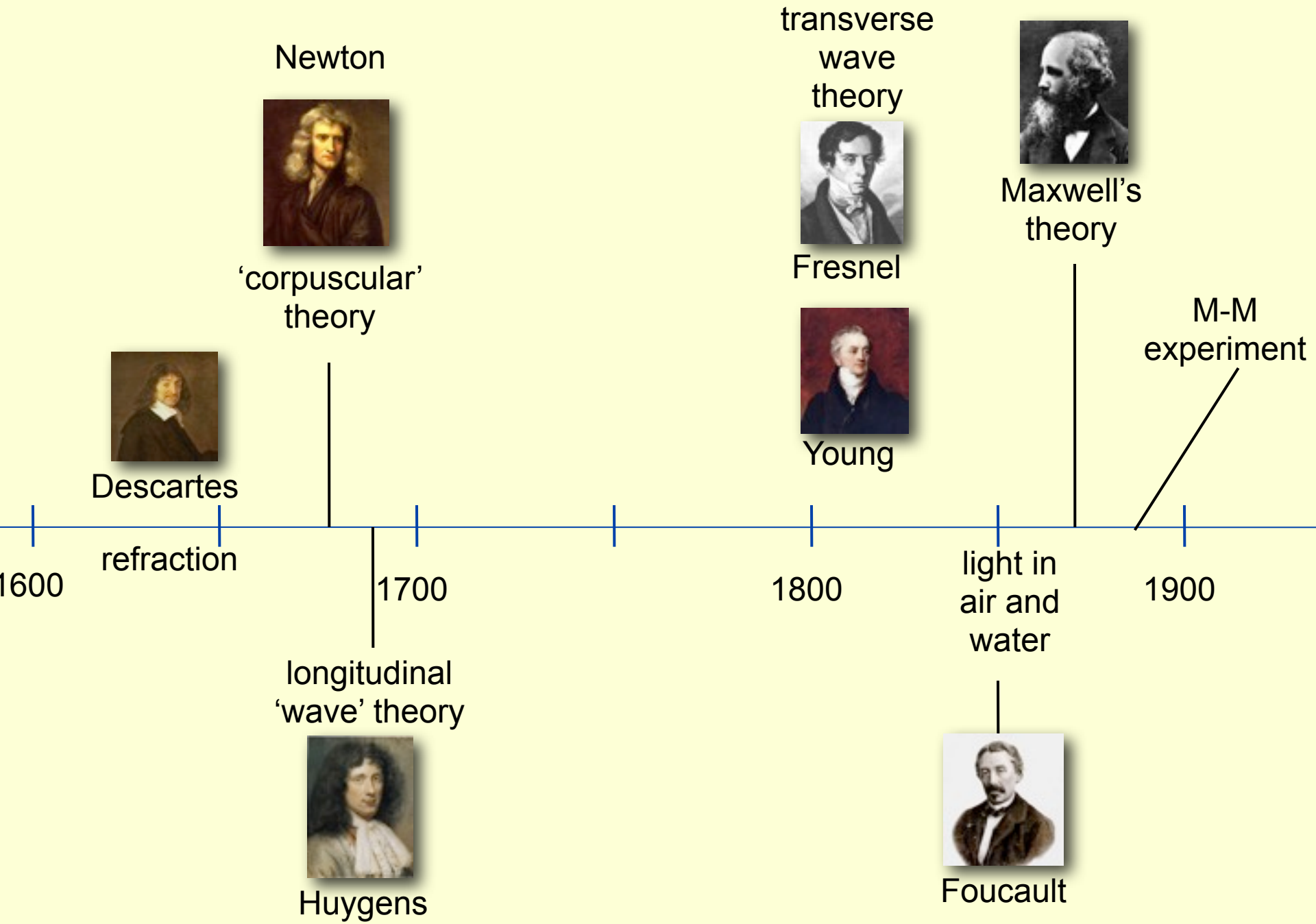
Optics

from Kepler to Newton


Newton
transverse wave theory
Maxwell's theory
'corpuscular' theory
Fresnel
M-M experiment
Descartes
Young
refraction
1600
1700
1800
light in air and water
1900
longitudinal 'wave' theory
Huygens
Foucault

Kepler's unsuccessful attempts to find the law of refraction of light (1604)

$$\alpha - \beta = n \sec \alpha$$

$$2\,\alpha - \beta = n \sin \alpha$$

$$\tan \alpha = n \sin (\alpha - \beta)$$

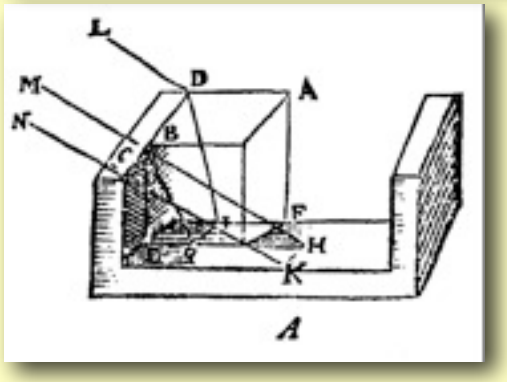
$$1 - \tan \alpha \cot (\alpha - \beta) = n \tan \alpha$$

$$1 - \tan \alpha \cot (\alpha - \beta) = n \sin \alpha$$

$$1 - \beta = n_1 + n_2 \sec \alpha$$

$$1 - \tan \alpha \cot (\alpha - \beta) = n_1 + n_2 \sin \alpha$$


AD VITELLIONEM
PARALIPOMENA,
Quibus
ASTRONOMIÆ
PARS OPTICA
TRADITVR:
Potissimùm
DE ARTIFICIOSA OBSERVATIONE ET ÆSTIMATIONE DIAMETRORVM deliquiorumq; Solis & Lunæ.
CVM EXEMPLIS INSIGNIVM ECLIPSIVM.
Habes hoc libro, Lector, inter alia multa nova,
Tractatum lucubratum de modo visionis, & humorum oculi vsu contra Opticos & Anatomicos,
AVTHORE
IOANNE KEPLERO, S. C. M
Mathematico.
FRANCOFVRTI,
Apud Claudium Marnium & Hæredes Ioannis Aubrii
Anno M. DCIV.
Cum Privilegio S. C. Maiestatis.



IOANNIS KEPLERI
Sr Cz. Mtis. MATHEMATICI

DIOPTRICE

SEV

Demonstratio eorum quæ visui & visibilibus propter Conspicillanon ita pridem inventa accidunt.

Præmissa Epistola Galilæi de iis, quæ post editionem Nuncij sibi ope Perspicilli, nova & admiranda in cælo deprehensa sunt.

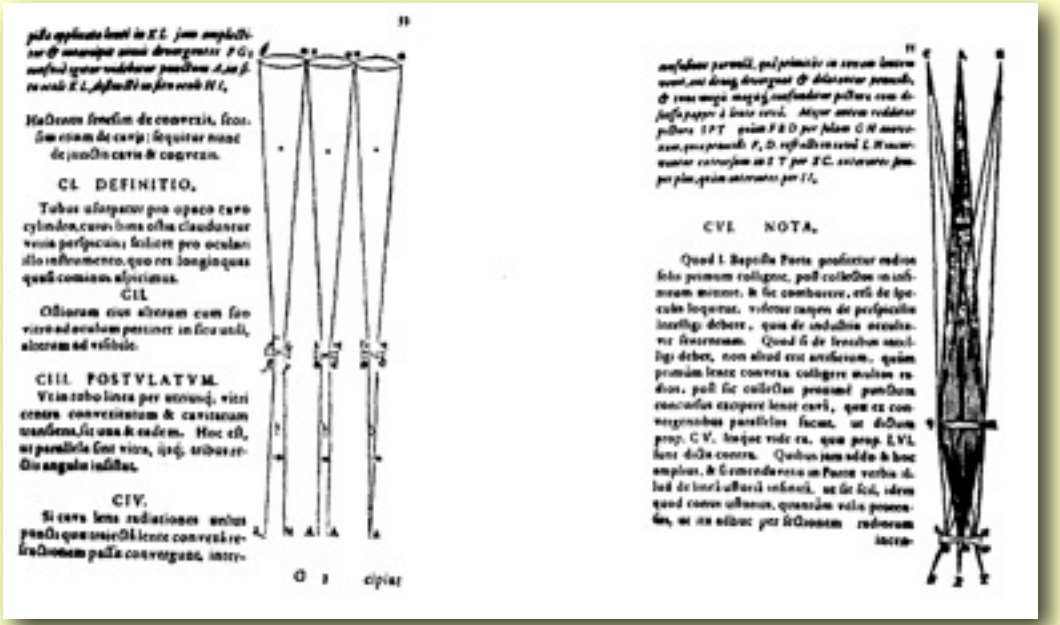
Item.

Examen præfationis Ioannis Pena Galli in Optica Euclidis, de usu Opticæ in philosophia.

AVGVSTAE VINDELICORVM,
typis Davidis Franci.

Cum priuilegio Cæsareo ad annos XV.

M. D CXI.



Argumentation without the law of refraction

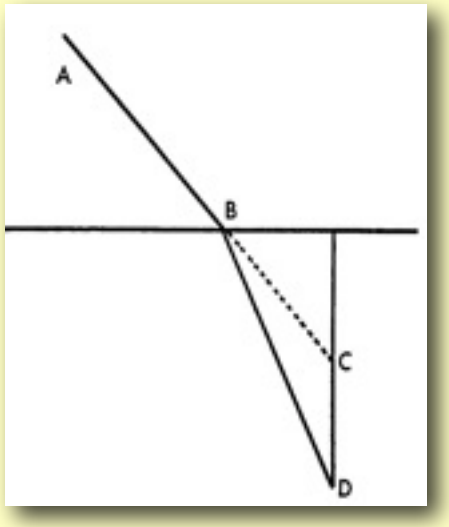
The correct relation $\sin \alpha = n \sin \beta$ found !

Thomas Harriot (< 1610)

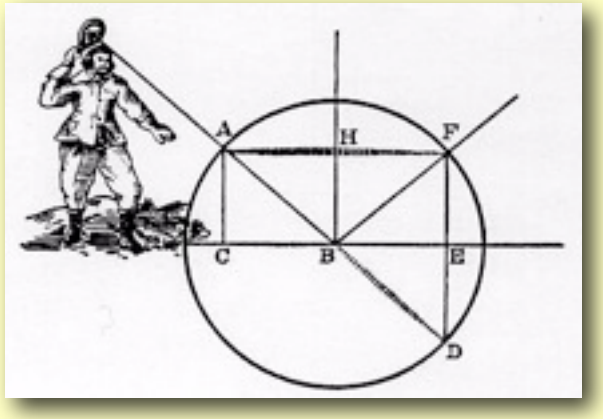
Willebrord Snell (1621)

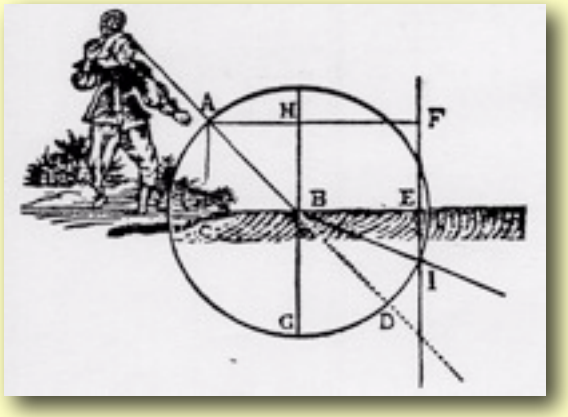
René Descartes (1637)

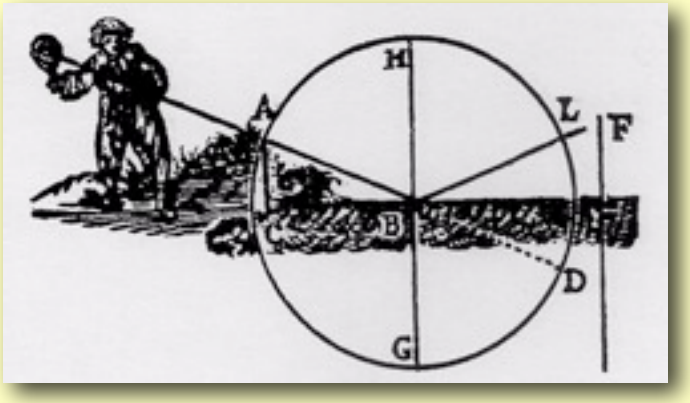
not published!



BC/BD = constans

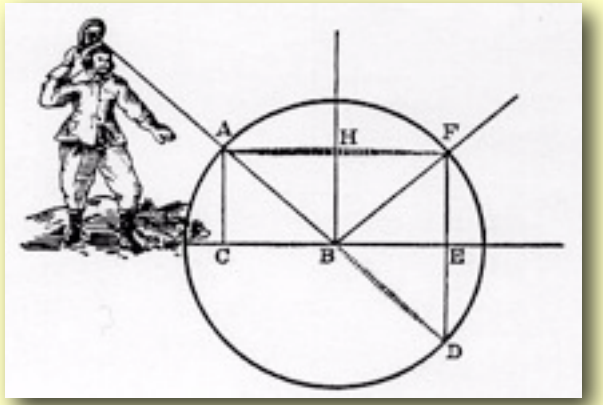


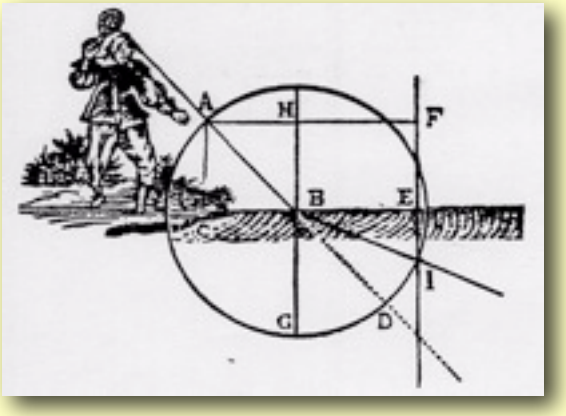


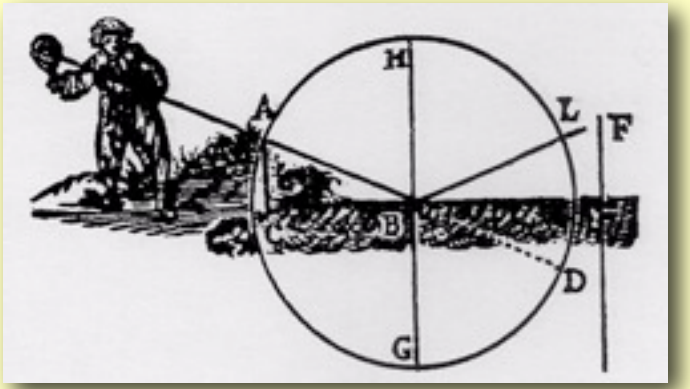


”We suppose that a ball driven from A to B, encounters at the point B, not the surface of the earth, but a cloth CBE, which is so weak and so thin that the ball can break it and pass entirely through it, losing only a part of its velocity, for example, half of it. Now, this being supposed, in order to determine what path it should follow, we notice first that its motion differs entirely from its tendency to move in one direction rather than in another; from which it follows that the quantities of these motions should be considered separately.”

Descartes, *La Dioptrique*

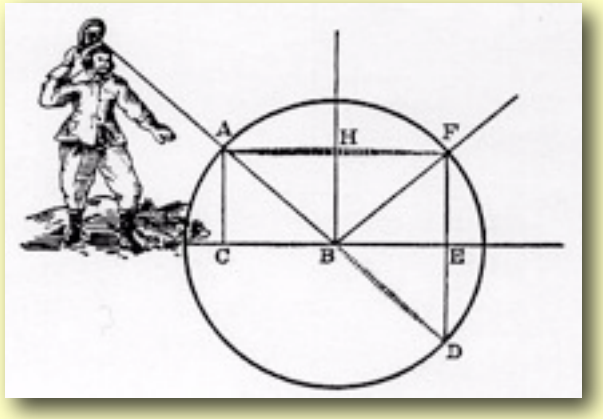


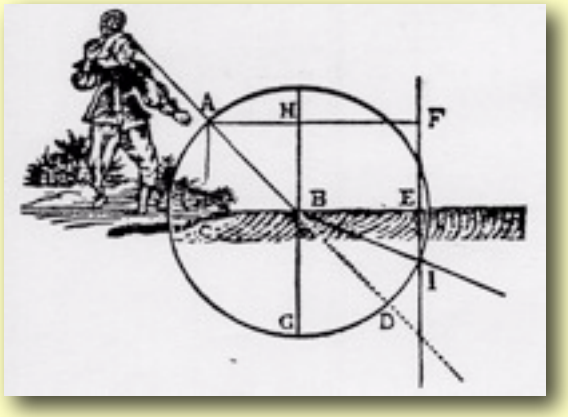


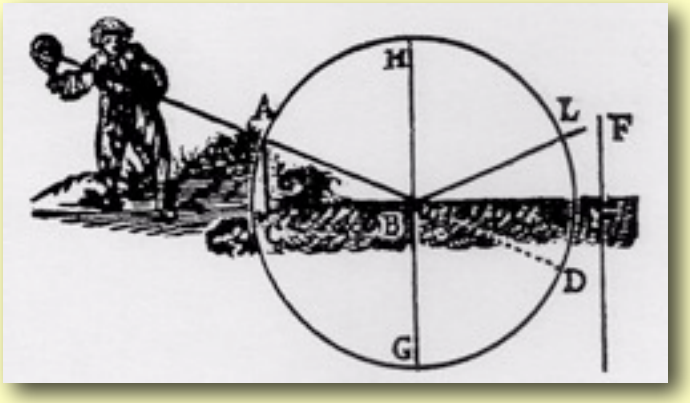


”And we notice that of the two parts of which we may imagine that this tendency is compounded, only that part which would make the ball move from above downward can be changed in any way by encountering the cloth; and that the tendency which made it move toward the right should always remain the same as it has been, because the cloth is in no way opposed to it in that sense.”

Descartes, *La Dioptrique*





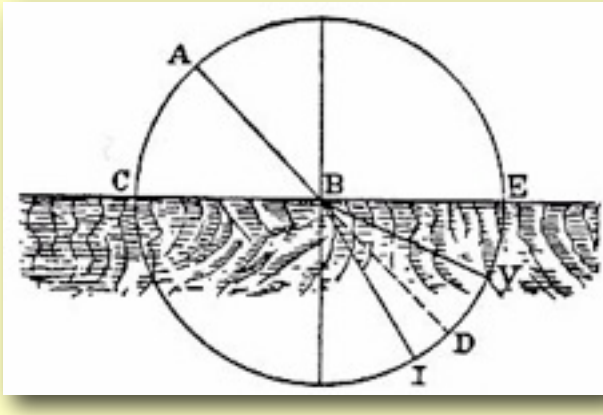


”Then having described from the centre B the circle AFD and drawn at right angles to CBE the three straight lines AC, HB, FE, in such a way that there is twice as much distance between FE and HB as between HB and AC, we see that the ball ought to move toward the point I. For since it loses half of its velocity when it passes through the cloth CDE, it ought to take twice as long to move downward from B to some point of the circumference of the circle AFD as it has taken above it to pass from A to B...”

Descartes, *La Dioptrique*

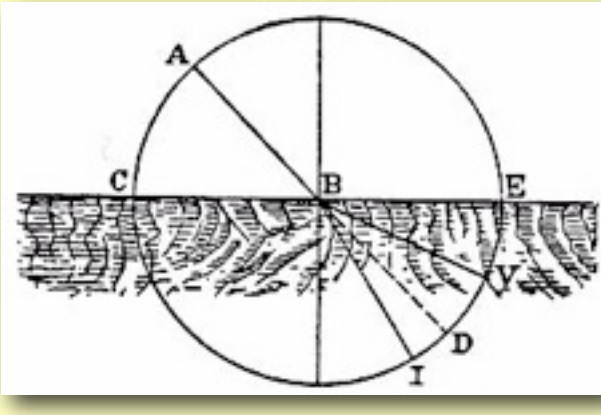
”...To conclude, inasmuch as the action of light follows in this respect the same laws as the motion of the ball, we must say that when its rays pass obliquely from one transparent body into another, which receives them more or less easily than the first body, they turn in such a way that they are always less inclined to the surface separating these bodies on the side where that body is which receives them more easily than on the side where the other body is, and this is just in proportion to that which receives them more easily than the other does.”

Descartes, *La Dioptrique*



”...If a ball, which is driven in air from A to B, encounters the surface of water CBE at the point B, it will be deflected from B toward V; and if it is a ray of light it will go on the contrary from B toward I. You will cease, however, to find this a strange effect, if you recall the nature that I have attributed to light, when I said that it is nothing other than a certain motion or an action conceived in a very subtle matter, which fills the pores of all other bodies; and when you consider that as a ball loses more of its motion when it strikes against a soft body than against a hard one, and that it rolls less easily on a table-cloth than on a bare table;”

Descartes, *La Dioptrique*

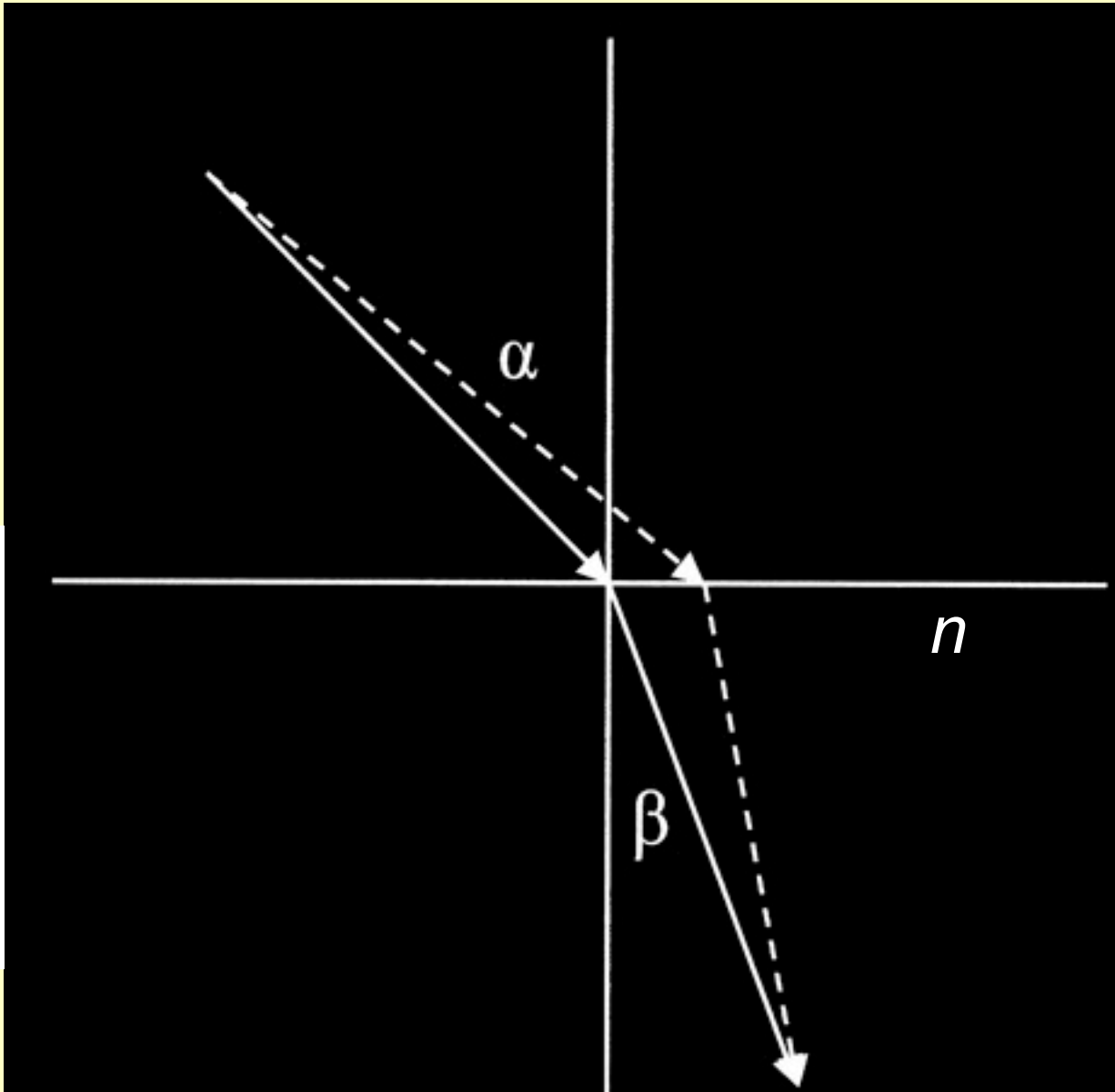


”so the action of this subtle matter may be much more constrained by the parts of the air which, being as they are soft and loosely joined together, do not offer much resistance to it than by the parts of the water, which offer more resistance, and still more by the parts of the water than by those of glass or crystal. Thus it happens that so much as the small parts of a transparent body are harder and firmer so much the more do they allow the light to pass more easily; for the light should not drive any of them out of their places, as a ball ought to drive out the parts of the water to find passage among them.”

Descartes, *La Dioptrique*

Pierre Fermat

The law of refraction
of light
$\sin \alpha = n \sin \beta$
is a consequence
of the principle
of the shortest time



René Descartes

DISCOURS

DE LA METHODE

Pour bien conduire sa raison, & chercher la verité dans les sciences.

PLUS

LA DIOPTRIQVE.

LES METEORES.

ET

LA GEOMETRIE.

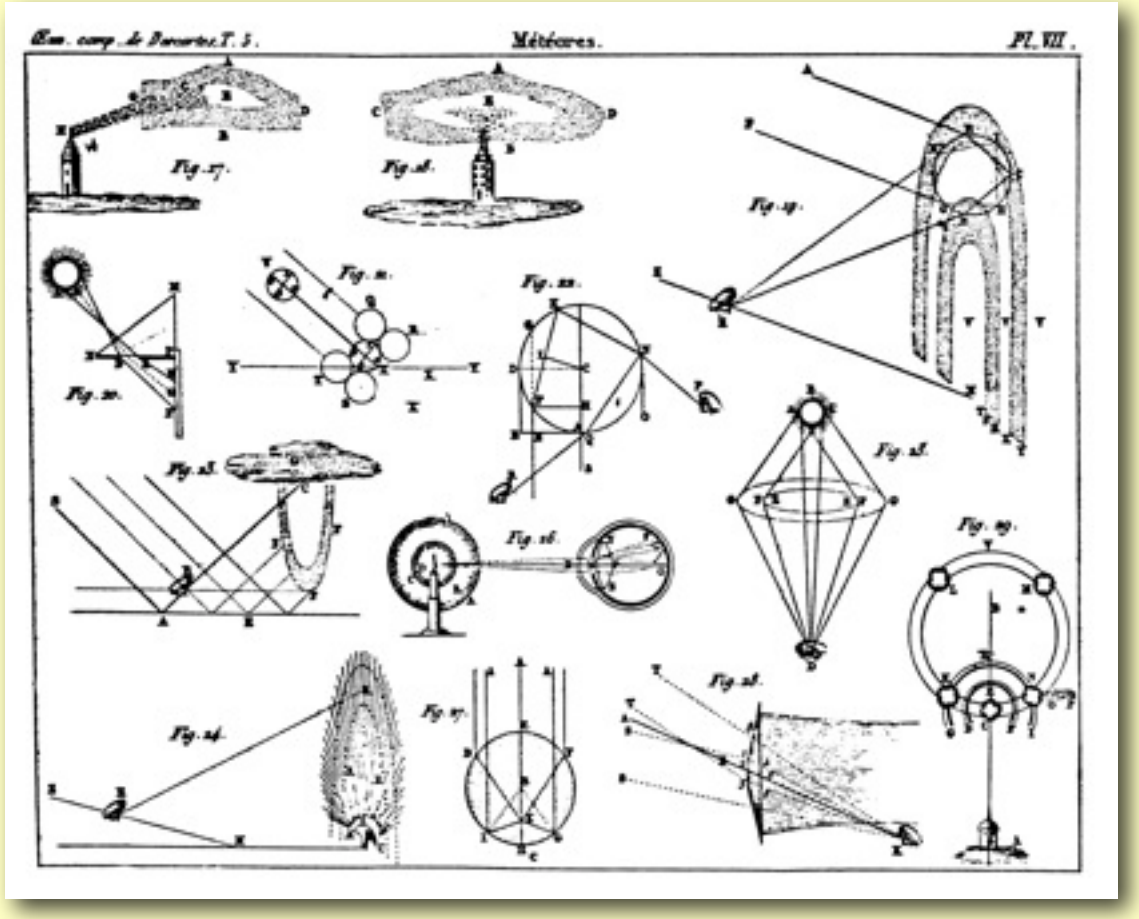
Qui sont des essais de cette METHODE.

A LEYDE

De l'Imprimerie de IAN MAIRE.

cIↃ IↃ C XXXVII

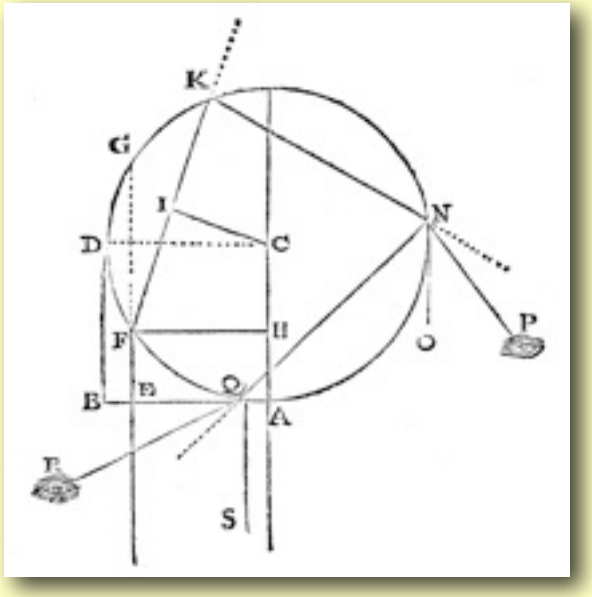
Avec Privilege.



”...I did not understand why the colours [of the rainbow] are visible only at certain angles; at last, I took my pen and performed exact calculation of trajectories of rays falling at the sphere of water, because I wanted to determine the angles at which they emerge and fall into the eye after two refractions and one or two reflections. Then I found that after one reflection and two refractions there are many more rays seen at angles between 41 and 42 degrees than at any smaller angle, and that there are no rays emerging at larger angles...
It shows clearly that the colours of the rainbow are produced by the same cause as that when we use a crystal...”

René Descartes, *Les meteores*, Chapter VIII

René Descartes



First quantitative explanation of the rainbow (1637)

La Ligne HF	La Ligne CI	L'Arc FG	L'Arc FK	L'Angle ONP	L'Angle SQR
1000	748	168,30	171,25	5,40	165,45
2000	1496	156,55	162,48	11,19	151,29
3000	2244	145, 4	154, 4	17,56	136, 8
4000	2992	132,50	145,10	22,30	122, 4
5000	3740	120	136, 4	27,52	108,12
6000	4488	106,16	126,40	32,56	93,44
7000	5236	91, 8	116,51	37,26	79,25
8000	5984	73,44	106,30	40,44	65,46
9000	6752	51,41	95,22	40,57	54,25
10000	7480	0	83,10	13,60	69,30
8000	5984	73,44	106,30	40,44	65,46
8100	6058	71,48	105,25	40,58	64,37
8200	6133	69,50	104,20	41,10	63,10
8300	6208	67,48	103,14	41,20	62,54
8400	6283	65,44	102, 9	41,26	61,43
8500	6358	63,34	101, 2	41,30	60,32
8600	6432	61,22	99,56	41,30	58,26
8700	6507	59, 4	98,48	41,28	57,20
8800	6582	56,42	97,40	41,22	56,18
8900	6657	54,16	96,32	41,12	55,20
9000	6732	51,41	95,22	40,57	54,25
9100	6806	49, 0	94,12	40,36	53,36
9200	6881	46, 8	93, 2	40, 4	52,58
9300	6956	43, 8	91,51	39,26	52,25
9400	7031	39,54	90,38	38,38	52, 0
9500	7106	36,24	89,26	3 ,32	51,54
9600	7180	32,30	88,12	36, 6	52, 6
9700	7255	28, 8	86,58	34,12	52,46
9800	7330	22,57	85,43	31,31	54,12

Experimental facts concerning light (mid-XVIIth cent.)

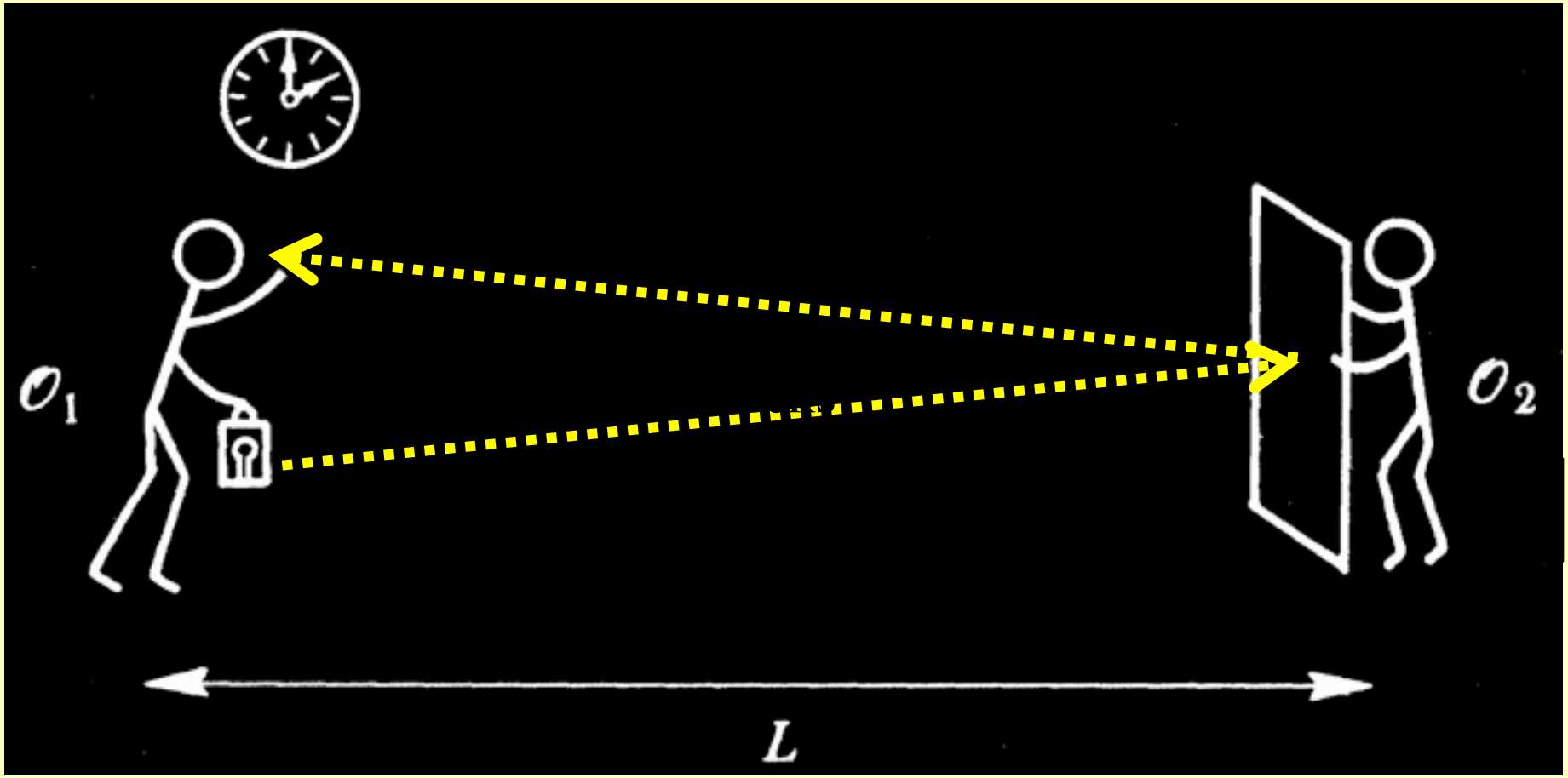
propagation in straight lines
reflection
refraction
rainbow/colours
geometrical optics → practical applications { mirrors, burning glasses, spectacles, physiological optics/vision

New results in Newton times

”Newton’s rings” (Robert Hooke, 1665)
Diffraction (Francesco Maria Grimaldi, 1665)
Double refraction (Erasmus Bartholinus, 1669)
Newton’s mirror telescope (1671)
Newton’s theory of light and colours (1672)
Finite speed of light (Ole Roemer, 1676)

Speed of light

Measuring the speed of light (Galileo, *Discorsi*, 1638)



Conclusion: light faster than about 30 km/s (in modern units)

Ole Roemer (1644-1710)

Demonstration touchant
le mouvement de la lumiere
(Journal des Sçavans, December 7, 1676)

A Demonſtration concerning the Motion of Light, *communicated from* Paris, *in the* Journal des Scavans, *and here made Engliſh.*

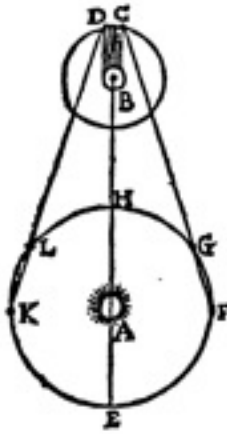
PHiloſophers have been labouring for many years to decide by ſome Experience, whether the action of Light be conveyed in an inſtance to diſtant places, or whether it requireth time. M. *Romer* of the *R. Academy* of the Sciences hath deviſed a way, taken from the Obſervations of the firſt Satellit of *Jupiter*, by which he demonſtrates, that for the diſtance of about 3000 leagues, ſuch as is very near the bigneſs of the Diameter of the Earth, Light needs not one ſecond of time.

Journal des Sçavans, December 7, 1676

DEMONSTRATION TOVCHANT LE mouvement de la lumiere trouvé par M. Römer de l'Academie Royale des Sciences.

IL y a long-temps que les Philoſophes ſont en peine de decider par quelque experience, ſi l'action de la lumiere ſe porte dans un inſtant à quelque diſtance que ce ſoit, ou ſi elle demande du temps. Mr Römer de l'Academie Royale des Sciences s'eſt aviſé d'un moyen tiré des obſervations du premier ſatellite de Jupiter, par lequel il démontre que pour une diſtance d'environ 3000 lieuës, telle qu'eſt à peu prés la grandeur du diametre de la terre, la lumiere n'a pas beſoin d'u-

ne ſeconde de temps.



Soit A le Soleil, B Jupiter, C le premier Satellite qui entre dans l'ombre de Jupiter pour en ſortir en D, & ſoit E F G H K L la Terre placée à diverſes diſtances de Jupiter.

Or ſuppoſé que la terre eſtant en L vers la ſeconde Quadrature de Jupiter, ait veu le premier Satellite, lors de ſon émerſion ou ſortie de l'ombre en D; & qu'en ſuite environ 42. heures & demie aprés, ſçavoir aprés une revolution de ce Satellite, la terre ſe trouvant en K, le voye de retour en D: Il eſt manifeſte que ſi la lumiere demande du temps pour traverſer l'intervalle L K, le Satellite ſera veu plus tard de retour en D, qu'il n'auroit eſté ſi la terre eſtoit demeurée en K, de ſorte que la revolution de ce Satellite, ainſi obſervée par les Emerſions, ſera retardée d'autant de temps que la lumiere en aura employé à paſſer de L en K, & qu'au contraire dans l'autre Quadrature F G, où la terre en s'approchant, va au devant de la lumiere, les revolutions des Immerſions paroiſtront autant accourcies, que celles des Emerſions avoient paru alongées. Et parce qu'en 42 heures & demy, que le Satellite employe à peu prés à faire chaque revolution, la diſtance entre la Terre & Iupiter dans l'un & l'autre Quadrature varie tout au moins de 210. diametres de la

Terre, il s'enſuit que ſi pour la valeur de chaque diametre de la Terre, il faloit une ſeconde de temps, la lumiere employeroit 3½ min. pour chacũ des intervalles GF, KL, ce qui cauſeroit une differẽce de prés d'un demy quart d'heure entre deux revolutions du premier Satellite, dont l'une auroit eſte obſervée en FG, & l'autre en KL, au lieu qu'on n'y remarque aucune difference ſenſible.

Il ne s'enſuit pas pourtant que la lumiere ne demande aucun temps : car apres avoir examiné la choſe de plus prés, il a trouvé que ce qui n'étoit pas ſenſible en deux revolutions, devenoit tres-conſiderable à l'égard de pluſieurs priſes enſemble, & que par exemple 40 revolutions obſervées du coſté F, eſtoient ſenſiblement plus courtes, que 40. autres obſervées de l'autre côté en quelque endroit du Zodiaque que Iupiter ſe ſoit rencontré ; & ce à raiſon de 22. pour tout l'intervalle H E, qui eſt le double de celuy qu'il y a d'icy au ſoleil.

La neceſſité de cette nouvelle Equation du retardement de la lumiere, eſt établie par toutes les obſervations qui ont eſté faites à l'Academie Royale, & à l'Obſervatoire depuis 8. ans, & nouvellement elle a eſté confirmée par l'Emerſion du premier Satellite obſervée à Paris le 9. Novembre dernier à 5. h. 35.′ 45.″ du ſoir, 10. minutes plus tard qu'on ne l'eût deû attendre, en la déduiſant de celles qui avoient eſté obſervées au mois d'Aouſt, lors que la terre eſtoit beaucoup plus proche de Iupiter ; ce que Mr Römer avoit predit à l'Academie dés le commencement de Septembre.

Mais pour oſter tout lieu de douter que cette inégalité ſoit cauſée par le retardement de la lumiere, il demontre qu'elle ne peut venir d'aucune excentricité, ou autre cauſe de celles qu'on apporte ordinairement, pour expliquer les irregularitez de la Lune & des autres Planetes : bien que neanmoins il ſe ſoit aperceu que le premier Satellite de Iupiter eſtoit excentrique, & que dailleurs ſes revolutions eſtoient avancées ou retardées à meſure que Iupiter s'aprochoit ou s'éloignoit du ſoleil, & même que les revolutions du premier Mobile eſtoient inégales ; ſans toutesfois que ces trois dernieres cauſes d'inégalité empêchent que la premiere ne ſoit manifeſte.

Many authors of physics textbooks give a false account of Roemer's discovery and its significance

Author	Value of c ascribed to Roemer in km/s	Year of Roemer's discovery
	(a) English texts	
French	$> 160\,000$	1675
Wolf	$\sim 193\,120$	1672–1676
Blum, Roller	$\sim 200\,000$	1675
Giancoli	200 000	1675
Halliday, Resnick	200 000	1675
Servay	$\sim 210\,000$	1675
Brandt, Maran	$\sim 210\,000$	1675
Froome, Essen	$\sim 214\,000$	1676
Sanders	$\sim 214\,000$	1676
Bergstrand	214 300	1676
Kittel, Knight, Ruderman	214 300	1676
Jeans	$\sim 222\,000$	1676
Jeans	...	1675
Kenworthy	$\sim 225\,000$	1675
Shortley, Williams	225 000	1675
Meyer-Arendt	227 000	1675
Arya	$\sim 230\,000$	1676
Monk	$\gtrsim 300\,000$	1676
Lipson and Lipson	$\sim 300\,000$	1678
Lipson	$< 320\,000$	1675
Fincham, Freeman	$\sim 306\,000$	1675
Morgan	309 000	1675
Dampier	309 000	1676
Longhurst	310 000	1676
Ditchburn	350 000	1676
Brown	350 000	1666
Wood	351 000	1876
Wood	308 000	1676
Weidner, Sells	...	1666
Feynman	...	1656

	(b) German texts	
Grimsehl	214 000	1673
Weller, Winkler	214 000	1676
Bergmann, Schaefer	214 450	1676
Pohl	230 000	1676
Feicht, Graf	~300 000	1676
Recknagel	300 000	...
Gerthsen, Kneser, Vogel	300 000	1676
	(c) Russian texts	
Frish, Timoryeva	215 000	1676
Landsberg	215 000	1676
Savyelyev	215 000	1676
Yavorsky, Detlaf	215 000	1676
Korolyev	215 000	1675
Putilov, Fabrikant	~220 000	1666
Govorkov	225 000	1676
Zisman, Todes	>225 000	...
	(d) Books on the history of science	
Dorfman	212 222	1676
Laue	300 000	1676
Spasski	300 870	1670–1679
Kudryavcev	311 300	...
Guaydier	327 000	1676

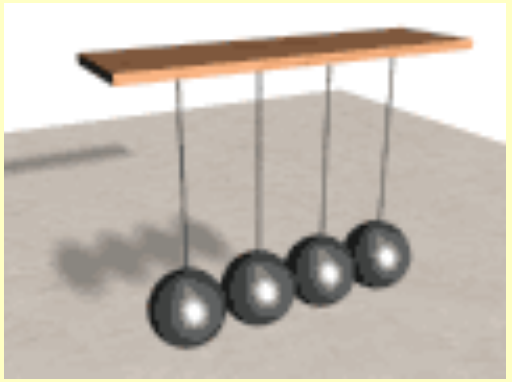
Robert Hooke, *Lectures on Light*, 1680

”But supposing this may prove it to be temporary, and not instantaneous, yet we find that ‘tis so exceeding swift that ‘tis beyond Imagination; for so far he thinks indubitable, that it moves a Space equal to the Diameter of the Earth, or near 8,000 Miles, in less than one single Second of the time, which is in as short time as one can well pronounce 1, 2, 3, 4; And if so, why it may not be as well instantaneous I know no reason...
...It makes it much more difficult to conceive so rapid a temporaneous Motion, than the Instantaneousness of that other. For the Motion of a Cannon Bullet is as much slower that this of Light, as the Motion of a Snail is than that of a Cannon Bullet.”

”It is true that we are here supposing a strange velocity that would be a hundred thousand times greater than that of Sound. For Sound, according to what I have observed, travels about 180 Toises in the time of one Second, or in about one beat of the pulse. But this supposition ought not to seem to be an impossibility; since it is not a question of the transport of a body with so great a speed, but of a successive movement which is passed on from some bodies to others. I have then made no difficulty, in meditating on these things, in supposing that the emanation of light is accomplished with time, seeing that in this way all its phenomena can be explained, and that in following the contrary opinion everything is incomprehensible... But the extreme velocity of Light, and other properties which it has, cannot admit of such a propagation of motion [as Sound], and I am about to show here the way in which I conceive it must occur. For this, it is needful to explain the property which hard bodies must possess to transmit movement from one to another.”

Christiaan Huygens, *Traité de la lumière*

”When one takes a number of spheres of equal size, made of some very hard substance, and arranges them in a straight line, so that they touch one another, one finds, on striking with a similar sphere against the first of these spheres, that the motion passes as in an instant to the last of them, which separates itself from the row, without one's being able to perceive that the others have been stirred. And even that one which was used to strike remains motionless with them. Whence one sees that the movement passes with an extreme velocity which is the greater, the greater the hardness of the substance of the spheres. But it is still certain that this progression of motion is not instantaneous, but successive, and therefore must take time. For if the movement, or the disposition to movement, if you will have it so, did not pass successively through all these spheres, they would all acquire the movement at the same time, and hence would all advance together, which does not happen. For the last one leaves the whole row and acquires the speed of the one which was pushed.”

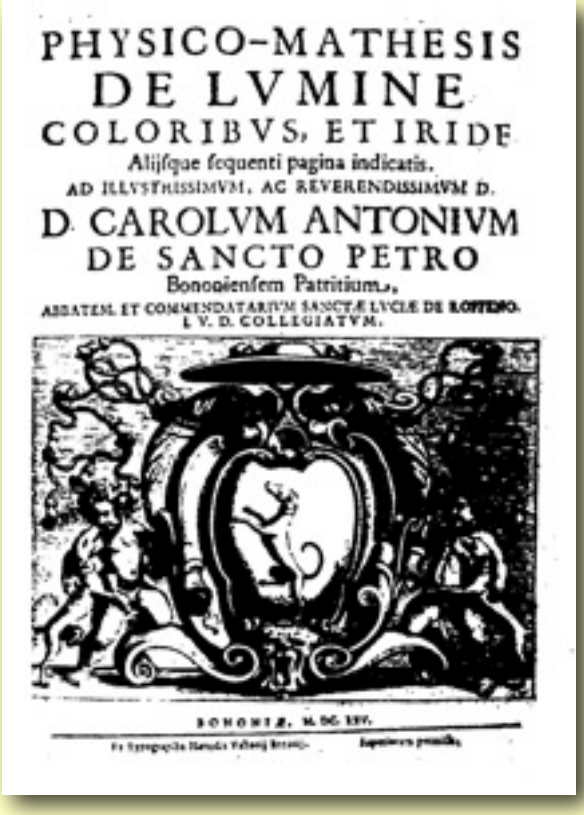


Christiaan Huygens, *Traité de la lumière*

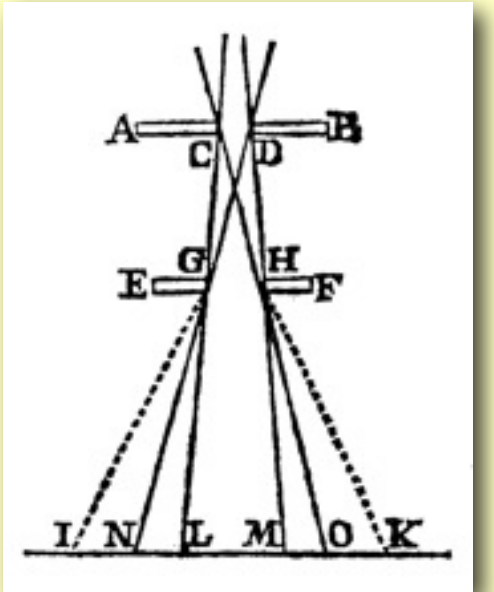
”That is to say they yield a little in themselves at the place where they are struck, and immediately regain their former figure. For I have found that on striking with a ball of glass or of agate against a large and quite thick piece of the same substance which had a flat surface, slightly soiled with breath or in some other way, there remained round marks, of smaller or larger size according as the blow had been weak or strong. This makes it evident that these substances yield where they meet, and spring back: and for this time must be required. Now in applying this kind of movement to that which produces Light there is nothing to hinder us from estimating the particles of the ether to be of a substance as nearly approaching to perfect hardness and possessing a springiness as prompt as we choose.”

Christiaan Huygens, *Traité de la lumière*

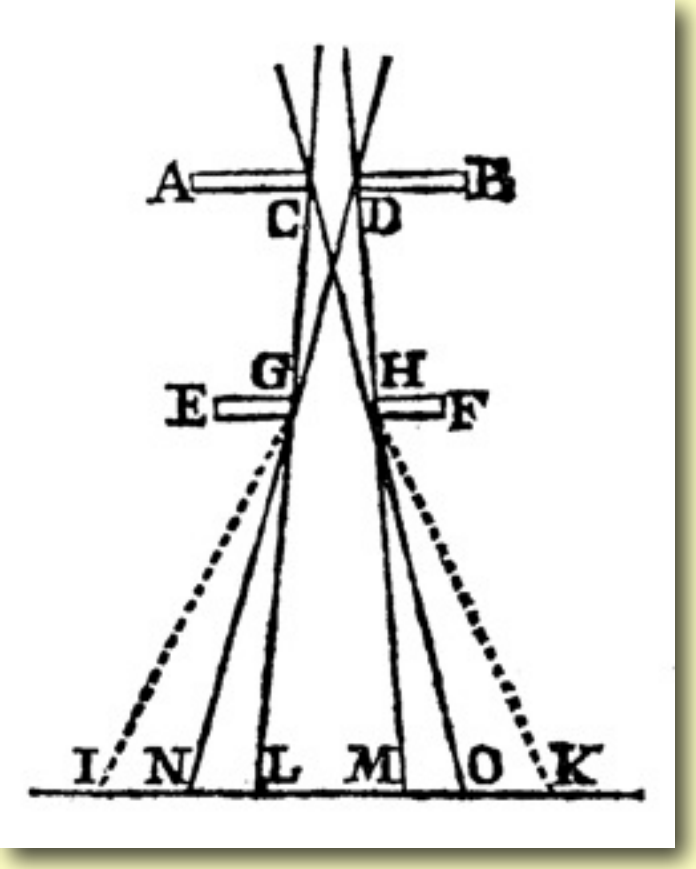
Francesco Maria Grimaldi
(1618-1663)

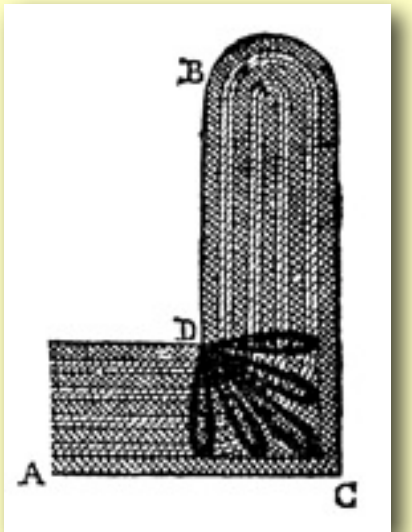

PHYSICO-MATHESIS
DE LVMINE
COLORIBVS, ET IRIDE
Alijſque ſequenti pagina indicatis.
AD ILLVSTRISSIMVM, AC REVERENDISSIMVM D.
D. CAROLVM ANTONIVM
DE SANCTO PETRO
Bononienſem Patritium,
ABBATEM, ET COMMENDATARIVM SANCTÆ LVCIÆ DE [illegible]
L. V. D. COLLEGIATVM.

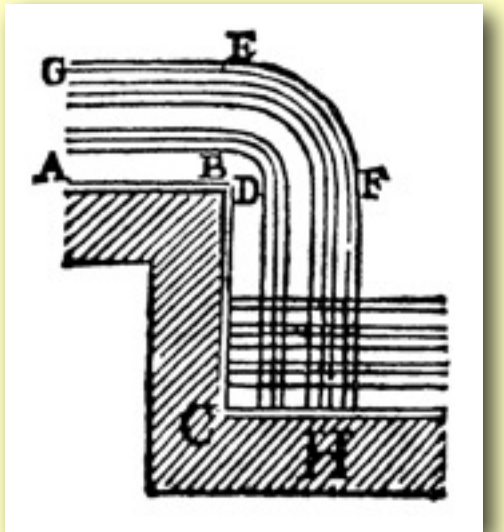
”Light is propagated or diffused not only directly, by refraction, and by reflection, but also in a fourth way - by diffraction...”

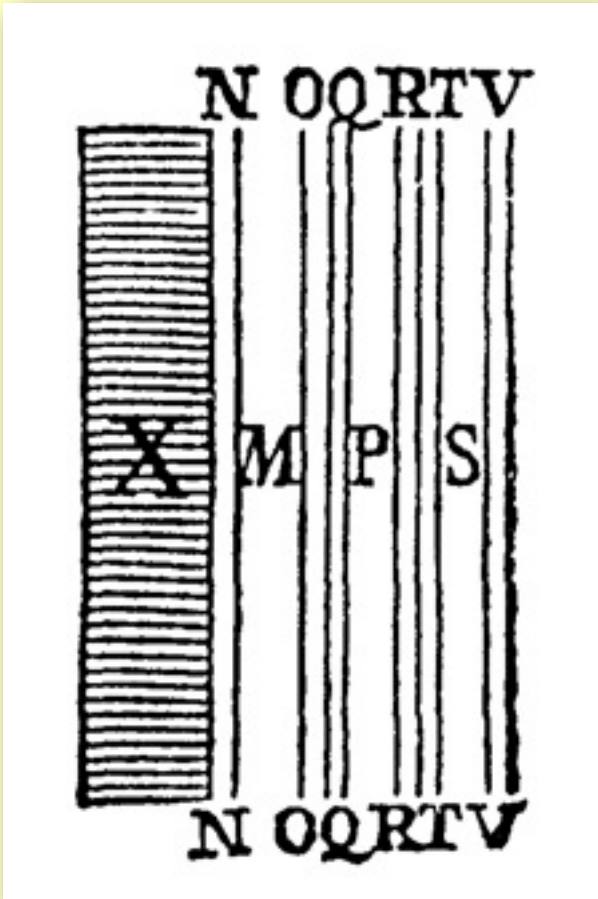


Diffraction of light illustrated by Grimaldi in *De lumine*









Huygens, Mariotte and de la Hire were sceptical about diffraction of light and after some experiments performed in Paris concluded that it is not a new phenomenon
but the outcome of sloppy experimentation!

”...in precise experiments one always finds that light propagates along straight lines without any diffraction”

Edme Mariotte (1681)


ERASMI BARTHOLINI
EXPERIMENTA
CRYSTALLI ISLANDICI
DISDIACLASTICI
Quibus mira & insolita
REFRACTIO
detegitur.

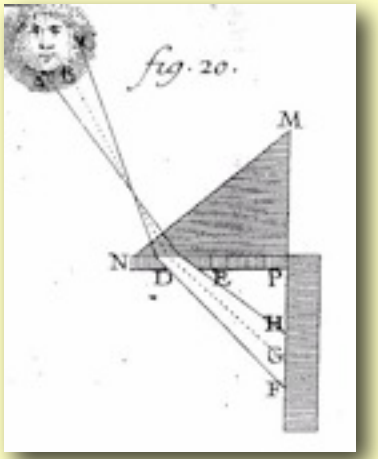
ANNO 1670.

HAFNIÆ,
Sumptibus DANIELIS PAULLI Reg. Bibl.

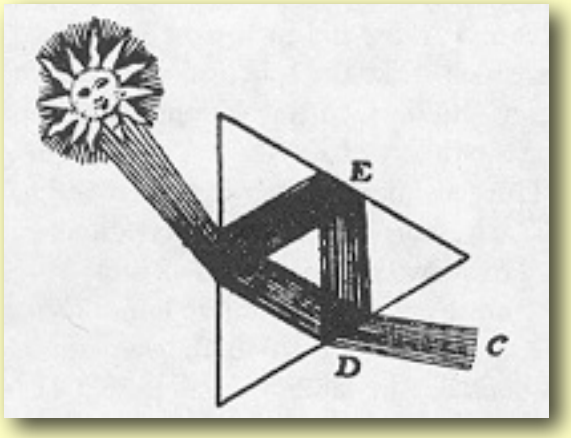
In 1669 Erasmus Bartholinus published a description of double refraction of light in crystals of calcite

The origin of colours

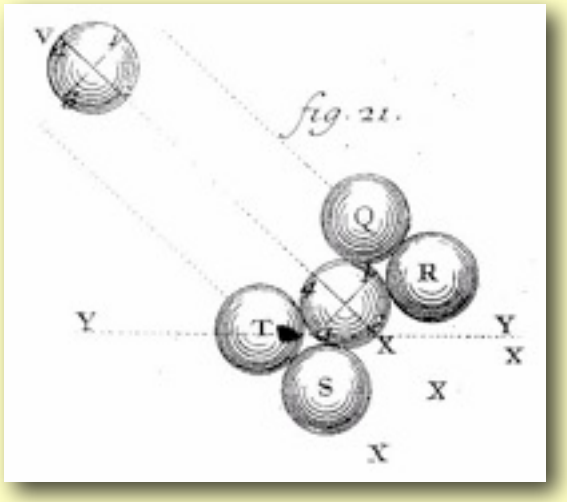
Opinions about the origin of colours

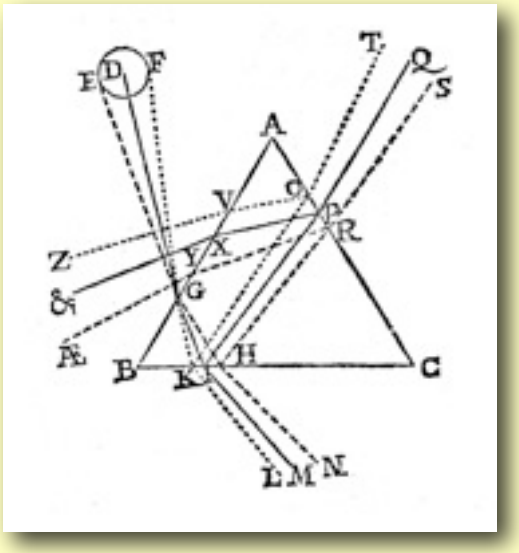


Descartes (1637)



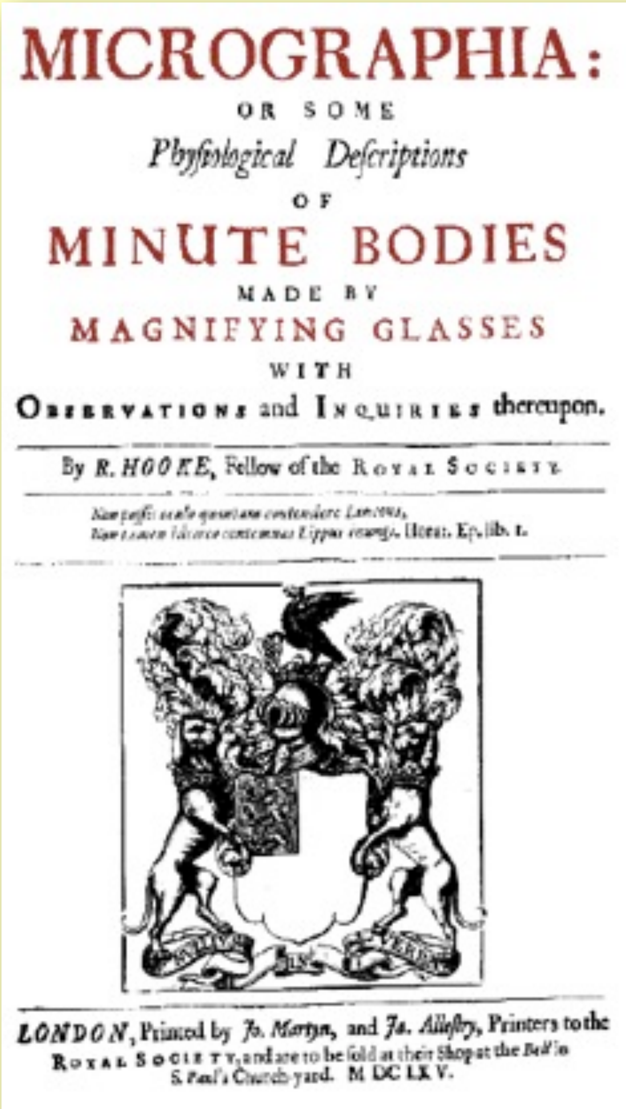
La Chambre (1650)





Grimaldi (1665)

Hooke's bestseller



MICROGRAPHIA:

OR SOME

Physiological Descriptions

OF

MINUTE BODIES

MADE BY

MAGNIFYING GLASSES

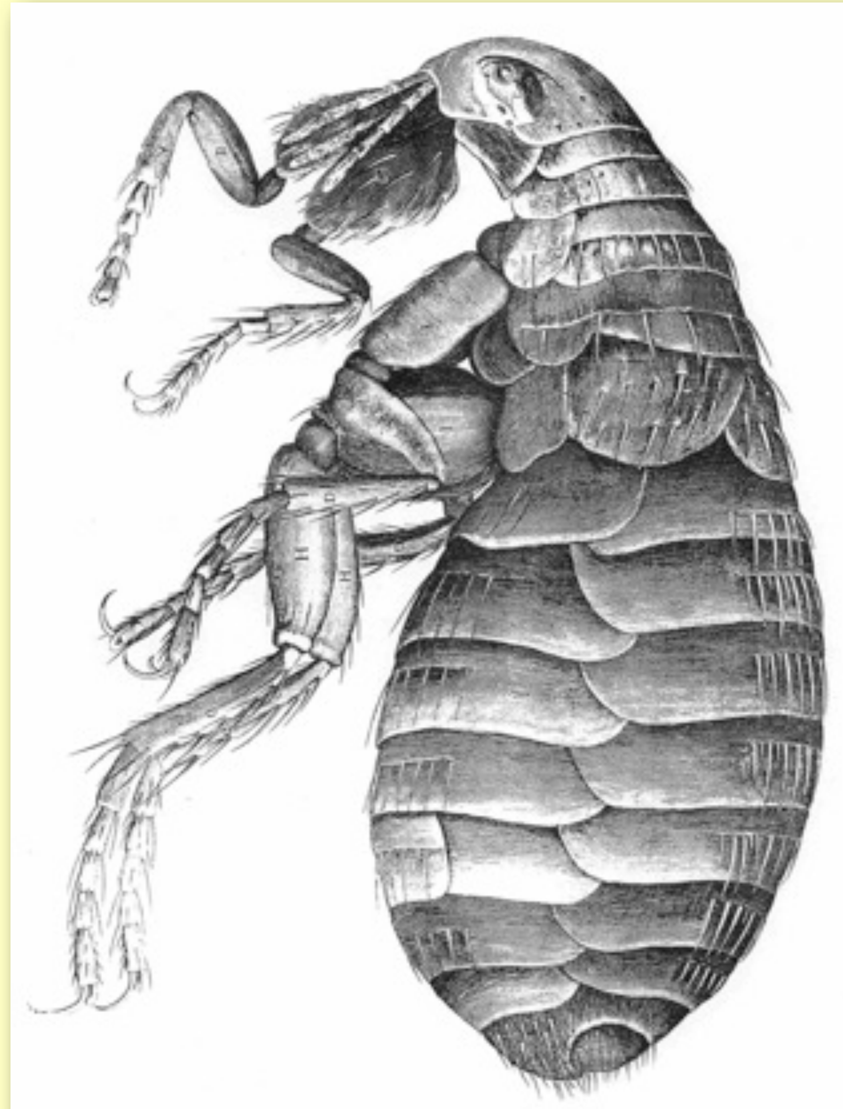
WITH

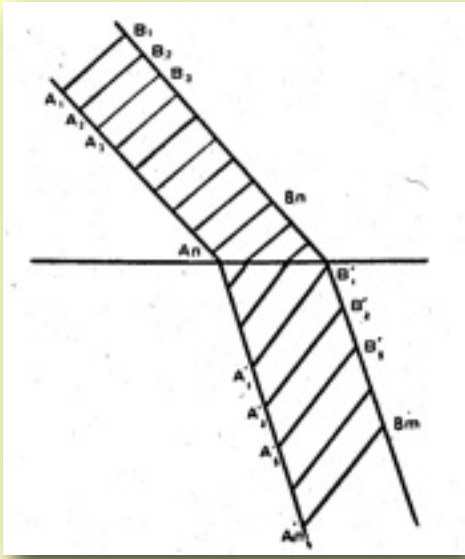
OBSERVATIONS and INQUIRIES thereupon.

By *R. HOOKE*, Fellow of the ROYAL SOCIETY.

Non possis oculo quantum contendere Linceus,
Non tamen idcirco contemnas Lippus inungi. Horat. Ep. lib. 1.

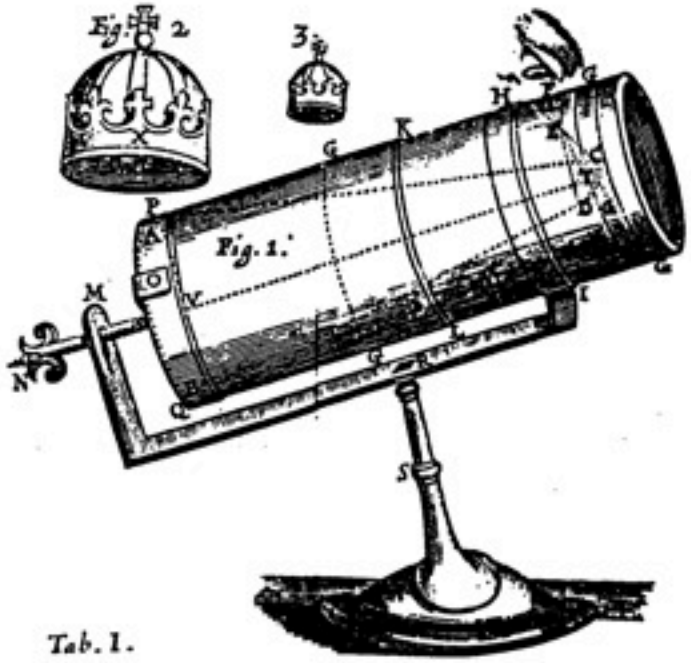
LONDON, Printed by *Jo. Martyn*, and *Ja. Allestry*, Printers to the ROYAL SOCIETY, and are to be sold at their Shop at the *Bell* in S. *Paul's* Church-yard. M DC LX V.





”…that white is nothing but a pulse or motion, propagated through homogenous, uniform and transparent medium; and that colour is nothing but a disturbance of that light, by the communication of that pulse to other transparent mediums, that is, by the refraction thereof; that whiteness and blackness are nothing but a plenty or scarcity of the undisturbed rays of light; and that the two colours (than the which that are not more uncompounded in nature) are nothing but the effects of a compounded pulse, or disturbed propagation of motion caused by refraction….”

Robert Hooke, Royal Society meeting, February 15, 1672



Tab. 1.

An Accompt of a New Catadioptrical Teleſcope invented by Mr. Newton, *Fellow of the* R. Society, *and Profeſſor of the Mathematiques in the Univerſity of Cambridge.*

THis Excellent Mathematician having given us, in the Tranſactions of *February* laſt, an account of the cauſe, which Induced him to think upon *Reflecting* Teleſcopes, inſtead of *Refracting* ones, hath thereupon preſented the Curious World with an *Eſſay* of what may be performed by ſuch Teleſcopes; by which it is found, that Teleſcopical Tubes may be conſiderably ſhortned without prejudice to their magnifying effect.

This new inſtrument is compoſed of two Metallin *ſpeculum's*, the one Concave, (inſtead of an Object-glaſs) the other Plain; and alſo of a ſmall plano-convex Eye-Glaſs.

By *Figure* I. of *Tab.* I. the ſtructure of it may be eaſily imagined; viz. *That* the Tube of this Teleſcope is open at the end which reſpects the object; *that* the other end is cloſe, where the ſaid Concave is laid, and *that* near the open end there is a flat oval *ſpeculum*, made as ſmall as may be, the leſs to obſtruct the entrance of the rays of Light, and inclined towards the upper part of the Tube, where is a little hole furniſh't with the ſaid Eye-glaſs. So that the rays coming from the object, do firſt fall on the Concave placed at the bottome of the Tube; and are thence reflected toward the other end of it, where they meet with the flat ſpeculum, obliquity poſited, by the reflection of which they are directed to the little plano-convex Glaſs, and ſo to the ſpectators Eye, who looking downwards ſees the Object, which the Teleſcope is turned to.

To underſtand this more diſtinctly and fully, the Reader may pleaſe to look upon the ſaid *Figure*, in which

A B is the Concave *ſpeculum*, of which the *radius* or ſemidiameter is $12\frac{1}{2}$ or 13 inches.

C D another metalline *ſpeculum*, whoſe ſurface is flat, and the circumference oval.

G D

PHILOSOPHICAL TRANSACTIONS.

February 19. 16$\frac{11}{12}$.

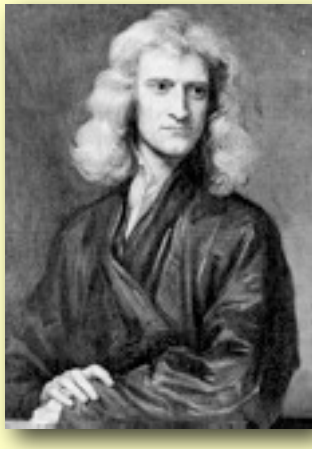
The CONTENTS.

A Letter of Mr. Isaac Newton, *Professor of the Mathematicks in the University of Cambridge: containing his New Theory about* Light *and* Colors: *sent by the Author to the Publisher from Cambridge, Febr.* 6. 16$\frac{11}{12}$; *in order to be communicated to the* R. Society.

SIR,

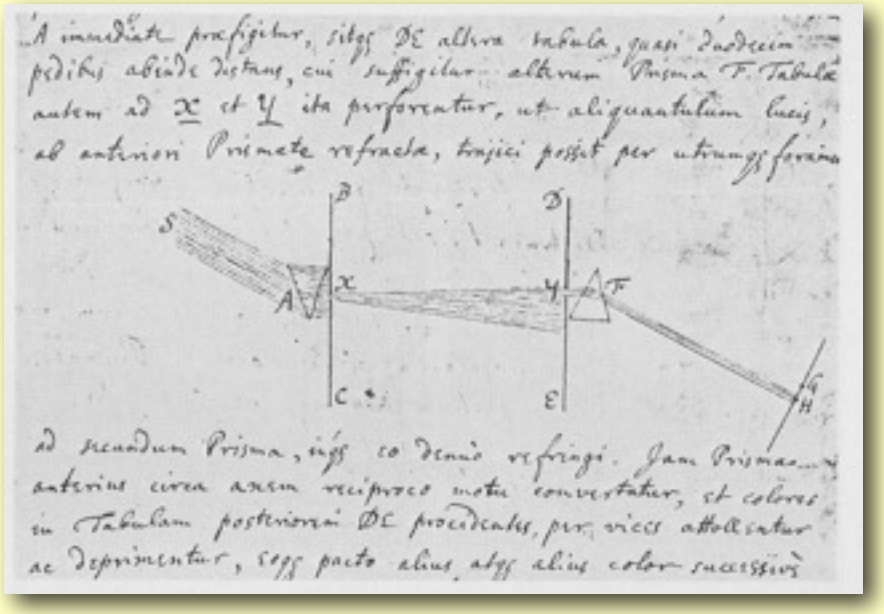
TO perform my late promise to you, I shall without further ceremony acquaint you, that in the beginning of the Year 1666 (at which time I applyed my self to the grinding of Optick glasses of other figures than *Spherical*,) I procured me a Triangular glass-Prisme, to try therewith the celebrated *Phænomena* of

”...*Experimentum crucis*...was this: I took two boards, and placed one of them close behind the Prisme at the window, so that the light might pass through a small hole, made in it for the purpose, and fall on the other board, which I placed at about 12 feet distance, having first made a small hole in it also, for some of that Incident light to pass through. Then I placed another Prisme behind this second board, so that the light, trajected through both the boards, might pass through that also, and be again refracted before it arrived at the wall.”

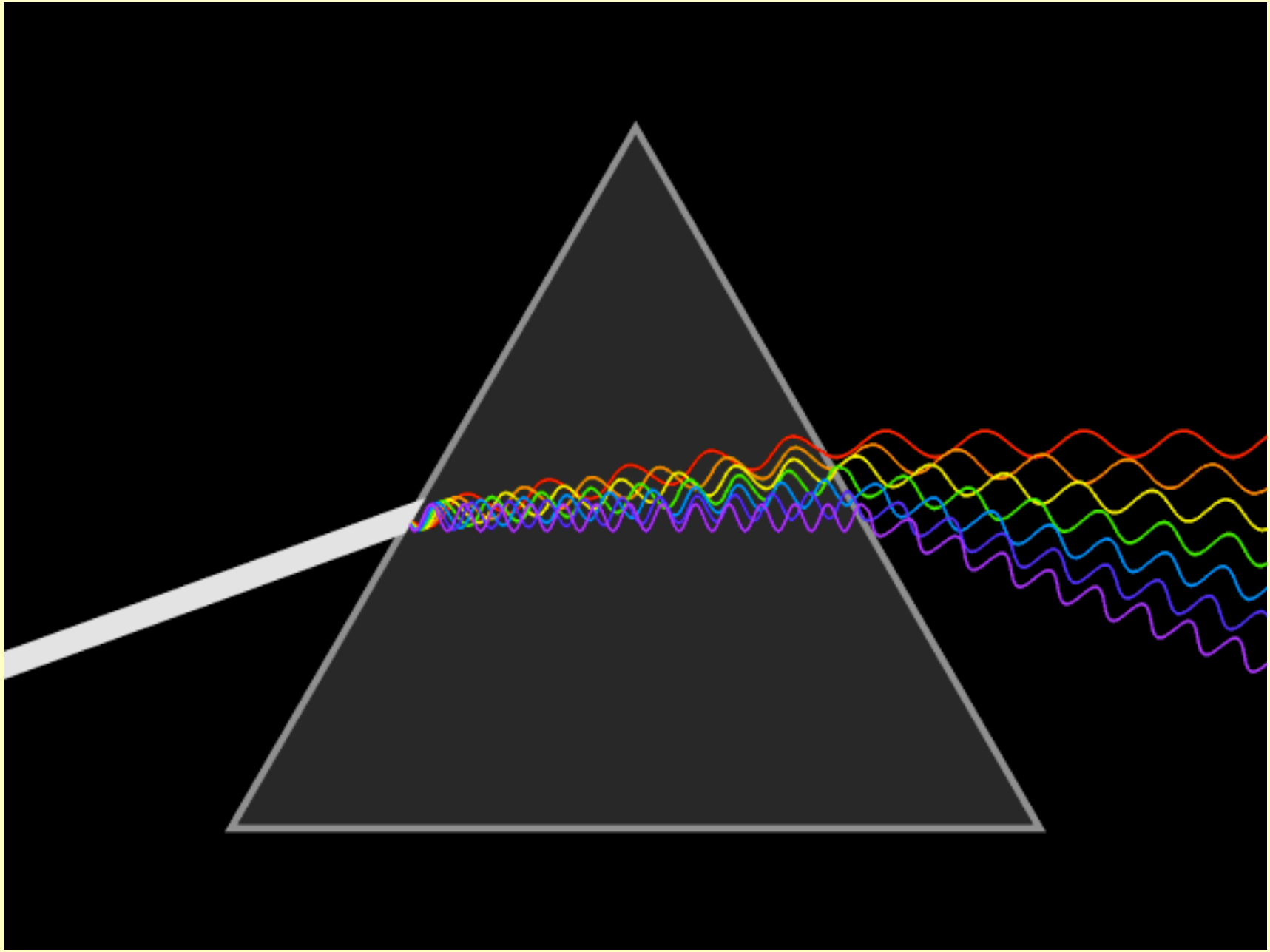


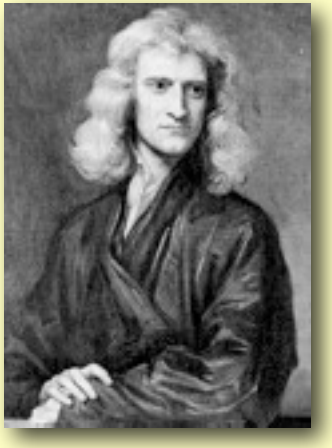
New Theory about Light and Colors

New Theory about Light and Colors (1672)



Newton's manuscript with the explanation of his *Experimentum crucis* concerning the nature of white light





”This done, I took the first Prisme in my hand, and turned it to and fro slowly about its *Axis*, so much as to make the several parts of the Image, cast on the second board, successively pass through the hole in it, that I might observe to what places on the wall the second Prisme would refract them. And I saw by the variation of those places, that the light, tending to that end of the Image, towards which the refraction of the first Prisme was made, did in the second Prisme suffer a Refraction considerably greater than the light tending to the other end. And so the true cause of the length of that Image was detected to be no other, then that *Light* consists of *Rays differently refrangible*, which, without any respect to a difference in their incidence, were, according to their degrees of refrangibility, transmitted towards divers parts of the wall.”

New Theory about Light and Colors

”As the rays of light differ in degrees of refrangibility so they also differ in their dispositions to exhibit this or that particular color. Colors are not qualifications of light derived from refractions, or reflections of natural bodies...

To the same degree of refrangibility ever belongs the same color, and to the same color ever belongs the same degree of refrangibility...

But the most surprising, and wonderful composition is that of *Whiteness*. There is no one sort of rays which alone can exhibit it. ‘Tis ever compounded, and to its composition, are requisite all the aforesaid primary colors, mixed in due proportion...

These things considered, the *manner* how colors are produced by the Prisme is evident...”

New Theory about Light and Colors

One has to remember that in 1672 Newton was a completely unknown person. Royal Society consisted largely of amateurs but included scientists of great authority and prestige, such as Robert Boyle, Robert Hooke, John Wallis, Christopher Wren.

Hooke underestimated Newton, as did the others (Christiaan Huygens, Robert Moray, Ignace Pardies, Francis Linus, John Gascoines, Anthony Lucas). Newton had to defend his views in a long and heated dispute, which however helped him to reformulate his theory of light.

Hooke's comment on the paper by Newton
(February 15, 1671/72)

"I have perused the discourse of Mr. Newton about colours and refractions and I was not a little pleased with the niceness and curiosity of his observations. But, tho' I wholly agree with him as to the truth of those he has alledged, as having, by many hundreds of trials, found them so; yet as to his hypothesis of solving the phenomena of colours thereby, I confess, I cannot see yet any undeniable arguments to convince me of the certainty thereof. For all the experiments and observations I have hitherto made, nay, and even those very experiments, which he alledgeth do seem to me to prove that white is nothing but a pulse or motion, propagated through an homogeneous, uniform, and transparent medium; and that colour is nothing but a disturbance of that light, by the communication of that pulse to other transparent mediums, that is, by the refraction thereof; that *whiteness* and *blackness* are nothing but the plenty or scarcity of the undisturbed rays of light..."

”...But why there is a necessity, that all those motions, or whatever else it be that makes colours, should be originally in the simple rays of light, I do not yet understand the necessity of; no more than that all those sounds must be in the air of the bellows, which are afterwards heard to issue from the organ-pipes, or in the string which are afterwards, by differing stoppings and strikings produced: which string (by the way) is a pretty representation of the shape of a refracted ray to the eye; and the manner of it may be somewhat imagined by the similitude there; for the ray is like the string, strained between the luminous object and the eye, and the stop or fingers is like the refracting surface, on the one side of which the string hath no motion, on the other
a vibrating one. Now we may say indeed and imagine that the rest or streightness of the string is caused by the cessation of motions or coalition of all vibrations, and that all the vibrations are dormant in it; but yet it seems more natural to me, to imagine it the other way...”

Hooke, February 15, 1671/72

”I received yours February 17, and, having considered Mr Hooke’s observations on my discourse, am glad, that so acute an objector hath nothing that can enervate any part of it: for I am still of the same judgement, and doubt not, but that upon severer examinations, it will be found as certain a truth as I have asserted it. You shall very suddenly have an answer...”

Newton to Oldenburg, 20 February 1672

”...But when Mr Hook would insinuate a difficulty in these things by alluding to sounds in the string of a musical Instrument before percussion, or in the Air of an Organ Bellowes before its arrival at the Pipes, I must confess I understand it as little as if one had spoken of Light in a piece of Wood before it be set on fire, or in the oyl of a Lamp before it ascend up the Match to feed the flame.”

Newton to Oldenburg, 11 June 1672;
(published *Phil. Trans*., Nov. 18)

Fragments of Newton's answer (Phil. Trans., Nov. 18, 1672)

"But supposing I had propounded that *Hypothesis*, I understand not, why the Objector should so much endeavour to oppose it. For certainly it has a much greater affinity with his own *Hypothesis*, than he seems to be aware of; the Vibrations of the *Aether* being as useful and necessary in this, as in *his*. For, assuming the Rays of Light to be small bodies, emitted every way from Shining substances, those, when they impinge on any Refracting or Reflecting superficies, must as necessarily excite Vibrations in the *aether*, as Stones do in water when thrown into it..."

Fragments of Newton's answer (Phil. Trans., Nov. 18, 1672)

"...The agitated parts of bodies, according to their several sizes, figures, and motions, do excite Vibrations in the aether of various depths or bignesses, which being promiscuously propagated through that Medium to our Eyes, effect in us a Sensation of Light of a White color; but if by any means those of unequal bignesses be separated from one another, the largest beget a Sensation of a red colour, the least or shortest, of a deep Violet, and the intermediat ones, of intermediat colors; much after the manner that bodies, according to their several sizes, shapes, and motions, excite vibrations in the Air of various bignesses, which according to those bignesses, make several Tones in Sound. That the largest Vibrations are best able to overcome the resistance of a Refracting superficies, and so break through it with least Refraction; whence the Vibrations of several bignesses, that is, the Rays of several Colors, which are blended together in Light, must be parted from one another by Refraction, and so cause the *Phenomena* of *Prismes* and other refracting substances."

Fragments of Newton's answer (Phil. Trans., Nov. 18, 1672)

"And it depends on the Plate or Buble, whether a Vibration shall be *reflected* at its further superficies, or *transmitted*, so that, according to the number of vibrations, interceding the two superficies, they may be reflected or transmitted for many successive thicknesses. And since the Vibrations which make *Blew* and *Violet*, are supposed shorter than those which make *Red* and *Yellow*, they must be reflected at a less thickness of the Plate: Which is sufficient to explicate all the ordinary phaenomena of those Plates or Bubles, and also of all natural bodies whose parts are like so many fragments of such Plates... For, to me, the Fundamental Supposition [of Hooke's hypothesis] it self seems impossible, namely that the *Waves* or Vibrations of any Fluid, can, like the Rays of Light, be propagated in *Streight* lines, without a continual and very extravagant spreading and bending every way into the quiescent Medium, where they are terminated by it..."

”First, it is to be supposed therein, that there is an aethereal Medium much of the same constitution with air, but far rarer, subtler and more strongly Elastic. Of the existence of this medium, the motion of a pendulum in a glass exhausted of air almost as quickly as in the open air, is no inconsiderable argument...

In the second place, it is to be supposed, that the Aether is a vibrating Medium like Air; only the vibrations far more swift and Minute; those of Air, made by a man’s ordinary voice succeeding one another at more than half a foot or a foot distance, but those of aether at a less distance than the hundred thousandth part of an inch. And, as in Air the Vibrations are some larger than others, but yet all equally Swift (for in a ring of Bells the Sound of every tone is heard at two or three miles distance, in the Same Order that the bells are Stroke); So I suppose the aethereal Vibrations differ in bigness but not in Swiftness...”

Newton’s text read in the Royal Society in December 1675 (unpublished)

”**I suppose light is neither aether, nor its vibrating motion, but something of a different kind propagated from lucid bodies.**

Whatever light be, I suppose, it consists of rays differing from one another in contingent circumstances, as bigness, form, or vigour; like as the sands on the shore, the waves of the sea, the faces of men, and all other natural things of the same kind differ... it is to be supposed, that light and aether mutually act upon one another, aether in refracting light, and light in warming aether; and that the densest aether acts most strongly...

Newton’s text read in the Royal Society in December 1675 (unpublished)

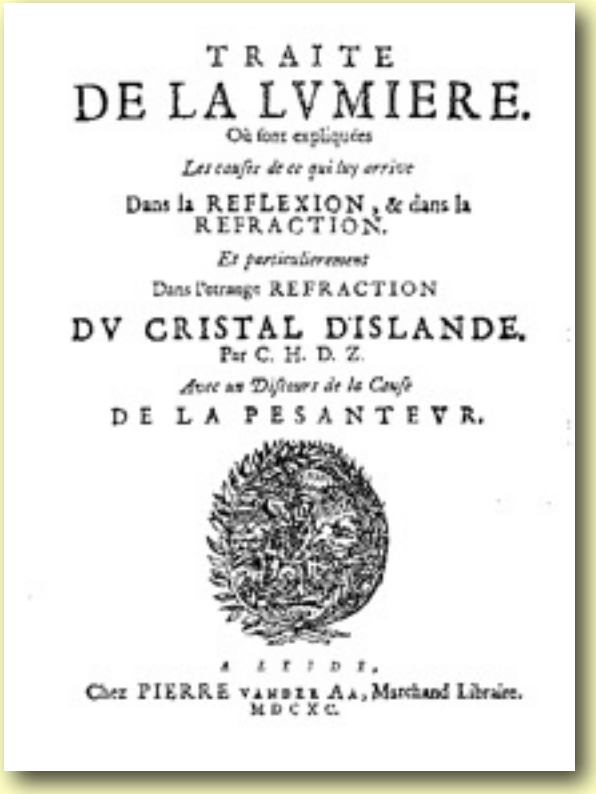
”But it remains further to be explained, how rays alike incident on the same superficies (suppose of crystal, glass or water) may be at the same time some refracted, others reflected. And for explaining this, I suppose, that the rays, when they impinge on the rigid resisting aethereal superficies, as they are acted upon by it, so they react upon it and cause vibrations in it, as stones thrown into water do in its surface; and that these vibrations are propagated every way into both the rarer and denser mediums... and alternately contract and dilate aether in that physical superficies...”

Newton’s text read in the Royal Society in December 1675 (unpublished)

”If a ray of light impinge upon it [aether] while it is much compressed, I suppose it is then too dense and stiff to let the ray pass through and so reflects it; but the rays, that impinge on it at other times, when it is either expanded by the interval of two vibrations, or not too much compressed and condensed, go through and are refracted...
I must further suppose, that, though light be unimaginably swift, yet the aethereal vibrations, excited by a ray, move faster than the ray itself, and so overtake and outrun it one after the other... It is possible light itself may not be so swift, as some are apt to think; for, notwithstanding any argument, that I know yet to the contrary, it may be an hour or two, if not more, for moving from the sun to us...”

Newton’s text read in the Royal Society in December 1675 (unpublished)

Treatise on Light by Christiaan Huygens



TRAITE
DE LA LVMIERE.
Où sont expliquées
Les causes de ce qui luy arrive
Dans la REFLEXION, & dans la REFRACTION.
Et particulierement
Dans l'etrange REFRACTION
DV CRISTAL D'ISLANDE.
Par C. H. D. Z.
Avec un Discours de la Cause
DE LA PESANTEVR.

A LEYDE,
Chez PIERRE VANDER AA, Marchand Libraire.
MDCXC.

”As happens in all the sciences in which Geometry is applied to matter, the demonstrations concerning Optics are founded on truths drawn from experience. Such are that the rays of light are propagated in straight lines; that the angles of reflexion and of incidence are equal; and that in refraction the ray is bent according to the law of sines, now so well known, and which is no less certain than the preceding laws. The majority of those who have written touching the various parts of Optics have contented themselves with presuming these truths...”

Christiaan Huygens, ***Traité de la lumière***, Chapter I

”I am astonished also that even here these have often been willing to offer, as assured and demonstrative, reasonings which were far from conclusive. For I do not find that any one has yet given a probable explanation of the first and most notable phenomena of light, namely why it is not propagated except in straight lines, and how visible rays, coming from an infinitude of diverse places, cross one another without hindering one another in any way.”

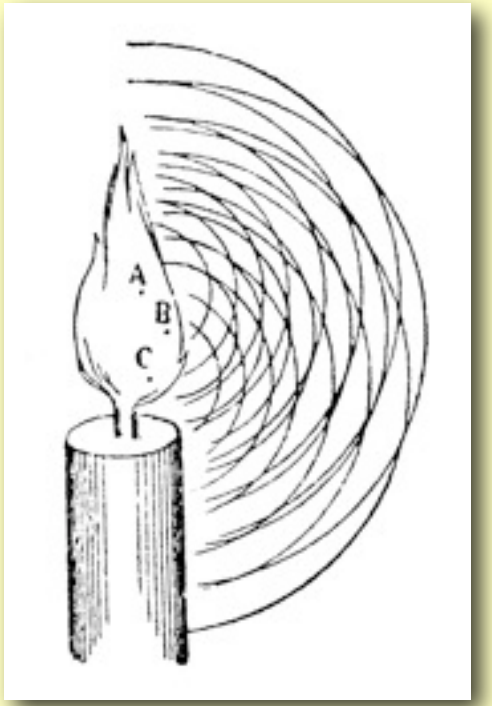
Christiaan Huygens, ***Traité de la lumière***, Chapter I

”I hope also that there will be some who by following these beginnings will penetrate much further into this question than I have been able to do, since the subject must be far from being exhausted. This appears from the passages which I have indicated where I leave certain difficulties without having resolved them, and still more from matters which I have not touched at all, such as Luminous Bodies of several sorts, and all that concerns Colours; in which no one until now can boast of having succeeded. Finally, there remains much more to be investigated touching the nature of Light which I do not pretend to have disclosed, and I shall owe much in return to him who shall be able to supplement that which is here lacking to me in knowledge.”

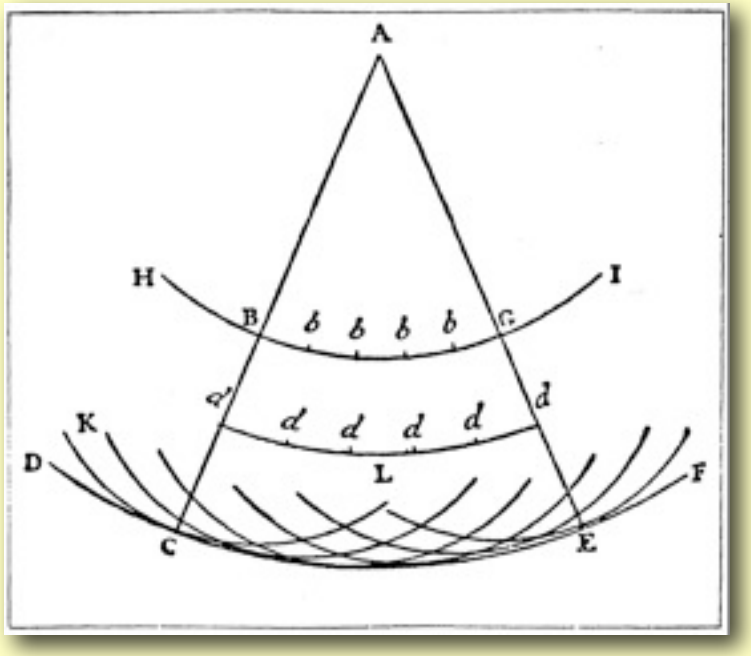
Christiaan Huygens, ***Traité de la lumière***, Preface

”...each little region of a luminous body, such as the Sun, a candle, or a burning coal, generates its own waves of which that region is the centre. Thus in the flame of a candle, having distinguished the points A, B,C, concentric circles described about each of these points represent the waves which come from them. And one must imagine the same about every point of the surface and of the part within the flame.

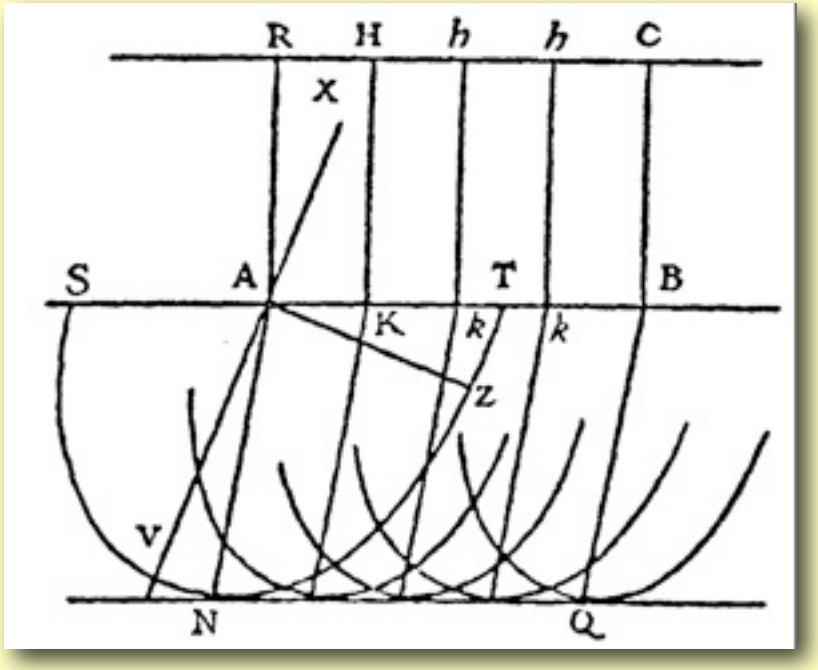
But as the percussions at the centres of these waves possess no regular succession, **it must not be supposed that the waves themselves follow one another at equal distances: and if the distances marked in the figure appear to be such, it is rather to mark the progression of one and the same wave at equal intervals of time than to represent several of them issuing from one and the same centre.**”



Christiaan Huygens, *Traité de la lumière*

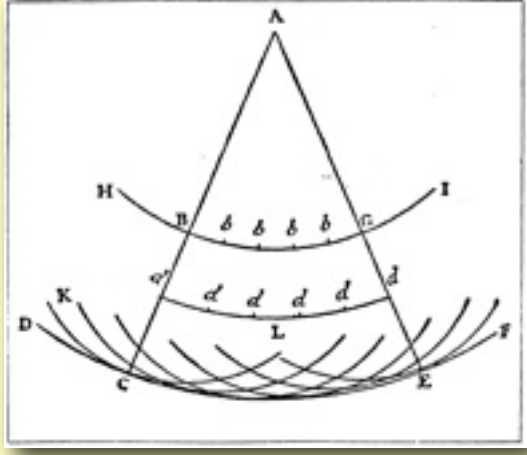


Huygens' principle



"Spheroidal" wave-fronts in a double-refracting crystal

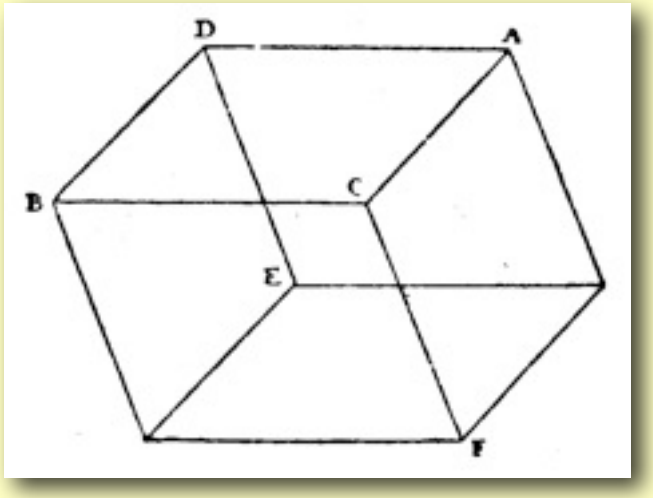
Strictly speaking, Huygens' theory of light was "pulse theory", not "wave theory"

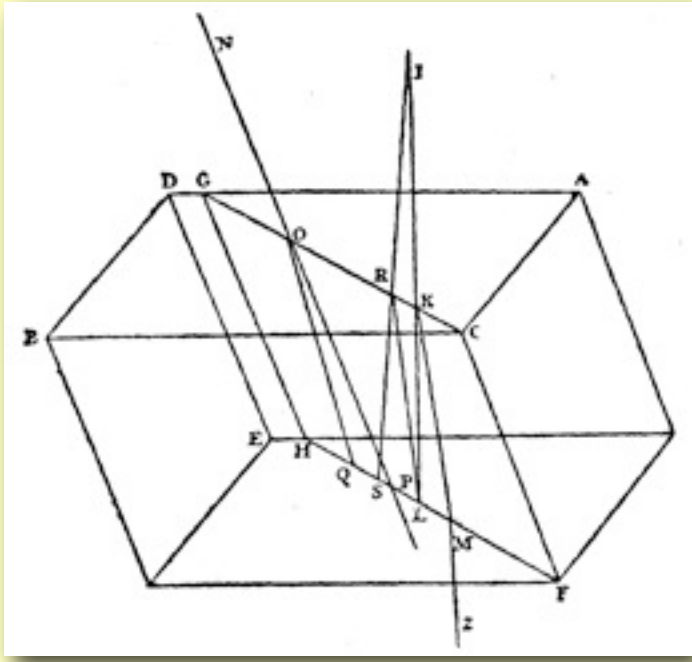


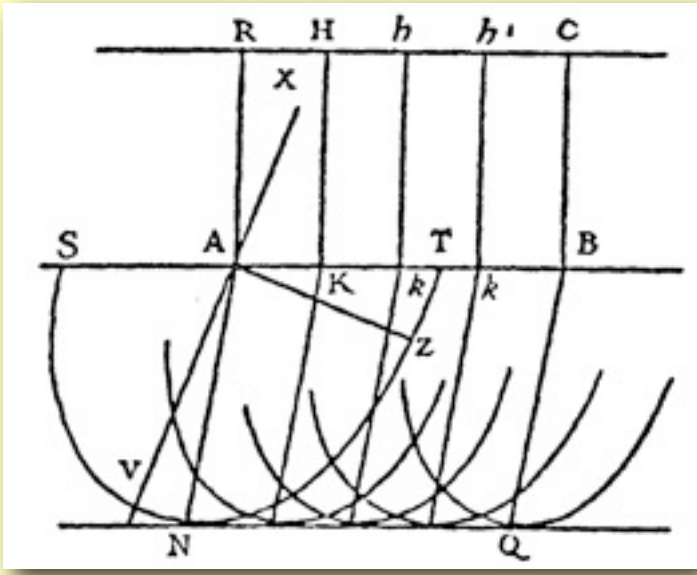
”There is the further consideration in the emanation of these waves, that each particle of matter in which a wave spreads, ought not to communicate its motion only to the next particle which is in the straight line drawn from the luminous point, but that it also imparts some of it necessarily to all the others which touch it and which oppose themselves to its movement. So it arises that around each particle there is made a wave of which that particle is the centre. Thus if DCF is a wave emanating from the luminous point A, which is its centre, the particle B, one of those comprised within the sphere DCF, will have made its particular or partial wave KCL, which will touch the wave DCF at C at the same moment that the principal wave emanating from the point A has arrived at DCF; and it is clear that it will be only the region C of the wave KCL which will touch the wave DCF, to wit, that which is in the straight line drawn through AB. Similarly the other particles of the sphere DCF, such as *bb*, *dd* etc., will each make its own wave. But each of these waves can be infinitely feeble only as compared with the wave DCF, to the composition of which all the others contribute by the part of their surface which is most distant from the centre A.”

Chapter V. On the Strange Refraction of Iceland Crystal

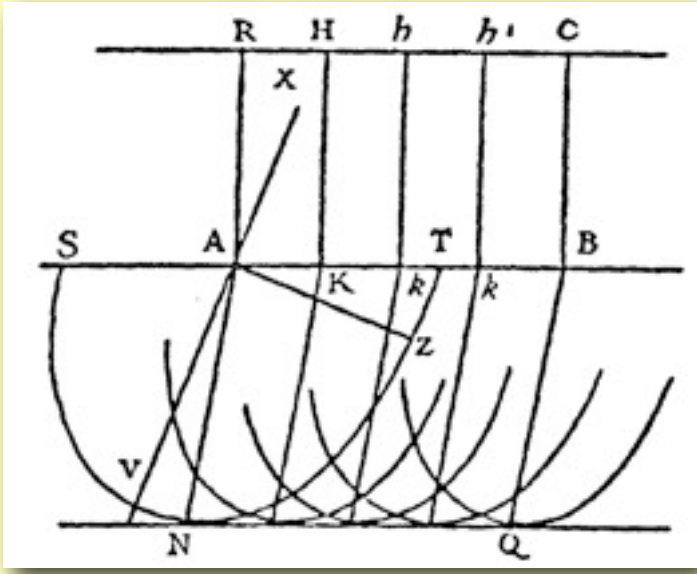
”There is brought from Iceland, which is an Island in the North Sea, in the latitude of 66 degrees, a kind of Crystal or transparent stone, very remarkable for its figure and other qualities, but above all for its strange refractions. The causes of this have seemed to me to be worthy of being carefully investigated, the more so because amongst transparent bodies this one alone does not follow the ordinary rules with respect to rays of light...The first knowledge which the public has had about it is due to Erasmus Bartholinus, who has given a description of Iceland Crystal and of its chief phenomena...”



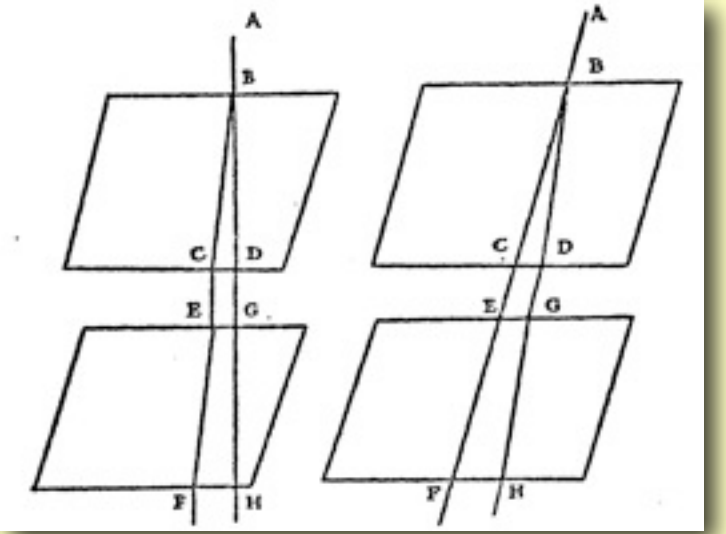




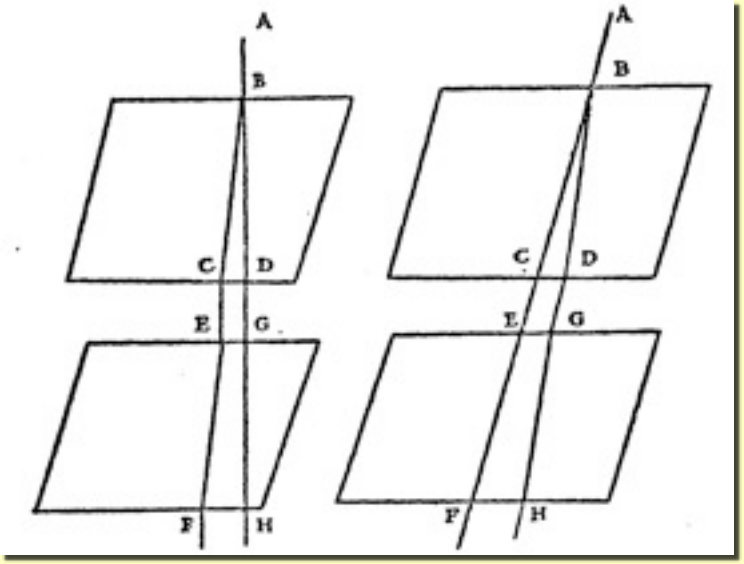
”As there were two different refractions, I conceived that there were also two different emanations of waves of light, and that one could occur in the ethereal matter extending through the body of the Crystal. I attributed to this emanation of waves the regular refraction which is observed in this stone, by supposing these waves to be ordinarily of spherical form, and having a slower progression within the Crystal than they have outside it; whence proceeds refraction as I have demonstrated.”



”As to the other emanation which should produce the irregular refraction, I wished to try what Elliptical waves, or rather spheroidal waves, would do; and these I supposed would spread indifferently both in the ethereal matter diffused throughout the crystal and in the particles of which it is composed, according to the last mode in which I have explained transparency. It seemed to me that the disposition or regular arrangement of these particles could contribute to form spheroidal waves (nothing more being required for this than that the successive movement of light should spread a little more quickly in one direction than in the other) and I scarcely doubted that there were in this crystal such an arrangement of equal and similar particles...”



”...Before finishing the treatise on this Crystal, I will add one more marvellous phenomenon which I discovered after having written all the foregoing. For though I have not been able till now to find its cause, I do not for that reason wish to desist from describing it, in order to give opportunity to others to investigate it... The phenomenon is, that by taking two pieces of this crystal and applying them one over the other, or rather holding them with a space between the two, if all the sides of one are parallel to those of the other, then a ray of light, such as AB, is divided into two in the first piece, namely into BD and BC, following the two refractions, regular and irregular. On penetrating thence into the other piece each ray will pass there without further dividing itself in two; but that one which underwent the regular refraction, as here DG, will undergo again only a regular refraction at GH; and the other, CE, an irregular refraction at EF. And the same thing occurs not only in this disposition, but also in all those cases in which the principal section of each of the pieces is situated in one and the same plane, without it being needful for the two neighbouring surfaces to be parallel. Now it is marvellous why the rays CE and DG, incident from the air on the lower crystal, do not divide themselves the same as the first ray AB.”



”One would say that it must be that the ray DG in passing through the upper piece has lost something which is necessary to move the matter which serves for the irregular refraction; and that likewise CE has lost that which was necessary to move the matter whicn serves for regular refraction: but there is yet another thing which upsets this reasoning. It is that when one disposes the two crystals in such a way that the planes which constitute the principal sections intersect one another at right angles, whether the neighbouring surfaces are parallel or not, then the ray which has come by the regular refraction, as DG, undergoes only an irregular refraction in the lower piece; and on the contrary the ray which has come by the irregular refraction, as CE, undergoes only a regular refraction.

But in all the infinite other positions, besides those which I have just stated, the rays DG, CE, divide themselves anew each one into two, by refraction in the lower crystal, so that from the single ray AB there are four, sometimes of equal brightness, sometimes some much less bright than others, according to the varying agreement in the positions of the crystals: but they do not appear to have all together more light than the single ray AB...

But to tell how this occurs, I have hitherto found nothing which satisfies me..”

OPTICKS:

OR, A

TREATISE

OF THE

REFLEXIONS, REFRACTIONS,
INFLEXIONS and COLOURS

OF

LIGHT.

ALSO

Two TREATISES

OF THE

SPECIES and MAGNITUDE

OF

Curvilinear Figures.

LONDON,
Printed for SAM. SMITH, and BENJ. WALFORD,
Printers to the Royal Society, at the *Prince's Arms* in
St. *Paul's* Church-yard. MDCCIV.

The FIRST BOOK OF OPTICKS.

PART I.

MY Design in this Book is not to explain the Properties of Light by Hypotheſes, but to propoſe and prove them by Reaſon and Experiments: In order to which, I ſhall premiſe the following Definitions and Axioms.

DEFINITIONS.

DEFIN. I.

BY the Rays of Light I underſtand its leaſt Parts, and thoſe as well Succeſſive in the ſame Lines as Contemporary in ſeveral Lines. For it is manifeſt that Light conſiſts of parts both Succeſſive and Contemporary; becauſe in the ſame place you may ſtop that which comes one moment, and let paſs that which comes preſently after; and in the ſame time you may ſtop it in any one place, and let it paſs in any other. For that part of Light which is ſtopt cannot be the ſame with that which is let paſs. The leaſt Light or part of Light, which may be ſtopt alone without the reſt of the Light, or propagated alone, or do or ſuffer any

Optics or a Treatise of the Reflections, Refractions, Inflections & Colours of Light

Three books without separate titles.

Book 1 (in two parts): principles of geometrical optics, description of the phenomena of reflection, refraction, and dispersion of light, the properties of white light, the explanation of the rainbow, and construction of telescopes.

Book 2 (in four parts): interference of light in thin layers.

Book 3 (has only Part 1!): diffraction and double refraction of light.

Fig. 65.

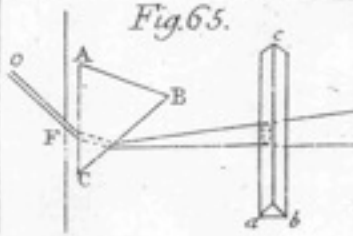


Fig. 66.

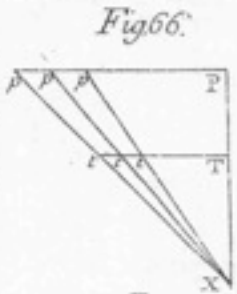


Fig. 67.

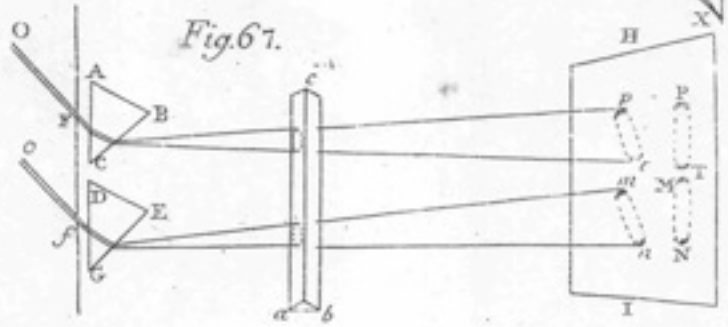


Fig. 68.

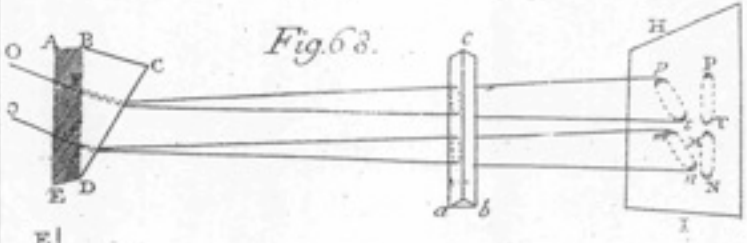


Fig. 69.

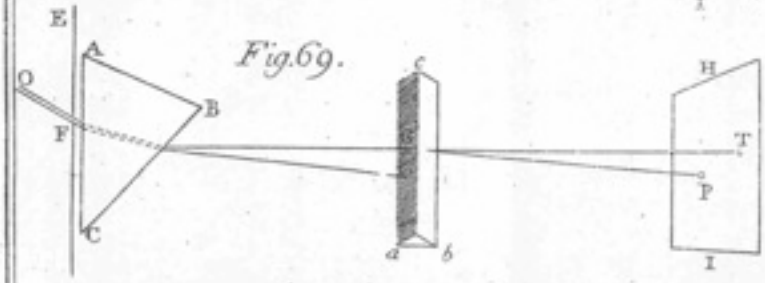


Fig. 124.

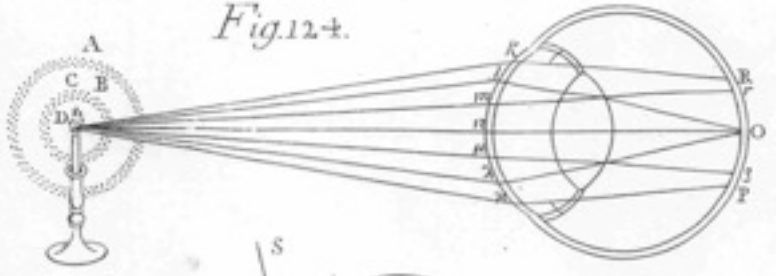


Fig. 125.

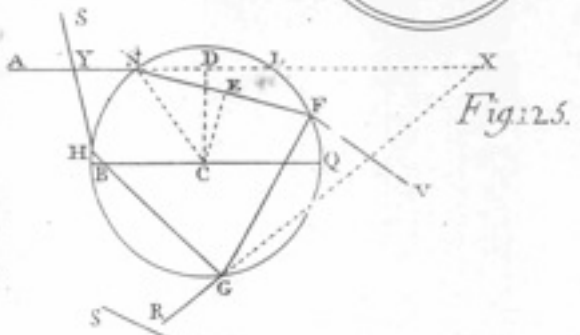
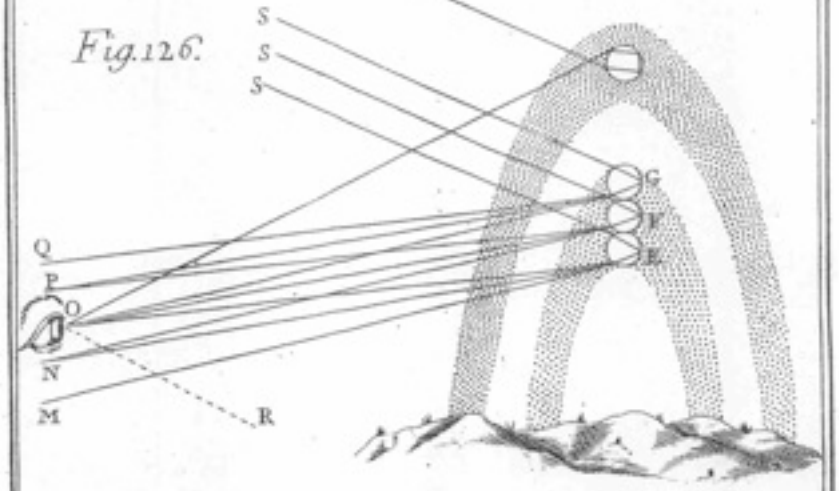


Fig. 126.



Newton – *Opticks* (end of Book 3)

”When I made the foregoing Observations, I designed to repeat most of them with more care and exactness, and to make some new ones for determining the manner how the Rays of Light are bent in their passage by Bodies for making the Fringes of Colours with the dark lines between them. But I was then interrupted, and cannot now think of taking these things into farther consideration. And since I have not finished this part of my Design, I shall conclude, with proposing only some Queries in order to a farther search to be made by others.”

Newton - *Opticks* (Query 12)

”Do not the Rays of Light in falling upon the bottom of the Eye excite Vibrations in the *Tunica Retina*? Which Vibrations, being propagated along the solid Fibres of the optick Nerves into the Brain, cause the Sense of seeing...”

Newton - *Opticks* (Query 13)

”Do not several sorts of Rays make Vibrations of several bignesses, which according to their bignesses excite Sensations of several Colours much after the manner that the Vibrations of the Air, according to their several bignesses excite Sensations of several sounds? And particularly do not the most refrangible Rays excite the shortest Vibrations for making a Sensation of deep violet, the least refrangible the largest for making a Sensation of deep red, and the several intermediate sorts of Rays, Vibrations of several intermediate bignesses to make Sensations of the several intermediate Colours?...”

Newton - *Opticks* (Query 28)

”Are not all Hypotheses erroneous, in which Light is supposed to consist in Pression or Motion, propagated through a fluid Medium? For in all these Hypotheses, the Phaenomena of Light have been hitherto explain'd by supposing that they arise from new Modifications of the Rays; which is an erroneous Supposition.

If Light consisted only in Pression propagated without actual Motion, it would not be able to agitate and heat the Bodies which refract and reflect it. If it consisted in Motion propagated to all distances in an instant, it would require an infinite force every moment, in every shining Particle, to generate that Motion. And if it consisted in Pression or Motion, propagated either in an instant or in time, it would bend into the Shadow. For Pression or Motion cannot be propagated in a Fluid in right Lines beyond an Obstacle which stops part of the Motion, but will bend and spread every way into the quiescent Medium which lies beyond the Obstacle...”

Newton - *Opticks* (Query 31)

”Have not the small Particles of Bodies certain Powers, Virtues or Forces, by which they act at a distance, not only upon the Rays of Light for reflecting, refracting and inflecting them, but also upon one another for producing a great part of the Phaenomena of Nature? For it's well known that Bodies act one upon another by the Attractions of Gravity, Magnetism and Electricity; and these Instances shew the Tenor and Course of Nature, and make it not improbable but that there may be more attractive Powers than these...

The Attractions of Gravity, Magnetism and Electricity, reach to very sensible distances, and so have been observed by vulgar Eyes, and there may be others which reach to so small distances as hitherto escape Observation; and perhaps electrical Attraction may reach to such small distances, even without being excited by Friction...”

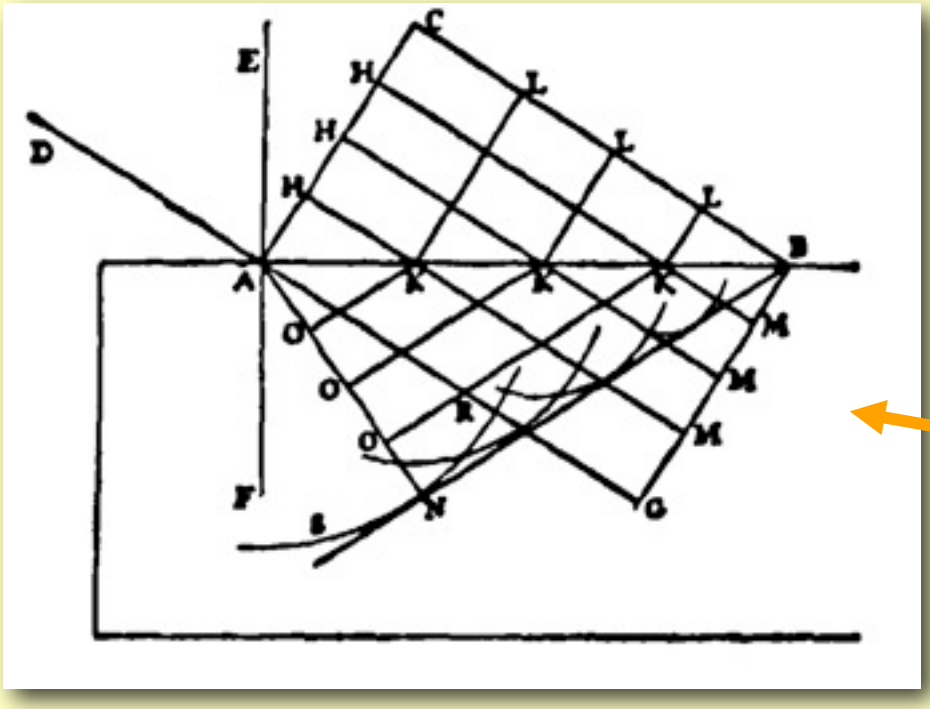
Newton - *Opticks* (Query 31) cont.

”...There are therefore Agents in Nature able to make the Particles of Bodies stick together by very strong Attractions. And it is the Business of experimental Philosophy to find them out.
Now the smallest Particles of Matter may cohere by the strongest Attractions, and compose bigger Particles of weaker Virtue; and many of these may cohere and compose bigger Particles whose Virtue is still weaker, and so on for divers Successions, until the Progression end in the biggest Particles on which the Operations in Chymistry, and the Colours of natural Bodies depend, and which by cohering compose Bodies of a sensible Magnitude.”

Newton - *Opticks* (Query 31) cont.

”All these things being consider'd, it seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and figures, and with such other Properties, and in such Proportion to Space, as most conduced to the End for which he form'd them; and that these primitive Particles being Solids, are incomparably harder than any porous Bodies compounded of them; even so very hard, as never to wear or break in pieces: No ordinary Power being able to divide what God himself made one in the first Creation. While the Particles continue entire, they may compose Bodies of one and the same Nature and Texture in all Ages: But should they wear away, or break in pieces, the Nature of Things depending on them, would be changed.
Water and Earth composed of old worn Particles and Fragments of Particles, would not be of the same Nature and Texture now, with Water and Earth composed of entire Particles, in the Beginning...”

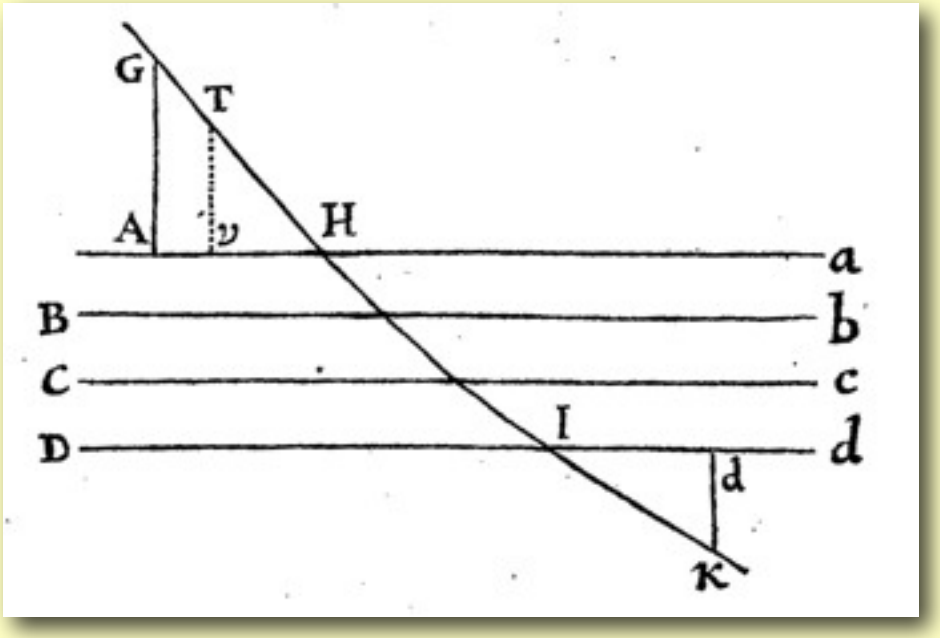
Refraction of light in Huygens' theory



medium of greater density

Hyugens assumed that the pulse of light moves with smaller speed in denser medium. Then from well-known construction of wave fronts at the boundary of two media of different density he obtained the law of refraction

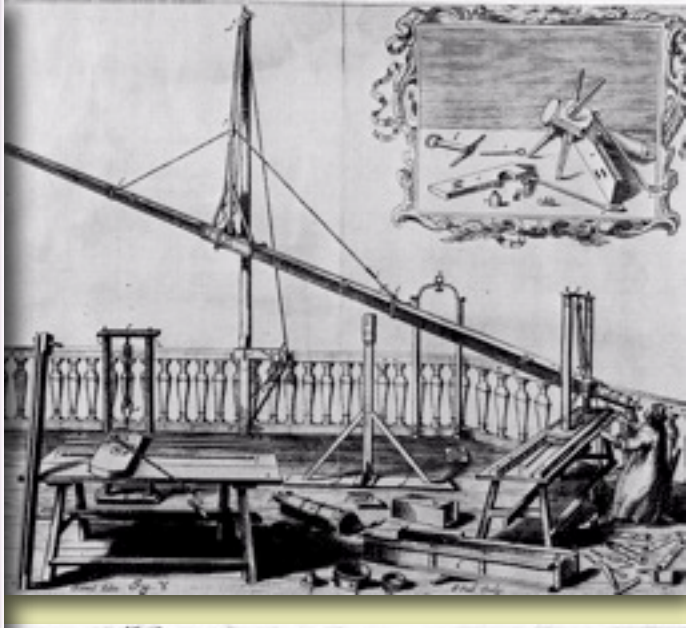
Refraction of light in Newton's theory

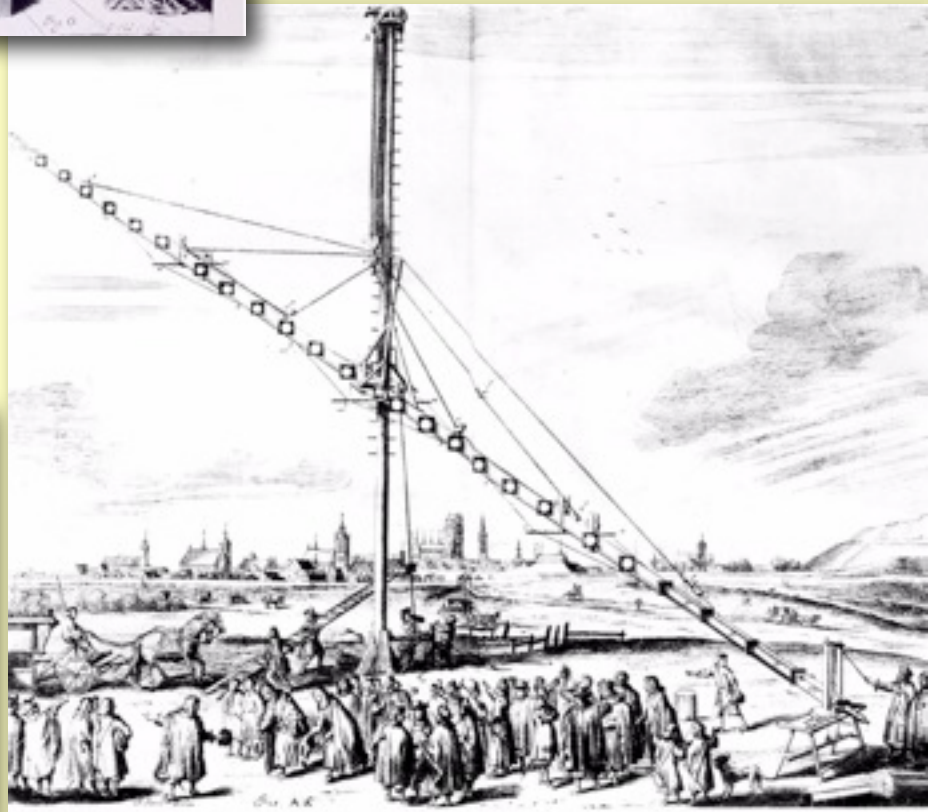


"…velocity of the body before its incidence is to its velocity after emergence as the sine of emergence is to the sine of incidence."
(*Principia*, Book I, Chapter XIV)

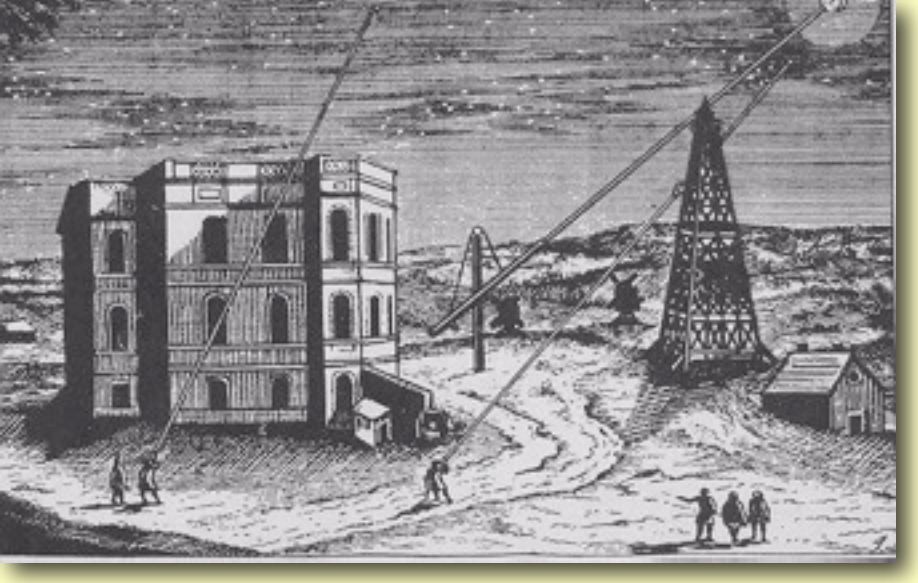
Progress in astronomy

Johannes (Jan) Hevelius (1611-1687)

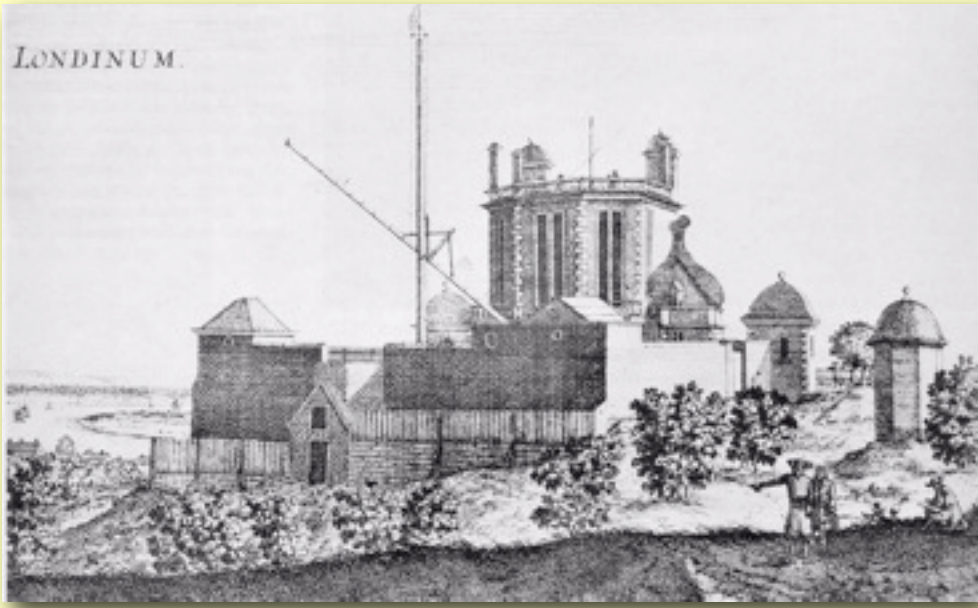




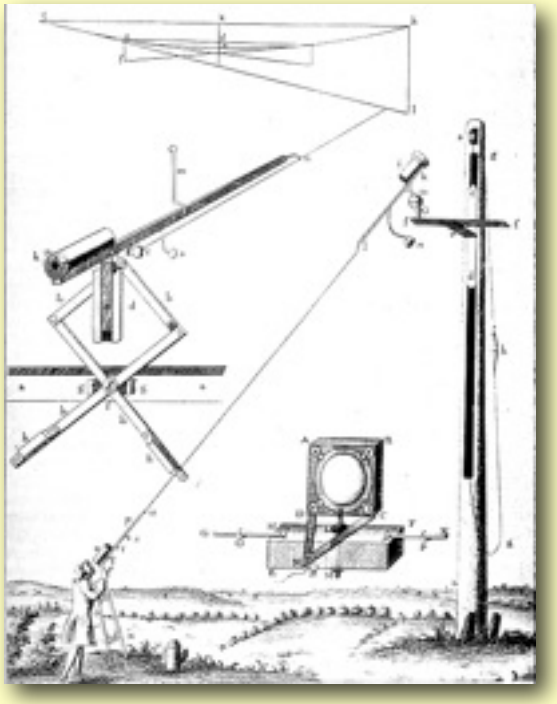

JOHANNIS HEVELII
MACHINA COELESTIS



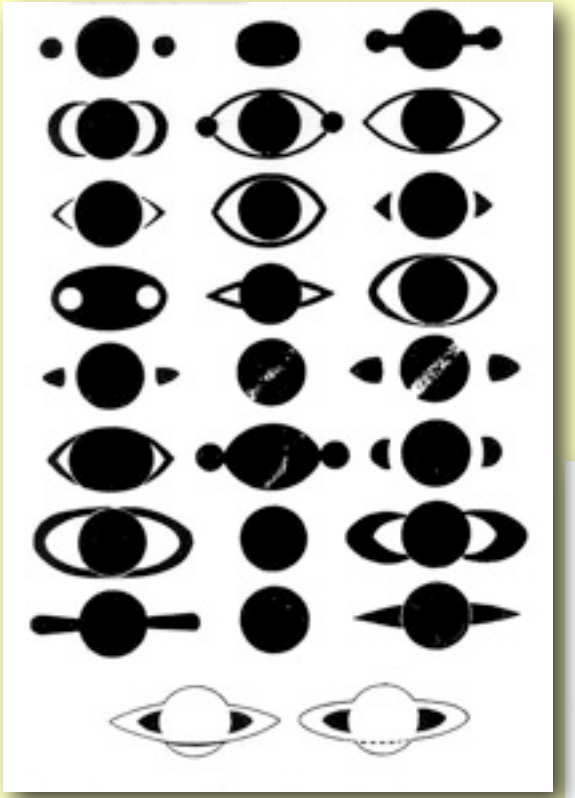
Paris Observatory



Greenwich Observatory



Saturn's ring from Galileo to Huygens



CRISTIANI HVGENII
ZVLICHEMII, CONST. F.

SYSTEMA SATVRNIVM,

SIVE

De causis mirandorum SATVRNI Phænomenôn,

ET

Comite ejus

PLANETA NOVO

HAGÆ-COMITIS,

Ex Typographia ADRIANI VLACQ.
M. DC. LIX.