A Companion to "Newton's Principia"

Slide 2. Newton's CV. Why the dates of birth and death of Isaac Newton are given in two versions?

It had been known since several centuries that Julian calendar was not exact and required correction. In the XVIth century the difference between natural phenomena and the Julian calendar grew to 10 days and was easily noticeable. Pope Gregory XIII asked a team of astronomers to prepare the reform. The Gregorian calendar was introduced by papal decree in October 1582. It was decided to advance the calendar by 10 days, so that after Oct. 4, 1582, the day following was reckoned as October 15 (see link to Slide 34).

Catholic states in Europe (France, Italy, Spain, Poland, Portugal etc.) accepted the reform within a year. However, England and other protestant countries lingered not to give the impression that they take orders from the papacy. England adopted the Gregorian calendar only in 1752.

Thus Isaac Newton was born on Christmas Day 1642 celebrated in England, while in France and many European countries it was already January 4, 1643. His father, a farmer, died three months before the birth of his son. Isaac's mother soon remarried and moved to another village. Isaac was raised by his grandmother. He seemed to be a bright boy, so that the family decided to send him to Cambridge. He began studies in 1661 as a subsizar, a poor student who had to be a servant of richer colleagues in order to support himself. He studied Greek, Latin, and philosophy. The first chair of mathematics in Cambridge was formed only in 1663. Newton became interested in mathematics a little earlier because he wanted to understand books on astrology and geometry which he had purchased at the nearby market.

At the end of 1664 the great bubonic plague hit Europe. The disease was extremely infectious and killed some twenty percent of the population. The universities were closed and the students sent home in order to lessen the effects of epidemic by avoiding large gatherings of people. Newton went back to his family house in Woolsthorpe (Slide 3, upper right picture) and stayed there until 1667. He later maintained that in that quiet period he made his great discoveries in mathematics, physics, and astronomy. He invented differential calculus (which he called method of fluxions) and integral calculus (inverse method of fluxions). He also began thinking about gravity - not yet universal, but only "...gravity extending to the orb of the Moon."

Please notice the expression: "…having found out how to estimate the force with which [a] **globe revolving within a sphere presses the surface of the sphere**". It clearly shows that Newton seriously considered Cartesian *conatus recedendi a centro*. He made use of Kepler's third law and decided that "the forces which keep the planets in their orbits must be reciprocally as the squares of their distance from the centers about which they revolve…" He wanted to check it by comparing the force acting on the Moon with the force of gravity at the surface of the Earth and "**found them answer pretty nearly**." It is a strange expression, indeed. We know from other documents that Newton found these two forces only approximately equal (the difference amounted to about fifteen percent) and decided that there must be also an additional contribution from the Cartesian mechanism. Anyway, he did not publish anything on that subject for the next twenty years. (read his full text in Slide 3).

In the spring of 1667 Newton returned to Cambridge, completed his studies and obtained the diploma. After a few months he was employed as professor of mathematics in the Trinity College thanks to the recommendation of the previous professor, Isaac Barrow, who decided to retire.

The buildings of the Trinity College, Cambridge, are shown in Slide 4; the rooms in which Newton lived are indicated with a red arrow in Slide 5.

Slide 6 pertains to the first published paper by Newton. It presented his new theory of light and colors. We shall discuss it in the Chapter "Optics from Kepler to Newton".

Slide 7. Origin of Newton's great work.

At that time several people discussed the question of planetary motions. Early in 1684 three members of the Royal Society, Edmond Halley, Robert Hooke, and Christopher Wren, met in a coffee-house in London to discuss celestial motions. Halley announced his belief that the force acting on a planet by the Sun must be inversely proportional to the distance between the two bodies. Hooke agreed and said that he had a proof of that conjecture, however, he could not produce it during the meeting. Christopher Wren was a bit doubtful and proposed to offer his book worth 40 shillings to anyone able to present the said proof.

After several months, probably in August, Halley decided to visit Newton in Cambridge and ask for his opinion. He asked him what would be the orbit of a planet if it was attracted to the Sun by gravitational force obeying an inverse square law. Newton immediately answered that it would be an ellipse. Halley asked him how he knew it, to which Newton replied that he had calculated it long ago. He could not, however, find the proof in his papers, but promised to send it to Halley soon.

After a few weeks Newton sent Halley a short proof of the elliptical orbits and soon after - a short treatise *De motu corporum (On the movements of bodies)*. Halley immediately saw the importance of Newton's work and asked him for permission to publish it in The "Philosophical Transactions of The Royal Society". Newton refused and informed Halley that he was working on a larger book. Halley pressed him to do it soon which Newton accepted. In April 1686 the manuscript of the first book of *Philosophiae naturalis principia mathematica* was sent to the Royal Society, the second and third books followed later. On July 5, 1686, Samuel Pepys, the President of The Royal Society, put his name to the imprimatur of that work in token of its approval by the Society. Exactly one year later, on July 5, 1687, the printing of *Principia* was completed. The exact number of copies is not known, it was probably between 250 and 350.

The first and three subsequent editions of *Principia* were in Latin, the language of all educated people at that time. The first English translation was initiated when Newton was still alive and completed in 1729. The excerpts presented in the following slides have been taken from that translation, slightly modernized in 1934.

It is worth remembering that in England and other English speaking countries "philosophia naturalis", or natural philosophy, was used instead of "physics", which was reserved for medicine.

That's why in the English language medical doctors are called physicians. The word "physicist" was coined as late as the XIX century, but there were textbooks of physics published under the traditional title until the beginning of the XXth century.

Slides 8 and 9 contain excerpts from the Preface to *Principia*. Slide 10 gives the contents of *Principia*; in the following we shall consider the main parts one after the other. It is easily seen that the book was full of mathematics, and contained large numer of lemmas, theorems, and problems in the first two books. The third book was originally planned to be more descriptive but after completing it Newton decided (see Slide 11) that it was too simple and could be misunderstood by the readers not sufficiently prepared to apprehend the novelty of his approach. He wrote another, mathematical version of Book 3, while the original Book 3 was published in 1728 under the title *De mundi systemate liber*.

Slides 13 and 14. Definitions.

The eight definitions at the beginning of *Principia* are very important. Newton realized that a mathematical discussion must be preceded by clear definitions of the quantities and entities to be discussed. Please notice the old English syntax. Instead of writing, as we do it now, that mass (or quantity of matter) *m* is the product of volume *V* and density ρ ($m = V \ge \rho$), Newton wrote that "the quantity of matter is the measure... arising from its density and bulk conjunctly". Similar old syntax was used in the remaining definitions.

Slide 15 contains Newton's definitions of absolute time and absolute space. They were modified only in the XXth century with the rise of the theory of relativity.

Slides 16 and 17 contain the famous Newton's law of dynamics which nowadays every child learns at school. Please remember that the first law, or the law of inertia, had been already stated by Descartes.

Slide 18 lists the topics discussed in the three books of Principia.

Slide 19. Famous Newton's law of gravitation did not appear in the *Principia* in the form we know it now. Instead we find it stated in three different parts in Book III.

Slide 20 is taken from *De mundi systemate liber* (*The system of the world*). It expresses Newton's thought experiment which originated the idea of artificial satellites of the Earth. It is one of the most important ideas which had been later expanded in the general theory of relaitivity; almost all bodies in the universe are in the state of free fall in gravitational field: the Moon is falling down to Earth, the Earth and the planets are falling down to the Sun, etc. The inertial frames can therefore exist only locally.

Slides 22 and 23 contain excerpts from the anonymous review of *Principia* in the French "Journal des sçavants". The author, obviously a follower of the Cartesian world picture, admitted the greatness and correctness of Newton's arguments but was very critical and concluded that it all did not pertain to physics but only to mechanics, i.e. a mathematical discipline. Such criticism was important and commonly expressed by most scholars, including the best minds of the XVIIth

century, Christiaan Hyugens and Gotfried Leibniz (Slide 24). The disturbing question was: how the two masses KNOW that they have to attract each other?

Newton answered that criticism in the General Scholium (Slide 21), which was added to the second edition of *Principia*. He clearly stated that his aim was to find and accurately describe HOW the world is organized, and not WHY it is so. His expression: "I frame no hypotheses" (Latin: *hypotheses non fingo*) became a famous motto of the new physics, so different from the physics of Aristotle for whom the principal question was WHY things are such as we perceive them.

Slide 25. The list of subjects which are not discussed in the *Principia*. In the following chapters of the course we shall see how these subjects were gradually introduced into mechanics.

Slides 26 and 27. The argumentation in the *Principia* had been purely geometrical. There were no equations nor other algebraic expressions. This was changed only in the fourth edition which was prepared by two Swiss monks, Thomas Le Suer and Franciscus Jacquier. They added extended commentaries explaining and translating geometrical arguments into the language of formulas. Because of these extended additions the total volume of the fourth edition published in Geneva in 1739-1742 was considerably larger than that of the previous three (see an example in Slide 29).

Slide 30. Newton had been fully aware that his work was only the beginning of the very long road to understand the world.

Slide 33. French philospher and scholar Voltaire was extremely intelligent and eager to learn. In 1726 he traveled to London in order to meet Newton and discuss with him problems of the new physics. Newton, however, was already seriosly ill and the meeting could not be arranged. Voltaire nevertheless wrote a superb book *Elements of Newton's philosophy* in which he explained in simple terms the essential points of Newton's discoveries. The book was translated into several languages and helped dissemination of Newton's ideas. In another of his books, *Lettres philosophiques* (1733) Voltaire explained in a witty way the differences between the physics of Descartes and Newton.

Slide 32. Two quotations summarizing the new approach to the study of nature by Galileo and Newton.