

Newton's *Principia*

Isaac Newton



4 I 1643
(25 XII 1642)

Born in Woolsthorpe

1661

Began studies in Cambridge

1665-1667

Woolsthorpe

1669

Professor of mathematics in Cambridge

1672

New Theory About Light and Colors

Entered Royal Society

1687

Philosophiae Naturalis

Principia Mathematica

1696

Began work in the London mint

1703

President of the Royal Society

1704

Opticks

31 III 1727
(20 III 1727)

Died in Kensington



"In the beginning of the year 1666 I found the method of approximating Series and the Rule for reducing any dignity of any Binomial into such a series. The same year in May I found the method of tangents of Gregory and Slusius, and in November had the direct method of fluxions, and the next year in January had the Theory of colours, and in May following I had entrance into the inverse method of fluxions. And the same year I began to think of gravity extending to the orb of the Moon, and having found out how to estimate the force with which [a] globe revolving within a sphere presses the surface of the sphere, from Kepler's Rule of the periodical times of the Planets being in a sesquialterate proportion of their distances from the centers of their Orbs I deduced that the forces which keep the Planets in their Orbs must [be] reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the earth, and found them answer pretty nearly. All this was in two plague years of 1665 and 1666, for in those years I was in the prime of my age for invention, and minded Mathematics and Philosophy more than at any time since."

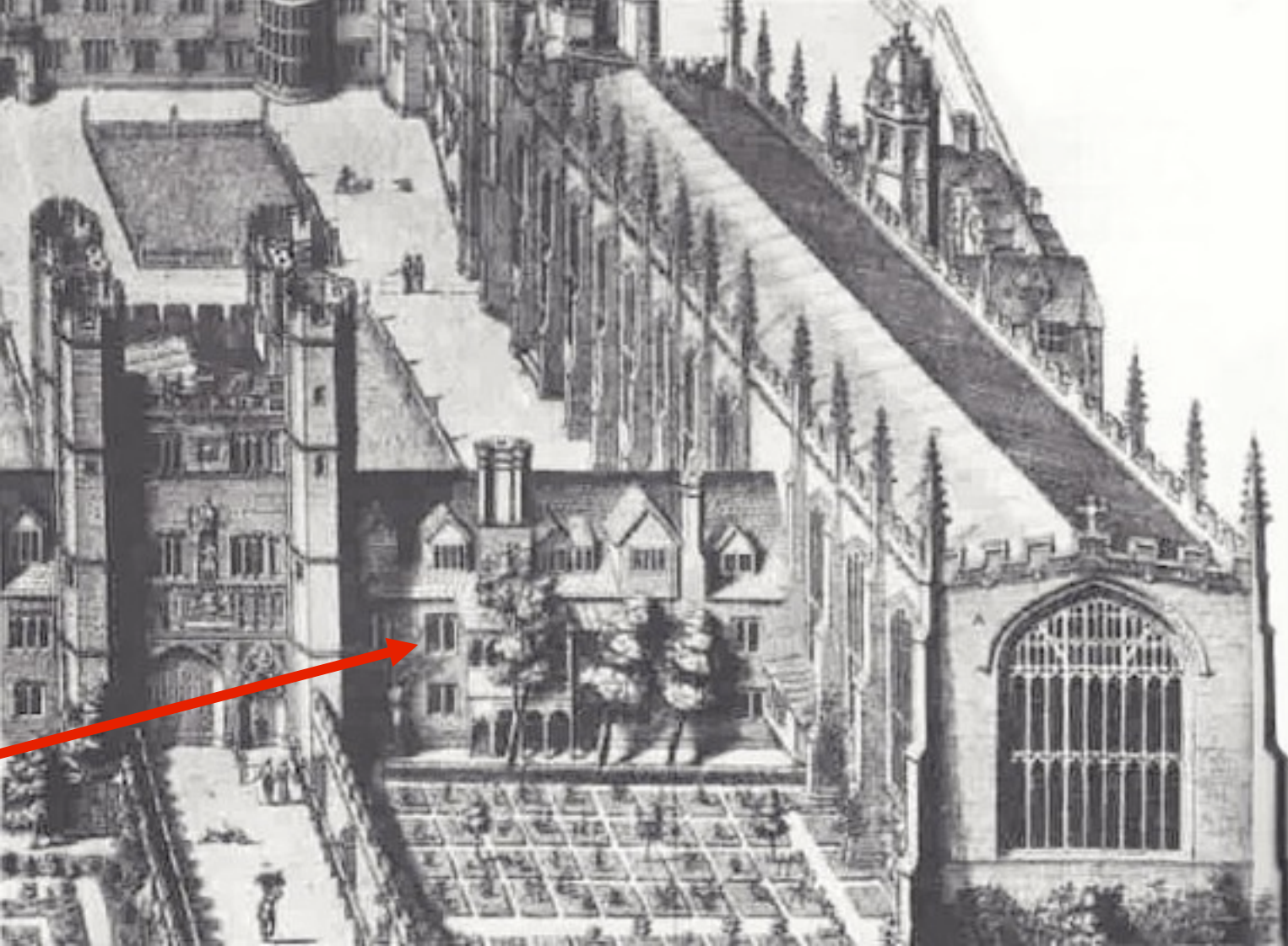


Isaac Newton, writing in 1717 or 1718

Trinity College, Cambridge, in the times of Newton



Trinity College, Cambridge, in the times of Newton



PHILOSOPHICAL TRANSACTIONS.

February 19. 1671.

THE CONTENTS.

A Letter of Mr. Isaac Newton, Mathematick Professor in the University of Cambridge; containing his New Theory about Light and Colors: Where Light is declared to be not Similar or Homogeneous, but consisting of differrant rays, some of which are more refrangible than others: And Colors are affirm'd to be not Qualifications of Light, deriv'd from Refractions of natural Bodies, (as 'tis generally believed;) but Original and Connate properties, which in divers rays are divers: Where several Observations and Experiments are alledged to prove the said Theory. An Account of some Books: I. A Description of the EAST-INDIAN COASTS, MALABAR, COROMANDEL, CEYLON, &c. in Dutch, by Phil. Baldæus. II. Antonii le Grand INSTITUTIO PHILOSOPHIÆ, secundum principia Renati Des-Cartes; novâ methodo adornata & explicata. III. An Essay to the Advancement of MUSICK; by Thomas Salmon M. A. Advertisement about Thæon Smyrnezus. An Index for the Traills of the Year 1671.

A Letter of Mr. Isaac Newton, Professor of the Mathematicks in the University of Cambridge; containing his New Theory about Light and Colors: sent by the Author to the Publisher from Cambridge, Febr. 6. 1671; in order to be communicated to the R. Society.

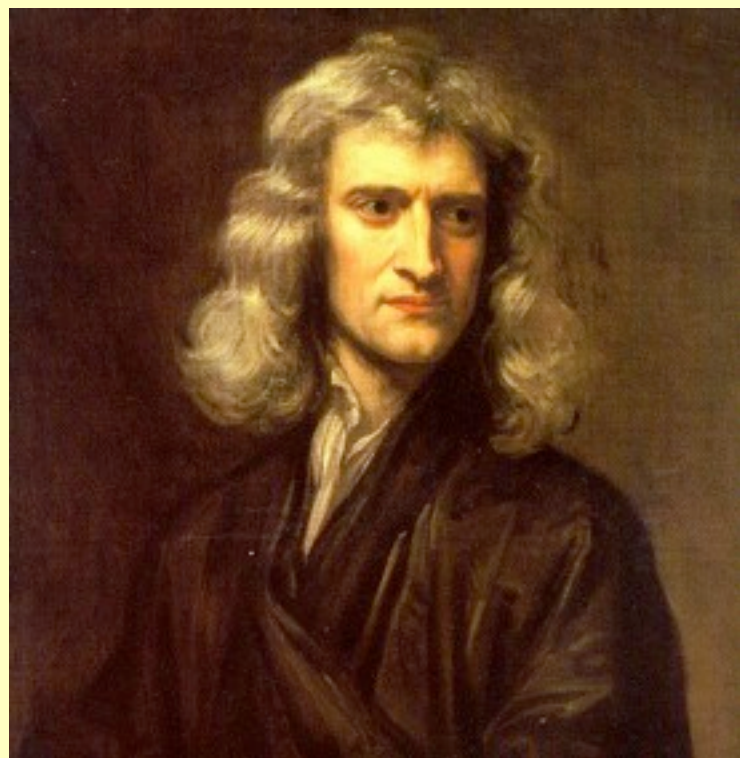
S I R,

TO perform my late promise to you, I shall without further ceremony acquaint you, that in the beginning of the Year 1666 (at which time I applyed my self to the grinding of Optick glasses of other figures than Spherical,) I procured me a Triangular glass-Prisme, to try therewith the celebrated *Phænomena* of

G g g g

Colours.





PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J. S. NEWTON*, *Trin. Coll. Cantab. Soc. Matheseos*
Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc. PRÆSES.*

Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostant Vena-
les apud Sam. Smith ad insignia Principis Walliæ in Cœmiterio
D. Pauli, aliosq; nonnullos Bibliopolas. Anno MDCLXXXVII.

”...I offer this work as the mathematical principles of philosophy, for the whole burden of philosophy seems to consists in this - from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena; and to this end the general propositions in the first and second Books are directed. In the third Book I give an example of this in the explication of the System of the World; for by the propositions mathematically demonstrated in the former Books, in the third I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and several planets. Then from these forces, by other propositions which are also mathematical, I deduce the motions of the planets, the comets, the moon, and the sea...”

Newton's Preface to the first edition of *Principia*

Newton's Preface to *Principia* (cont.)

"I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are either mutually impelled towards one another, and cohere in regular figures, or are repelled and recede from one another. These forces being unknown, philosophers have hitherto attempted the search of Nature in vain; but I hope the principles here laid down will afford some light either to this or some truer method of philosophy..."

Mathematical Principles of Natural Philosophy

DEFINITIONS

Scholium

AXIOMS, or **Laws of Motion**

BOOK I (*DE MOTU CORPORUM*)

14 chapters, 11 lemmas, 50 theorems, 48 problems

BOOK II (*DE MOTU CORPORUM*)

9 chapters, 7 lemmas, 41 theorems, 12 problems

BOOK III (*DE MUNDI SYSTEMATE*)

Regulae philosophandi

11 lemmas, 20 theorems, 22 problems

Scholium generale

Principia, Book III, Introduction

"In the preceding books I have laid down the principles of philosophy; principles not philosophical but mathematical; such namely, as we may build our reasonings upon in philosophical inquiries. These principles are the laws and conditions of certain motions, and powers or forces, which chiefly have respect to philosophy...

It remains that, from the same principles, I now demonstrate the frame of the System of the World. Upon this subject I had, indeed, composed the third book in a popular method, that it might be read by many; but afterwards, considering that such as had not sufficiently entered into the principles could not easily discern the strength of the consequences, nor lay aside the prejudices to which they had been many years accustomed, therefore, to prevent the disputes which might be raised upon such accounts, I chose to reduce the substance of this Book into the form of Propositions (in the mathematical form), which should be read by those only who had first made themselves masters of the principles established in the preceding Books..."



PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

DEFINITIONES.

DEFINITIO I. (²)

*Quantitas Materiae est mensura ejusdem ora ex illius Densitate
& Magnitudine conjunctim.*

¶ ER, densitate duplicata, in spatio etiam duplicato fit
quaduplus; in triplicato sextuplus. Idem intellige de
Nive & Pulveribus per compressionem vel liquefac-
tionem

*Licet primæ definitiones NEWTONIANÆ via aliquam possidere videantur expli-
cationem; in ipso tamen operis nostri hunc, nonnulla leviter momenti præmittenda indi-
cantes, quæ ad majorem viam flectunt. Præmissa quæ in posterum sæpius revertent Me-
chanicæ principia interserere non aliud erit, nisi ut Lectorem saltem præcavimus, cum ut
magis continua seruetur nostrarum demonstrationum series.*

(²) 1. Materia est substantia cinis di-
mensaque prædicta, solida seu incompressi-
bilis, mobilis, divisibilis. Spacium pu-
rum est illa impenetrabile, pene-
trabile, ubique

Definitions

„**Definition I.** The quantity of matter is the measure of the same, arising from its density and bulk conjunctly...

Definition II. The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjunctly ...

Definition III. The *vis insita*, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line....

Definition IV. An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.

This force consists in the action only; and remains no longer in the body, when the action is over...”

Definitions

”**Definition V.** A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre...

Definition VI. The absolute quantity of a centripetal force is the measure of the same, proportional to the efficacy of the cause that propagates it from the centre, through the spaces round about...

Definition VII. The accelerative quantity of a centripetal force is the measure of the same, proportional to the velocity which it generates in a given time....

Definition VIII. The motive quantity of a centripetal force is the measure of the same, proportional to the motion which it generates in a given time. Thus the weight is greater in a greater body, less in a less body; and, in the same body, it is greater near to the earth, and less at remoter distances...”

Scholium (An explanatory note)

"I. Absolute, true, and mathematical time, of itself, and from its own nature flows equably without regard to anything external, and by another name is called duration...

II. Absolute space, in its own nature, without regard to anything external, remains always similar and immovable...

III. Place is a part of space which a body takes up, and is according to the space, either absolute or relative. I say, a part of space; not the situation, nor the external surface of the body. For the places of equal solids are always equal; but their superficies, by reason of their dissimilar figures, are often unequal...

IV. Absolute motion is the translation of a body from one absolute place into another; and relative motion, the translation from one relative place into another..."

AXIOMATA SIVE LEGES MOTUS

Lex. I.

Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

Projectilia perseverant in motibus suis nisi quatenus a resistencia aeris retardantur & vi gravitatis impelluntur deorsum. Trochus, cujus partes cohærendo perpetuo retrahunt sese a motibus rectilineis, non cessat rotari nisi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus suos & progressivos & circulares in spatiis minus resistentibus factos conservant diutius.

Lex. II.

Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimitur.

Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, sive simul & semel, sive gradatim & successive impressa fuerit. Et hic motus quoniam in eandem semper plagam cum vi generatrice determinatur, si corpus antea movebatur, motui ejus vel conspiranti additur, vel contrario subducitur, vel obliquo oblique adjicitur, & cum eo secundum utriusq; determinationem componitur.

Lex. III.

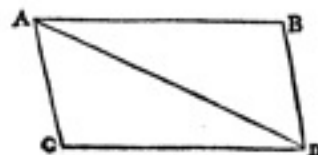
Actioni contrariam semper & æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales & in partes contrarias dirigi.

Quicquid premit vel trahit alterum, tantundem ab eo premitur vel trahitur. Siquis lapidem digito premit, premitur & hujus digitus a lapide. Si equus lapidem funi allegatum trahit, retrahetur etiam & equus æqualiter in lapidem: nam funis utrinq; distensus eodem relaxandi se conatu urgebit Equum versus lapidem, ac lapidem versus equum, tantumq; impedit progressum unius quantum promovet progressum alterius. Si corpus aliquod in corpus aliud impingens, motum ejus vi sua quomodocunq; mutaverit, idem quoque vicissim in motu proprio eandem mutationem in partem contrariam vi alterius (ob æqualitatem pressionis mutue) subibit. His actionibus æquales fiunt mutationes non velocitatum sed motuum, (scilicet in corporibus non aliunde impeditis:) Mutationes enim velocitatum, in contrarias itidem partes factæ, quia motus æqualiter mutantur, sunt corporibus reciproce proportionales.

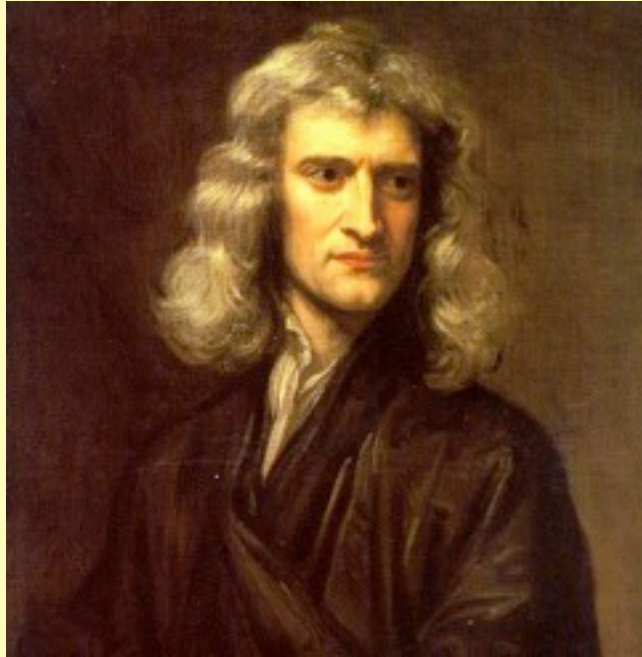
Corol. I.

Corpus viribus conjunctis diagonalem parallelogrammi eodem tempore describere, quo latera separatim.

Si corpus dato tempore, vi sola *M*, ferretur ab *A* ad *B*, & vi sola *N*, ab *A* ad *C*, compleatur parallelogrammum *ABDC*, & vi utraq; feretur id eodem tempore ab *A* ad *D*. Nam quoniam vis *N* agit secundum lineam *AC* ipsi *BD* parallelam, hæc vis nihil mutabit velocitatem accedendi ad lineam illam *BD* a vi altera genitam. Accedet igitur corpus eodem tempore ad lineam *BD* sive vis *N* imprimatur, sive non, atq; adeo in fine illius temporis reperietur alicubi in linea illa *BD*. Eodem argumento in fine temporis ejusdem reperietur alicubi in linea *CD*, & idcirco in utriusq; lineæ concursu *D* reperiri necesse est.



Axioms, or laws of motion



I Law: Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.

II Law: The change of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

III Law: To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.”

Selected problems discussed in *Principia*

Book I: motion of bodies under central and other forces, movement of the line of apsides, attraction by spherical and nonspherical bodies, three-body problem, corpuscular optics

Book II: motion of bodies in resisting media, aerodynamics, hydrodynamics, pendulum, refutation of the Cartesian theory of vortices

Book III: law of universal gravitation, orbits of planets and comets, polar flattening of the earth and planets, perturbed motion of the moon, tidal phenomena, inertial and gravitational mass, precession

Newton's law of gravitation appears in Book III in the form of several statements

Proposition I. Theorem I.

"That the forces by which the circumjovial planets are continually drawn off from rectilinear motions, and retained on their proper orbits, tend to Jupiter's centre; and are inversely as the squares of the distances of those planets from that centre"

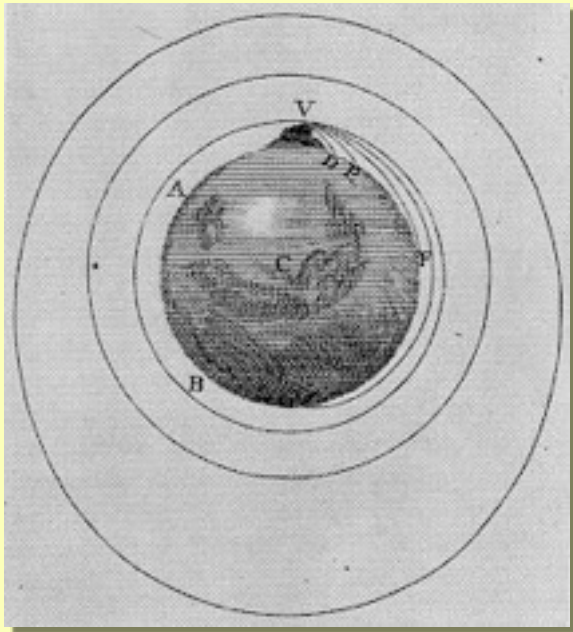
Proposition II. Theorem II.

"That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the sun; and are inversely as the squares of the distances of the places of those planets from the sun's centre."

Proposition VIII. Theorem VII

"That there is a power of gravity pertaining to all bodies, proportional to the several quantities of matter which they contain."

Newton's thought experiment originating the idea of launching artificial satellites of the earth



"A stone that is projected is by the pressure of its own weight forced out of the rectilinear path, which by the initial projection alone it should have pursued, and made to describe a curved line in the air; and through that crooked way is at last brought down to the ground; and the greater the velocity is with which it is projected, the further it goes before it falls to the earth. We may therefore suppose the velocity to be so increased, that it would describe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the earth, till at last, exceeding the limits of the earth, it should pass into space without touching it."

From General Scholium:

"Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the Sun and planets, without suffering the least diminution of its force; that operates...according to the quantity of solid matter which they contain, and propagates its virtue on all sides to immense distances, decreasing always as the inverse square of the distances... But hitherto I have not been able to discover the cause of these properties of gravity from phenomena and **I frame no hypotheses**. Hypotheses...have no place in experimental philosophy...to us it is enough that gravity does really exist...and abundantly serves to account for all the motions of the celestial bodies, and of our sea..."

Anonymous review of *Principia*
in *Journal des Sçavans*, 2 August 1688

"The work of M. Newton is a mechanics, the most perfect that one could imagine, as it is not possible to make demonstrations more precise or more exact than those that he gives in the first two books on lightness, on springiness, on the resistance of fluid bodies, and on the attractive and repulsive forces that are the principal basis of Physics. But one has to confess that one cannot regard these demonstrations otherwise than as only mechanical; indeed, the author recognizes himself at the end of page four and the beginning of page five that he has not considered their Principles as a Physicist, but as a mere Geometer."

Anonymous review of *Principia*
in *Journal des Sçavans*, 2 August 1688 (cont.)

"He confesses the same thing at the beginning of the third book, where he endeavours nevertheless to explain the System of the World. But it is [done] only by hypotheses that are, most of them, arbitrary, and that, consequently, can serve as foundation only to a treatise of pure mechanics. He bases the explanation of the inequality of the tides on the principle that all the planets gravitate reciprocally toward each other...But this supposition is arbitrary as it has not been proved; the demonstration that depends on it can therefore only be mechanics.

In order to make an *opus* as perfect as possible, M. Newton has only to give us a Physics as exact as his Mechanics. He will give it when he substitutes true motions for those that he has supposed."

"Concerning the Cause of the flux given by M. Newton, I am by no means satisfied [by it], nor by all other Theories that he builds upon his Principle of Attraction, which to me seems absurd... And I have often wondered how he could have given himself all the trouble of making such a number of investigations and difficult calculations that have no other foundation than this very principle."

Huygens in a letter to Leibniz (18 XI 1690)



What one does not find in *Principia*?

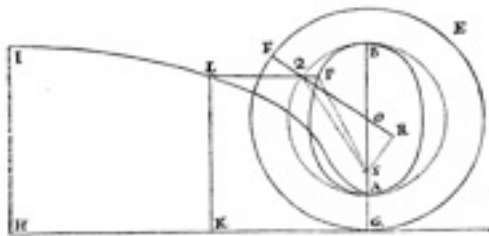
The argumentation in *Principia* is entirely geometrical.

There are no Newton's equations of motions, nor any other equations of mechanics, hydrodynamics and aerodynamics. There are no considerations of energy (the principle of conservation of energy was still unknown), there are no vectors, no definition of velocity and acceleration in the present sense.

Purely geometric argumentation in *Principia*

[108]

ad OA ut OA ad OS . Erige perpendiculum GH , centroq; O & intervallo OG describe circulum EFG , & super regula GH , cefundo, progrediatur rota GEF revolvendo circa axem suum, & interea puncto suo A describendo Trochoidem ALL . Quo facto, cape GK in ratione ad rotæ perimetrum $GEFG$, ut est tempus quo corpus progrediendo ab A descripsit arcum AP , ad tempus



revolutionis unius in Ellipti. Erigatur perpendiculum KL occurrens Trochoidi in L , & acta LP ipsi KG parallela occurret Ellipti in corporis loco quaesito P .

Nam centro O , intervallo OA describatur semicirculus AQB , & arcui AQ occurrat LP producta in Q , junganturq; SQ , OQ . Arcui EFG occurrat OQ in F , & in eandem OQ demittatur perpendiculum SR . Area APS est ut area AQS , id est, ut differentia inter sectorem OQA & triangulum OQS , sive ut differentia rectangulorum $OQ \times AQ$ & $OQ \times SR$, hoc est, ob datam OQ , ut differentia inter arcum AQ & rectam SR , adeoque (ob aequalitatem rationum SR ad finem arcus AQ , OS ad OA , OA ad OG , AQ ad GF , & divisim $AQ - SR$ ad $GF - fin. arc. AQ$) ut GK differentia inter arcum GF & finem arcus AQ . *Q. E. D.*

Sch.

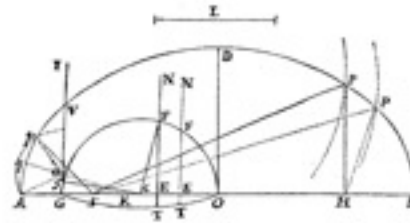
[109]

Solutio.

Ceterum ob difficultatem describendi hanc curvam praestat constructiones vero proximas in praxi Mechanica adhibere. Ellipseos cujusvis APB sit AB axis major, O centrum, S umbilicus, OD semiaxis minor, & AK dimidium lateris recti. Secetur AS in G , ut sit AG ad AS ut BO ad BS ; & quadratur longitudo L , quæ sit ad $\frac{1}{2}GK$ ut est AO quad. ad rectangulum $AS \times OD$. Bifecetur OG in C , centroq; C & intervallo CG describatur semicirculus GFO . Deniq; capiatur angulus GCF in ea ratione ad angulos quatuor rectos, quam habet tempus datum, quo corpus descripsit arcum quaesitum AP , ad tempus periodicum seu revolutionis unius in Ellipti.

Ad AO demittatur normalis FE , & producat eadem versus F ad usq; N , ut sit EN ad longitudinem L , ut anguli illius sinus EF ad radium CF ; centroq; N & intervallo AN descriptus circulus secabit Elliptin in corporis loco quaesito P quam proxime.

Nam completo dimidio temporis periodici, corpus P semper reperietur in Apfide summa B , & completo altero temporis dimidio, redibit ad Apfidem imam, ut oportet. Ubi vero proxime abest ab Apfidibus, ratio prima nascentium sectorum ASP , GCF , & ratio ultima evanescentium BSP & OCF , eadem est rationi Ellipseos totius ad circulum totum. Nam punctis

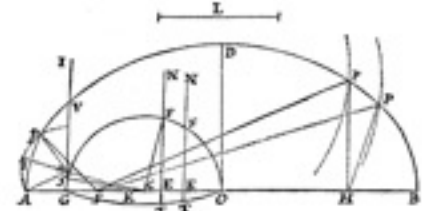


[110]

P , F & N incidentibus in loca p , f & n axi AB quam proximis; ob aequales An , $p n$, recta nq , quæ ad arcum Ap perpendicularis est, adeoque concurret cum axe in puncto K , bifecat arcum Ap . Proinde est: Ap ad Gn ut AK ad GK , & Ap ad Gn ut $\frac{1}{2}AK$ ad GK . Est & Gn ad Gf ut EN ad EF , seu L ad CF , id est, ut $\frac{GK \times AO q.}{2AS \times OD} \frac{2d}{CF}$, seu $GK \times AO q. \frac{2d}{2AS \times OD \times CF}$,

& ex æquo Ap ad Gf ut $\frac{1}{2}AK$ ad $GK + GK \times AO q. \frac{2d}{2AS \times OD \times CF}$, id est, ut $AK \times AO q. \frac{2d}{2AS \times OD \times CF}$, hoc est, ob aequalia $AK \times AO$ & $OD q.$ ut $AO \times OD$ ad $AS \times CF$. Proinde $Ap \times \frac{1}{2}AS$ est ad $Gf \times \frac{1}{2}GC$ ut $AO \times OD$ ad $AS \times CF \times GC$, seu $AO \times OD$ ad $CG q.$ id est, sector nascenti AS p ad sectorem nascentem GCF ut $AO \times OD$ ad $CG q.$ & propterea ut area Ellipseos totius ad aream circuli totius. *Q. E. D.* Argumento prolixiore probari potest analogia ultima in Sectoribus evanescentibus BSP , OCF : ideoq; locus puncti P prope Apfides satis accurate inventus est. In quadraturis error quasi quingentesima pars arcus Ellipseos totius vel paulo major obvenire solet: qui tamen propetudum evanescet per ulteriorem Constructionem sequentem.

Per puncta G , O , duc arcum circulem GTO justæ magnitudinis; dein produc EF hinc inde ad T & N ut sit EN ad FT ut $\frac{1}{2}L$ ad CF ; centroq; N & intervallo AN describe circulum qui secet Elliptin in P , ut supra. Arcus autem GTO determinatur quæ-



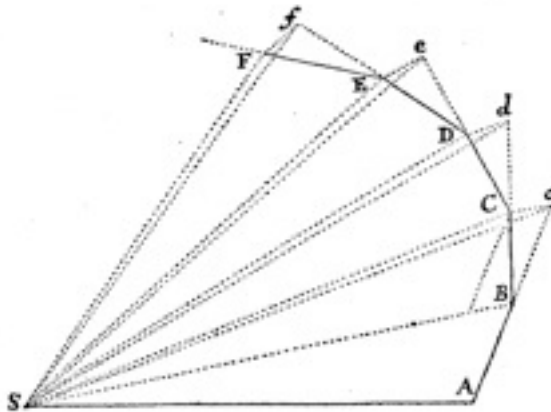
S E C T. II.

De Inventione Virium Centripetarum.

Prop. I. Theorema. I.

Areas quas corpora in gyros acta radiis ad immobile centrum virium ductis describunt, & in planis immobilibus consistere, & esse temporibus proportionales.

Dividatur tempus in partes æquales, & prima temporis parte describat corpus vi insita rectam AB . Idem secunda temporis parte, si nil impediret, recta pergeret ad c , (per Leg. I) describens lineam Bc æqualem ipsi AB , adeo ut radiis AS , BS , cS ad centrum actis, confectæ forent æquales areæ ASB , $BS c$. Verum ubi corpus venit ad B , agat vis centripeta impulsu unico sed magno, faciatq; corpus a recta Bc deflectere & pergere in recta BC . Ipsi BS parallela agatur cC occurrens BC in C , & completa secunda temporis parte, corpus (per Legum Corol. 1) reperietur in C , in eodem plano cum triangulo ASB . Junge SC , & triangulum SBC , ob parallelas SB , Cc , æquale erit triangulo $S B c$, atq; adeo etiam triangulo SAB . Simili argumento si

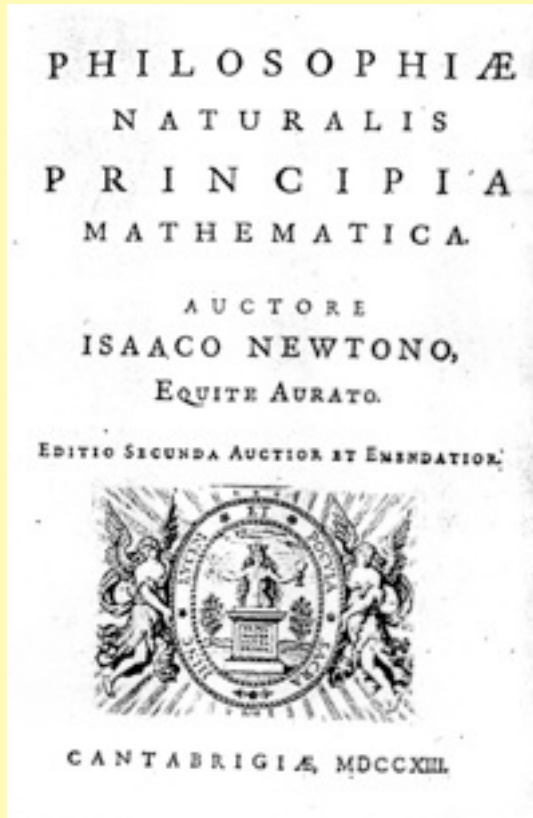


vis

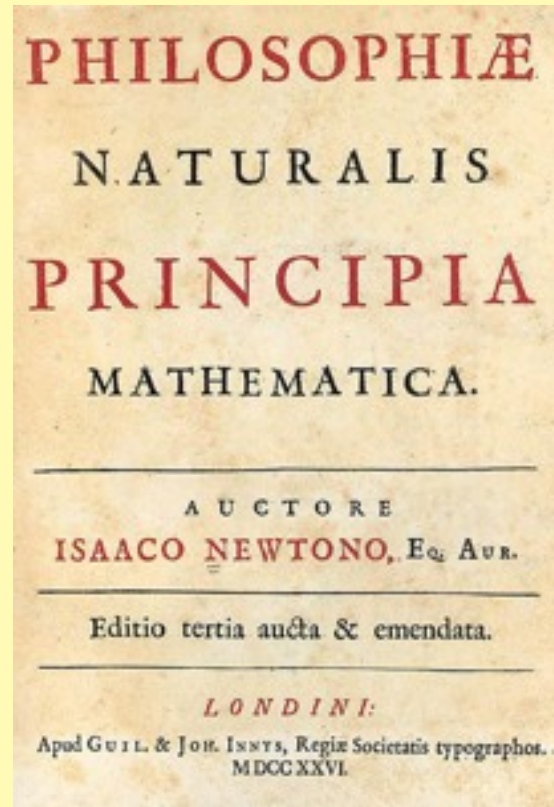
An example of geometric argumentation in *Principia* Book I

The proof of Kepler's law of areas

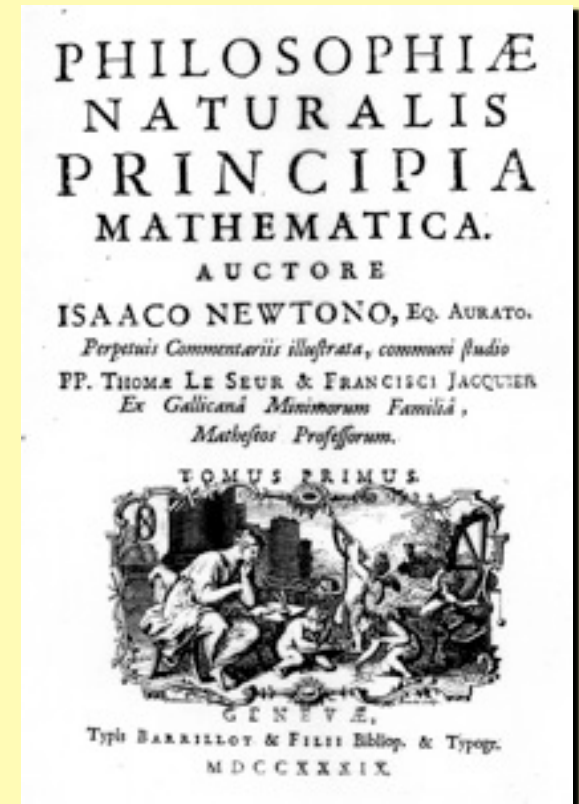
Subsequent editions of *Principia*



Cambridge 1713



London 1726



Geneva 1739

Newton's Principia

[104]

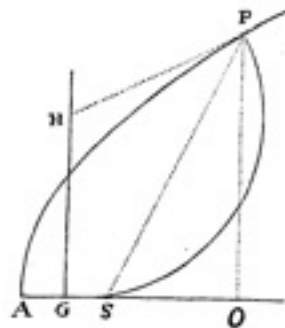
SECT. VI.

De inventione motuum in Orbibus datis.

Prop. XXX. Prob. XXII.

Corporis in data Trajectoria Parabolica moventis, invenire locum ad tempus assignatum.

Sit S umbilicus & A vertex principalis Parabolæ, sitq; $4AS \times M$ area Parabolica APS , quæ radio SP , vel post excessum corporis de vertice descripta fuit, vel ante appulsum ejus ad verticem describenda est. Innotescit area illa ex tempore ipsi proportionali. Bisecca AS in G , erigeq; perpendicularum GH æquale $3M$, & circulus centro H , intervallo HS descriptus secabit Parabolam in loco qua sito P . Nam demissa ad axem perpendiculari PO , est $HGq. + GSq. (= HSq. = G$



$Oq. + HG \times PO + POq. = GOq. + HGq. - 2HG \times PO + POq.$ Et delecto utrinq; $HGq.$ fiet $GSq. = GOq. - 2HG \times PO + POq.$ seu $2HG \times PO (= GOq. + POq. - GSq. = AOq. - 2GAO + POq.) = AOq. + \frac{1}{2}POq.$ Pro $AOq.$ scribe $AO \times \frac{POq.}{4AS}$, & applicatis terminis omnibus ad $3PO$, ductisq; in $2AS$, fiet $\frac{1}{2}GH \times AS (= \frac{1}{2}AO \times PO + \frac{1}{2}AS \times PO = \frac{AO + 3AS}{6} \times PO = \frac{4AO - 3SO}{6} \times PO =$ area $APQ - SPQ =$ area APS . Sed GH erat $3M$, & inde $\frac{1}{2}HG$

PRINCIPIA MATHEMATICA.

259

LIBER PRIMUS.

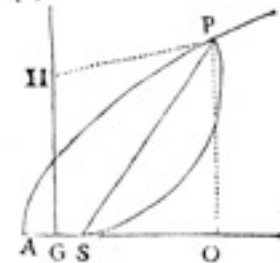
SECTIO VI.

De inventione motuum in orbibus datis.

PROPOSITIO XXX. PROBLEMA XXII.

Corporis in datâ trajectoriâ parabolicâ moti invenire locum ad tempus assignatum. (1)

(1) Sit S umbilicus & A vertex principalis parabolæ, sitque $4AS \times M$ æquale areae parabolicae abscindendæ APS , quæ radio SP , vel post excessum corporis de vertice descripta fuit, vel ante appulsum ejus ad verticem def-



rectæ Bh , parallela, ob similia triângula fgh , fAP erit $fg:hg = Af:PA$ ob sim. tri. fgi , fAQ $gi:fg = QA:Af$ ob sim. tri. ghi , AKM $hg:gi = AK:AM$ ob sim. tri. AKL , PAR $AK:AL = PA:PR$ ob sim. tri. fQi , fPh $fh:fi = Ph:Qh$ sed ob sim. tri. fhi , ALN , $fh:fi = AL:AN$, ergo $AL:AN = Ph:Qi$ & $AL:AN = Ph:AL:Qi:AN$ & quia $AL = Rh$ & $AL:AN = PR:Qi - AN$ unde per compositionem rationum & ex æquo, $AK:AN = QA \times AK:AM \times (Qi - AN)$ quare $AK \times AM:AN \times AM = QA \times AK:AM \times Qi - AN$, ac proinde $AM:AN = QA:Qi - AN$, atque $AM:AN = QM$ seu $QA + AM:Qi$ seu $Qi = AN + AN$. Quosiam igitur rectæ AN , Qi , sunt parallelae (per constr.) patet puncta M , N , esse in una rectâ, atque hæc est prima pars constructionis Newtonianæ quæ erat demonstranda.

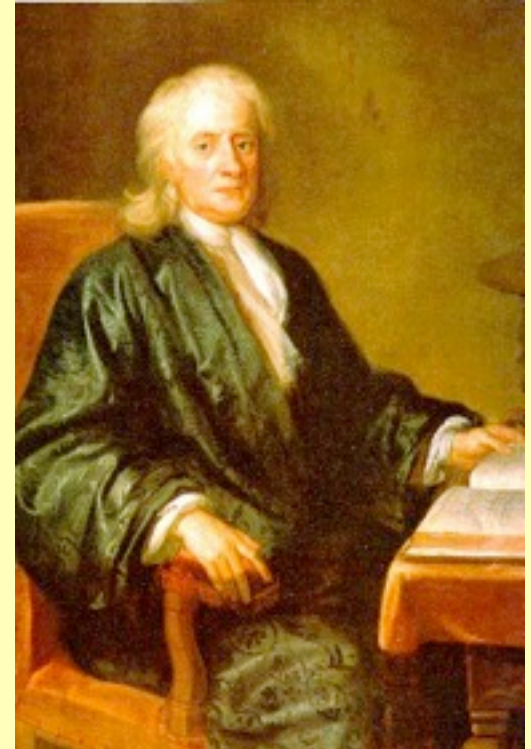
2. illius pars facile ostenditur. Nam (vid. fig. Newt.) punctâ Pi , erit (per constr.) triângulum PiE , super rectâ Ei constructum simile triângulo fIg , ad eundem angulum i & g . ductæ sunt ex puncto Pi rectæ Ei , Eg ; quare (356), si per

punctum P agatur recta PQ , quæ cum rectâ Eg , continet angulum PQE æqualem angulo fIg , recta illa PQ , producta tanget arcum fE triânguli fIg , seu trapezii $fghi$. $Q. e. D.$

(1) 338. Newtonus in hac totâ sectione supponit corpus in trajectoriâ conicâ datâ ita moveri, ut radiis ad trajectoriæ umbilicum ductis areas seu sectores describat temporibus proportionales; eâ enim lege planetæ omnes in orbitis conicis revolvuntur ex phenomenis lib. 1. ostendit. Præterea supponit novum esse tempus quo corpus ex puncto trajectoriæ dato v. g. ex vertice illius principali ad aliud ejusdem trajectoriæ punctum datum pervenit, durante esse aream seu trajectoriæ sectorem huic tempori correspondentem, atque ex his datis quaerit locum mobilem in trajectoriâ ad aliud quodvis tempus datum, quæ contra quaerit tempus quo mobile datum quodvis trajectoriæ punctum attingit; nam cum sint areae temporibus proportionales, dato tempore quovis, data area hoc tempore descripta, & vicissim data area descripta datur tempus quo describitur.

(2) * Sit S umbilicus, & A vertex principalis parabolæ, durante sit tempus quo

"I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than arbitrary, whilst the great ocean of truth lay all undiscovered before me."





"A Frenchman who arrives in London finds a great change in philosophy, as in everything else. He left the world full, he finds it empty. In Paris one sees the Universe composed of vortices of subtle matter. In London one sees nothing of it. In Paris it is the pressure of the moon that causes the flux of the sea; in England it is the sea that gravitates toward the moon.

With your Carthesians, everything is done by an impulsion that nobody understands; with Mr. Newton, it is by an attraction, the cause of which is not better known."

Voltaire, *Lettres philosophiques*



"Philosophy is written in that grand book – I mean the universe – which stands forever open before our eyes; but it cannot be read until we have learnt to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to comprehend a single word of it; without these, one is wandering about in a dark labyrinth."

Hypotheses non fingo...



Additional explanatory slide

CALENDARIVM GREGORIANVM PERPETVVM.

Orbi Christiano vniuerso à GREGORIO XIII. P. M. pro-
positum. ANNO M. D. LXXXII.

GREGORIVS EPISCOPVS SERVVS SERVORVM DEI AD PERPETVAM REI MEMORIAM.

NTE si graviorum pastoralis officij nostri curas, et postrema monit. ut quæ sa-
cta Traditione Cunctis Sæculis Apostolica referata sunt, illa ad suum optatum, Uer-
aditatem perducatur. Sane cunctis Censibus Patres, cum ad reliquum negotio-
rum breuitatem quæque curam aduenerint, tempore tamen exiliis rem latam ex-
ipsum Censibus decreta ad iustitiam & iudicium Romani Pontificis reuoluerunt.
Duo autem de ceteris præcipue considerantur, quoniam etiam preter, laudatque diui-
na scripta præf. sique debet per fiduciam compleretur, etiam persister ad annos
solus, sed etiam quæ ex pendunt non retuler, sed, et Luce mutumetendos: Atque illud quidem
solus recordatus vniuersi pro brevior noster absoluentis curam, atque edidit. Hoc vero, quod no-
uorum ceteris legimus Calendarij reformationem, et modum à Romanis Pontificibus predecessores no-
stros, et seipsum testatur est, verum absoluit, et ad exitum perducit ad hoc usque tempus non potui, quod
virescentes emendandi Calendarij, quæ a cunctis mutam peritis præparabantur, propter magnas, et
fere intractabiles difficultates, quæ huiusmodi commendatio semper habuit, neque perennes erant, neque
antiqui Pastores rectos inciderant (quod in primis his in re curadatum erat) seruabant. Tum
magis nos quædam, credidimus, huiusmodi, à Deo dispensatione freti, in hac cogitatione, curaque
perseruum, aduenit nobis liber à dilecto filio Antonio, et medicus doctore, quæ quæ-
dam Alapianus iam germanus frater conscripserat, in qua per nouum quendam Cyclum Calendarij
exemplum, et ad certum usque numeri numerum diuisionem, atque ad quancunque anni solaria
mutatam diuisionem accedendum, omnia, quæ in Calendario colligenda sunt, constantiatum, et scilicet
nouum diuisionem, si rectius possiderent, ut Calendarium ipsum nulli iniquam mutationem in poste-
riorem ceteris esse videretur. Nouum hanc reformationem Calendarij rationem, etiam volumus com-
prehensum ad Censibus, Principes, ecclesiasticosque vniuersitates prius ante annos missimus, et res,
quæ antea in ceteris erant, cum omni et conuenientia consilio perseruer. Illi cum, quæ maxime opta-
bamus, ceteris de reformatione, curam nos omnino consensum adhiberi, curas ad Calendarij emen-
dationem adhiberemus in illius Verba harum rerum permissimus, quæ longe ante ex primariis Christianis
arbitrationibus discrepamus: In enim multum tempore, et diligenter ad eam faculationem addi-
bant, et Cyclum veterem, quoniam recentiorum vniuersis conscriptis, et diligentissime perpesus
inter se contulerunt, seu, et doctorem huiusmodi, qui de ea rescripserant, iudicio bonis pro ceteris eleger-
unt Episcopus Cyclum, cum nonnulli etiam adiecerunt, quæ ex accurata et coniectura visus ad
Calendarij perfectionem maxime pertinere.

Cyclus. Epact. An. Cor. 1582.	Litteræ Domi- cal.	Dies Men- sis.	OCTOBER. Cui defunt decem dies pro Correctionis Anni Solaris.
xxj	A	Kal.	1 Remigij Episcopi & Confes.
xxj	b	vj	2
xx	c	v	3
xix	d	4. No.	4 Francisci Confes. dupl.
viii	A	Idus	15 Dionysij, Rustici, & Eleuthe- rij mart. semi. cum com. S. Marci Papæ & confes. & SS. Sergij, Bachi, Marcelli, & Apuleij martyr.
vii	b	xvij	16 Calisti Papæ & mar. semid.
vj	c	xvj	17
v	d	xv	18 Lucæ Euangelistæ. dupl.
iiii	e	xiiii	19
iii	f	xiii	20
ii	g	xij	21 Hilarionis Abbatis. & com. SS. Vrsulæ & soc. Virgin. & martyr.
i	A	xj	22
30	b	x	23
29	c	ix	24
28	d	viii	25 Chrysanti & Dariz martyx.
27	e	vij	26 Euaristi Papæ & marty.
26	f	vj	27 Vigilia.
25	g	v	28 Simonis & Iudæ Apostolo- rum. duplex.
24	A	iiii	29
23	b	iii	30
22	c	ii	31 Vigilia.

NOVEM