = V
$$

$$2V = 1v_1 + 1v_2 = 2v_3$$

$$1 \quad + 1 \quad = 2 \qquad / \otimes m$$

$$mv_1 + mv_2 = m_3 v_3$$

$$m \quad + m \quad = m_3$$

czteroprędkość

$$\frac{1}{\sqrt{1-C{v_{1/2}}^2}}=\frac{1}{\sqrt{1-C\left(\dfrac{V\pm u}{1\pm CVu}\right)^2}}=$$

$$\frac{1\pm CVu}{\sqrt{(1\pm CVu)^2-C(V\pm u)^2}}=\frac{1\pm CVu}{\sqrt{1-CV^2}\sqrt{1-Cu^2}}$$

$$\frac{v_{1/2}}{\sqrt{1-C{v_{1/2}}^2}}=\frac{(1\pm CVu)\dfrac{V\pm u}{1\pm CVu}}{\sqrt{1-CV^2}\sqrt{1-Cu^2}}=\frac{V\pm u}{\sqrt{1-CV^2}\sqrt{1-Cu^2}}$$

$$\frac{v_1}{\sqrt{1-v_1{}^2/c^2}}+\frac{v_2}{\sqrt{1-v_2{}^2/c^2}}=\frac{2}{\sqrt{1-u^2/c^2}}\frac{V}{\sqrt{1-V^2/c^2}}$$

$$V=v_3$$

$$\frac{1}{\sqrt{1-v_1{}^2/c^2}}+\frac{1}{\sqrt{1-v_2{}^2/c^2}}=\frac{2}{\sqrt{1-u^2/c^2}}\frac{1}{\sqrt{1-V^2/c^2}}$$

$$\frac{mv_1}{\sqrt{1-v_1{}^2/c^2}}+\frac{mv_2}{\sqrt{1-v_2{}^2/c^2}}=\frac{m_3v_3}{\sqrt{1-v_3{}^2/c^2}}$$

$$\frac{m}{\sqrt{1-v_1{}^2/c^2}}+\frac{m}{\sqrt{1-v_2{}^2/c^2}}=\frac{m_3}{\sqrt{1-v_3{}^2/c^2}}$$

$$\frac{v_1}{\sqrt{1-v_1{}^2/c^2}}+\frac{v_2}{\sqrt{1-v_2{}^2/c^2}}=\frac{2}{\sqrt{1-u^2/c^2}}\frac{V}{\sqrt{1-V^2/c^2}}$$

$$V=v_3$$

$$\frac{1}{\sqrt{1-v_1{}^2/c^2}}+\frac{1}{\sqrt{1-v_2{}^2/c^2}}=\frac{2}{\sqrt{1-u^2/c^2}}\frac{1}{\sqrt{1-V^2/c^2}}$$

$$\frac{mv_1}{\sqrt{1-v_1{}^2/c^2}}+\frac{mv_2}{\sqrt{1-v_2{}^2/c^2}}=\frac{m_3v_3}{\sqrt{1-v_3{}^2/c^2}}$$

$$\frac{m\ c^2}{\sqrt{1-v_1{}^2/c^2}}+\frac{m\,c^2}{\sqrt{1-v_2{}^2/c^2}}=\frac{m_3\,c^2}{\sqrt{1-v_3{}^2/c^2}}$$

Pęd i energia

$$p = \frac{mv}{\sqrt{1 - v^2 / c^2}}$$

$$\frac{mc^2}{\sqrt{1 - v^2 / c^2}} = mc^2 + \left(\frac{mc^2}{\sqrt{1 - v^2 / c^2}} - mc^2 \right) = mc^2 + \frac{mv^2}{\sqrt{1 - v^2 / c^2}\left(1 + \sqrt{1 - v^2 / c^2}\right)}$$

$$T = \frac{mc^2}{\sqrt{1 - v^2 / c^2}} - mc^2$$

$$T + mc^2 = T + E_{\mathrm{wewn}} \equiv E = \frac{mc^2}{\sqrt{1 - v^2 / c^2}}$$

$$p = \frac{mv}{\sqrt{1 - v^2 / c^2}}$$

$$E = \frac{mc^2}{\sqrt{1 - v^2 / c^2}}$$

$$v = \frac{p}{E / c^2}$$

$$E = \sqrt{m^2 c^4 + c^2 p^2}$$

mc^2

$$\sum_{\text{konc}} (T + mc^2) = \sum_{\text{pocz.}} (T + mc^2)$$

$$\sum_{\text{konc}} T - \sum_{\text{pocz.}} T = \sum_{\text{pocz.}} mc^2 - \sum_{\text{konc}} mc^2 = c^2 \Delta m = \frac{\Delta m}{C}$$

STW 100 lat potem