Petersen Diagrams for RR Lyræ Stars in the Magellanic Clouds

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1. Introduction

The importance of double-mode pulsators in astrophysics has been realized many years ago by Petersen (1973). He showed that star positions in the $P_1/P_0 - P_0$ diagram (P_0 and P_1 are, respectively, fundamental mode and first overtone periods) is a useful probe of stellar properties. The diagram is now called the *Petersen diagram*. Petersen (1973) found that Cepheid positions were in a very significant disagreement with the theoretical predictions for typical Cepheid masses. The discrepancy led Simon (1982) to the seminal suggestion that the discrepancy could be eliminated if the metal contribution to stellar opacities at temperatures $\sim 2 \cdot 10^5$ K is augmented by a large factor. This suggestion became the crucial impulse to the OPAL (Iglesias et al., 1992) and OP (Seaton et al., 1994) projects. With the new opacity data the observed period ratios could be reproduced within the acceptable range of stellar parameters. The important question now is whether the two period values could be used to reliably constrain these parameters.

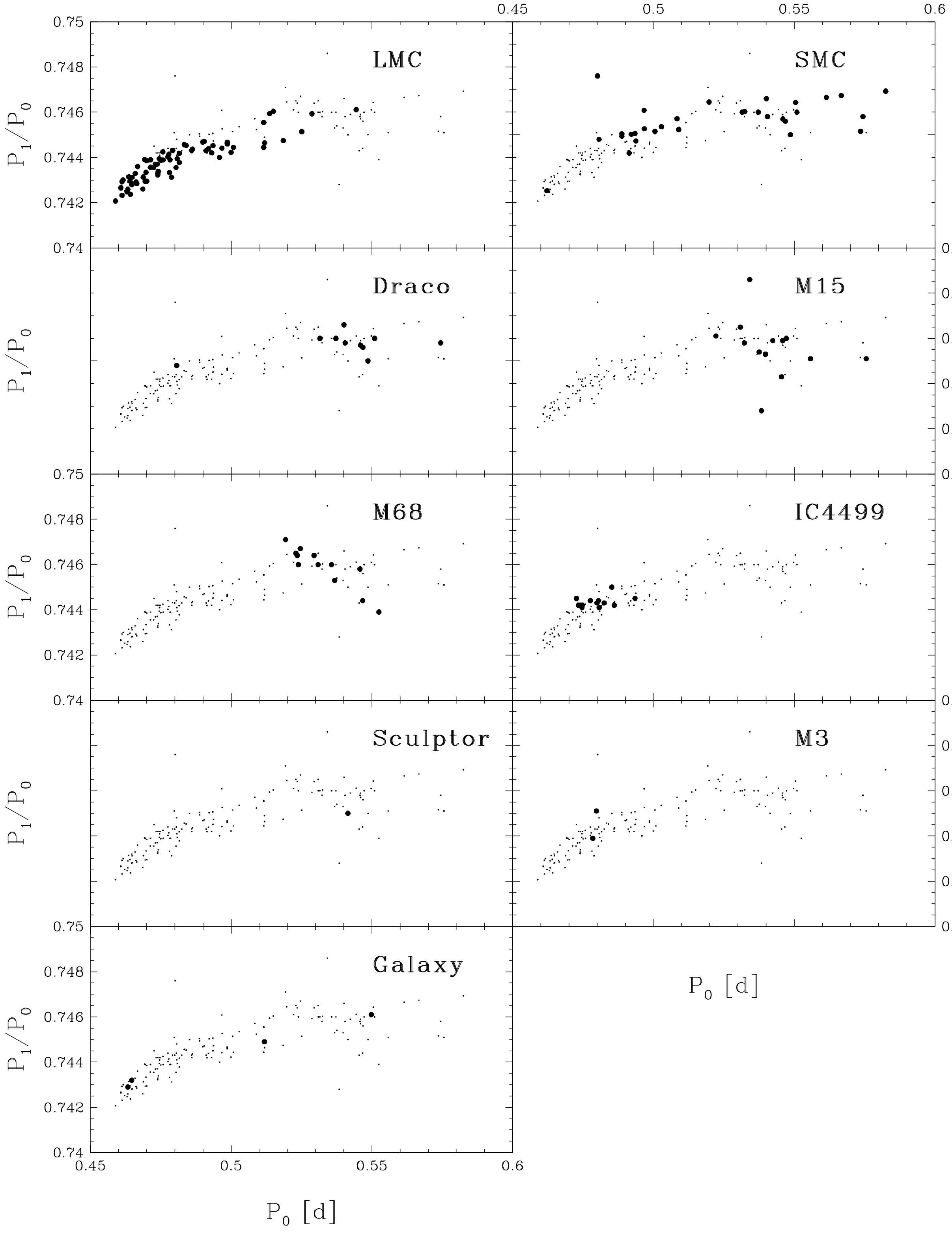

LMC
SMC
Draco
M15
M68
IC4499
Sculptor
M3
Galaxy
P_1/P_0
P_0 [d]
0.45
0.5
0.55
0.6
0.74
0.742
0.744
0.746
0.748
0.75

 Figure 1:

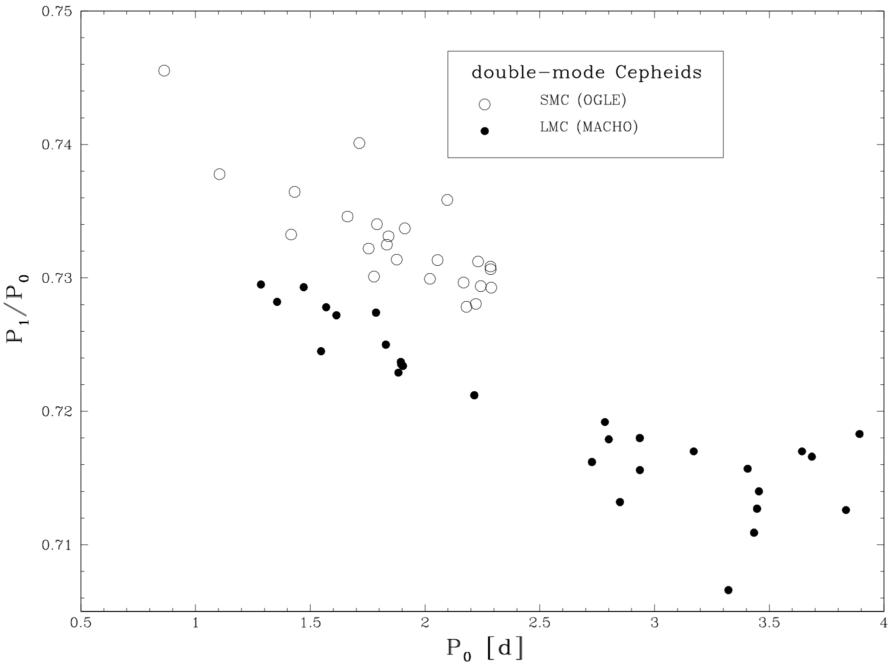


Figure 2: Observations of double-mode Cepheids from Magellanic Clouds.

Massive photometric surveys resulted in a dramatic increase of the number of double-mode pulsators. In our work we focus attention on the *Petersen diagrams* for RRd stars in the Magellanic Clouds. RR Lyræ stars belong to the oldest objects thus data about them may teach us something about the early history of star formation in these systems. We also discuss reliability of theoretical predictions of the $P_1/P_0 - P_0$ dependence.

Figure 1: *Petersen diagrams* for observed RR Lyræ stars belonging to different clusters or galaxies: small dots represent all of observed RRd stars.

In figure 1 we show the band of the RRd stars in the *Petersen diagram*. The Magellanic Clouds objects populate essentially the whole band while those from the other dwarf galaxies and the galactic globular clusters occur only in isolated areas. There is a striking difference in the shape of the band for RRd stars and that for double-mode Cepheids shown in figure 2. In the latter case we see a monotonic decrease while in the former, after an initially fast increase, we see a flattening of the dependence.

Explaining the shape of the RRd band in the Petersen diagram was our main goal.

2. Theoretical Petersen diagram Main parameters:

We considered unfitted models of RR Lyræ star envelopes. The models depend on the following stellar parameters: mass(M), temperature(T_{eff}), luminosity(L) and chemical abundances(X, Z). The least important for our study was X which we fixed at the value of 0.76. Thus, we considered a four-parameter family of the envelopes for which the periods of the first two radial modes covered the range for RRd stars. The models were calculated with the use of OPAL (Rogers and Iglesias, 1995) opacities and EOS. The pulsation periods were determined with a linear nonadiabatic approximation.

In figures 3 and 4 we show the influence of the four stellar parameters on the star position in the *Petersen diagram*.

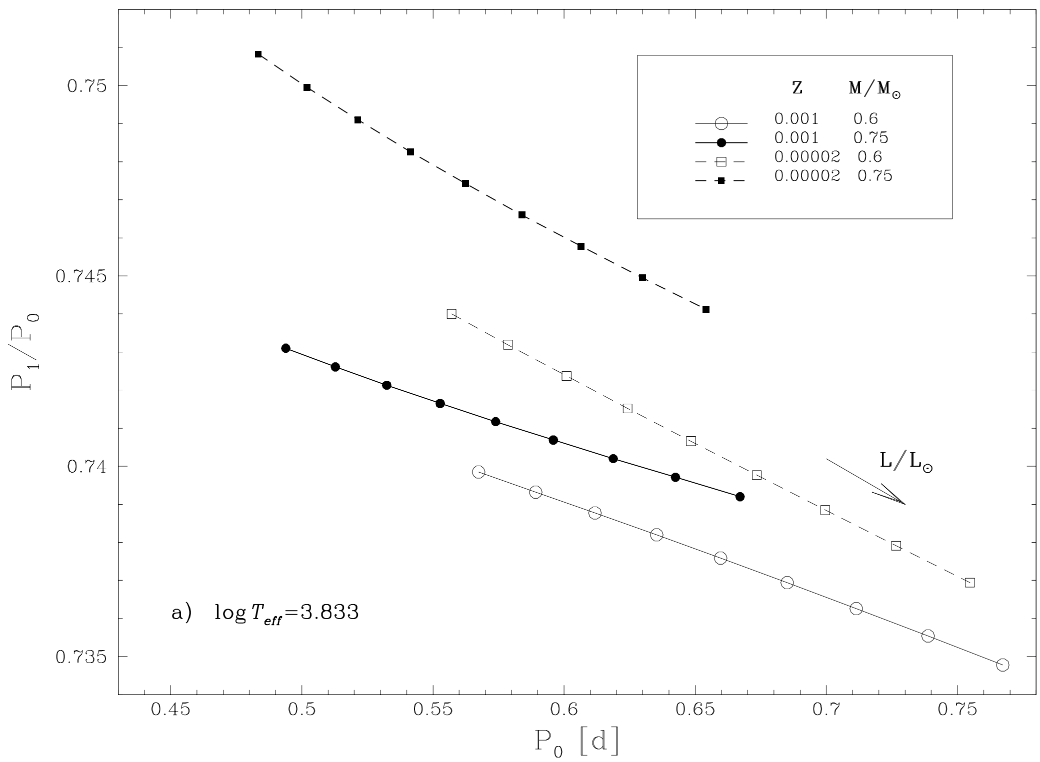


Figure 3: Theoretical *Petersen diagram* for models calculated at fixed T_{eff} and various $L/L_{\odot}$. Two values of mass and metal abundance are shown.

We see a well-known fact that Z is the parameter which causes primarily a vertical shift while the remaining three parameters cause primarily a horizontal shifts. All of them however have significant vertical components. Clearly, since we utilize here only two observables we cannot determine all the parameters.

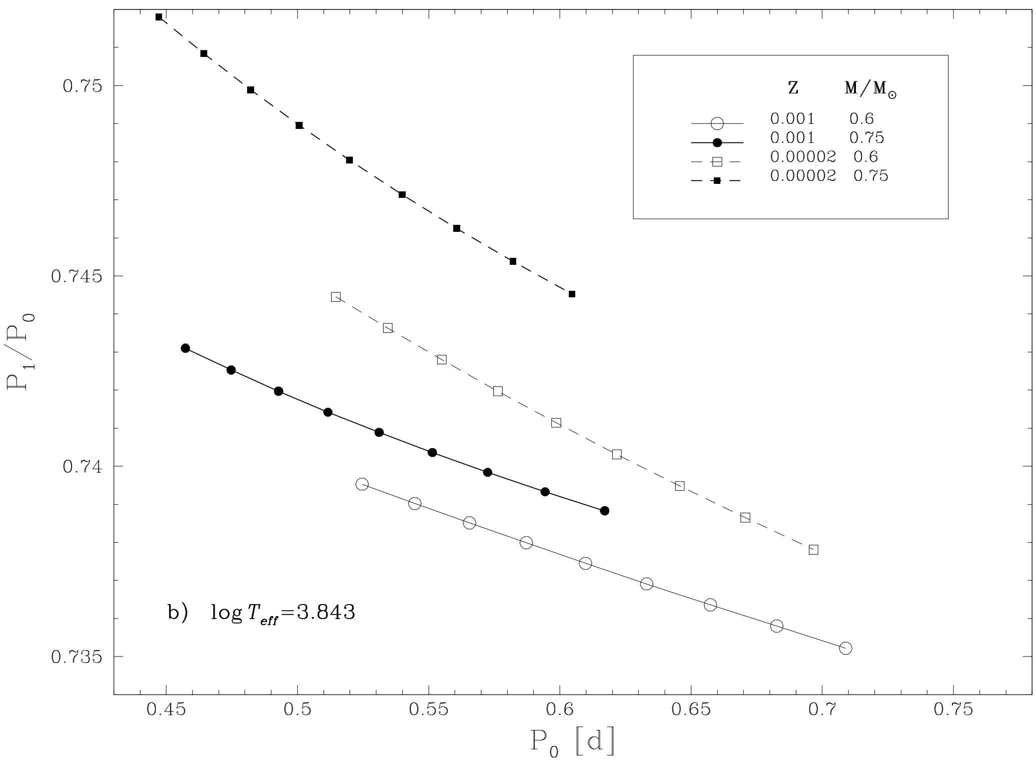


Figure 4: Same as in figure a) but with higher temperature. The increase of $\log T_{eff}$ by 0.01 results in decrease of P_1/P_0 at fixed P_0 by 0.001.

Uncertainties:

We believe that the main uncertainty in model calculations is connected with the treatment of convection. We use the standard MLT approximation. To asses the uncertainty we carried calculation with two values of the parameter α: 1 (our standard value) and 2. The effect, especially at lower T_{eff} is quite significant. The increase of α at the lower T_{eff} value is similar to a Z increase by about 0.0005.

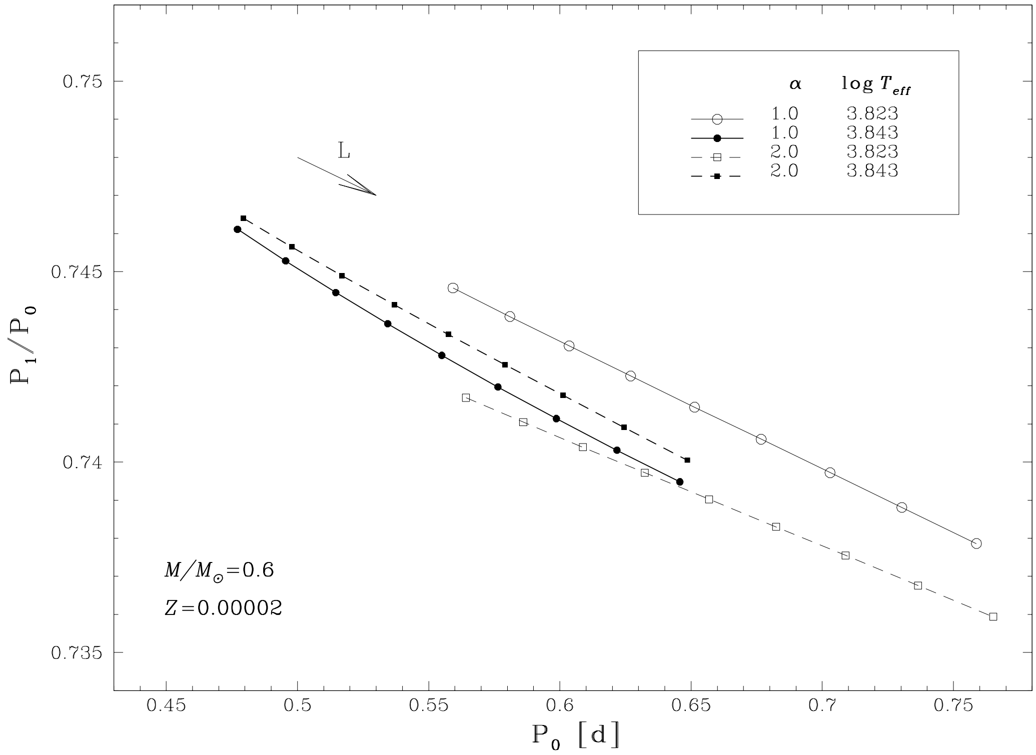
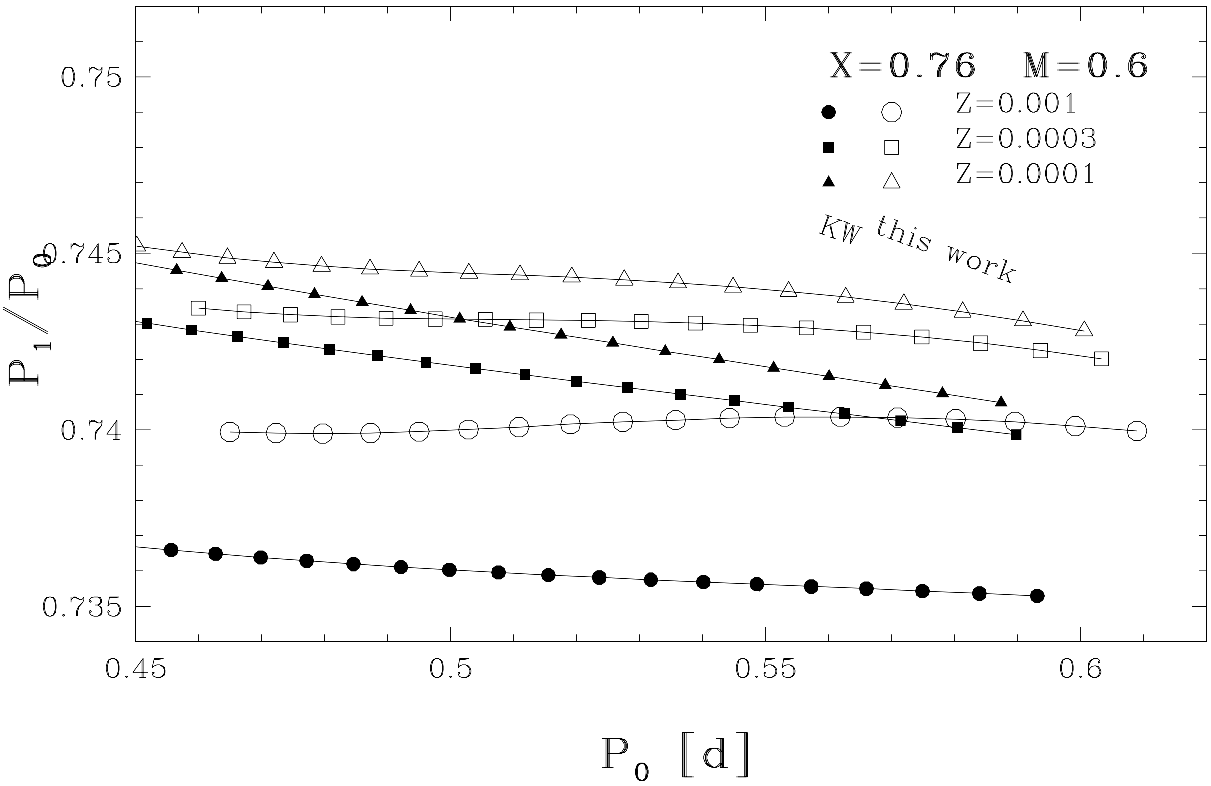
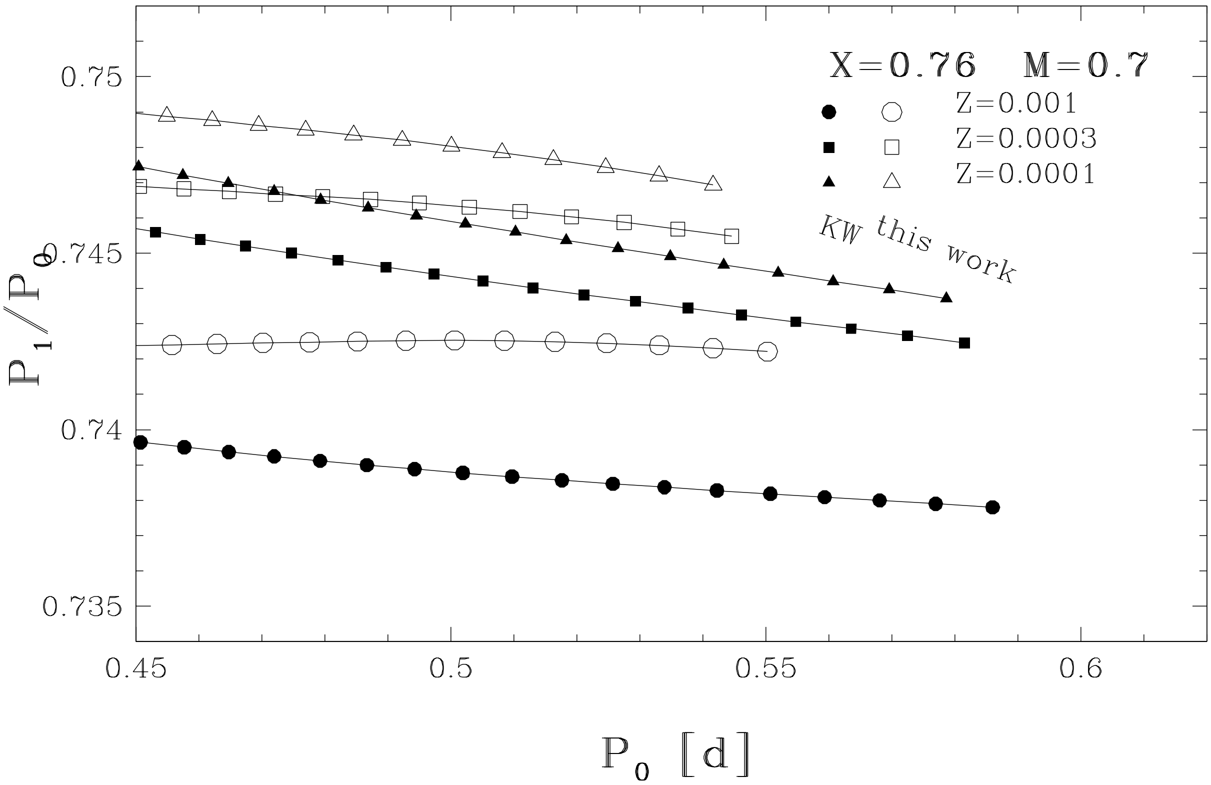


Figure 5: Effect of the MLT parameter α.

There are also uncertainties involved in treatment of pulsation. The most important are connected with nonlinear effects which alter the periods. The problem was addressed by Bono et al. (1996). They calculated period of nonlinear monomode pulsation of a star in the EO range. They found that the nonlinear value of P_1/P_0 is lower by about 0.005. However, it is not clear whether the same difference applies to genuine double-mode pulsators.

Figure 6: Comparison of the models obtained in this work with those of Kovács and Walker (KW,1999).


X=0.76 M=0.6
Z=0.001
Z=0.0003
Z=0.0001
KW this work
P_1/P_0
0.75
0.745
0.74
0.735
0.45
0.5
0.55
0.6
P_0 [d]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0.735
0.45
0.5
0.55
0.6
P_0 [d]

Figure 6:

Feuchtinger (1998) who was able to reproduce double-mode pulsation with his nonlinear code found $P_1/P_0 = 0.7468$ for a model with $M = 0.65\ M_\odot$, $T_{eff} = 6820\ K$, $\log L/L_\odot = 1.72$, $X = 0.76$ and $Z = 0.001$. For the same model our linear value was less by 0.006. In this case we do not know whether the difference may be attributed to the nonlinear effect.

There are certainly other uncertainties in the P_1/P_0 for which we do not have explanation. They are revealed by a comparison of the results obtained by different teams which apparently use the same physics. Here we illustrate it in figure 6 where we compare our results with those of Kovács and Walker (1999).

The uncertainties in the calculated values of the P_1/P_0 ratios are too large to enable a full use of the precise observational data. Improvements in modeling are needed. Therefore we regard it premature to attach significance to inferences regarding specific values of the metallicity parameter Z. However, since the form of the dependence of the P_1/P_0 on stellar parameters is not very much affected by the uncertainties we will proceed to our main goal that is to the interpretation of the RRd band in the *Petersen diagram*.

The RRd band in the Petersen diagram

We will show here that the pattern of the RRd band shown in figure 1 may be reproduced by considering a single stellar parameter Z. The theoretical $P_1/P_0 - P_0$ relation may be constrained with the use of evolutionary model calculations and/or empirical relations such as the *luminosity-metallicity* dependence. We will use both.

Evolutionary models of horizontal branch stars yield the following relations:

$$\begin{cases} L_{HB} = L_{HB}(Z, X, \log T_{eff}) \\ M_{HB} = M_{HB}(Z, X, \log T_{eff}) \end{cases}$$

Since evolutionary tracks in the RR Lyræ range make loops, each of the relations may have two branches. We will see that the first relation is far more important and for that we prefer to rely on empirical formulæ. For the second relation we use calibration of Dorman's (1992) modeling. We selected two values of $\log T_{eff}$: 3.8 and 3.9 which generously encompass the RR Lyræ range. If there was a loop we selected the lower luminosity models but we checked that the choice is not essential. The *mass-metallicity* relation has the following form:

$$M/M_{\odot} = \alpha \cdot \log Z + \beta$$

The coefficient are given in the table below.

name	$\log T_{eff}$	α	β
MZ1	3.8	-0.15493	0.239176
MZ2	3.9	-0.09534	0.347183

The *luminosity-metallicity* relation is usually represented in the form:

$$\mathcal{M}_V[mag] = \epsilon \cdot [Fe/H] + \zeta,$$

where ϵ and ζ coefficients are:

name	source	ϵ	ζ
LZ1	(Buonanno et al., 1990)	0.39	1.3
LZ2	(Walker, 1992)	0.15	0.73

Adopting $BC = -0.1$ (after (Smith, 1995)) and $Z = (0.015555 - 0.019031 \cdot [Fe/H]) \cdot 10^{[Fe/H]}$ (a fit of Dorman's (1992) tabulated data) we determined the *luminosity-metallicity* relation shown in figure 7. In the same figure we also show the relations based on the evolutionary model calculations.

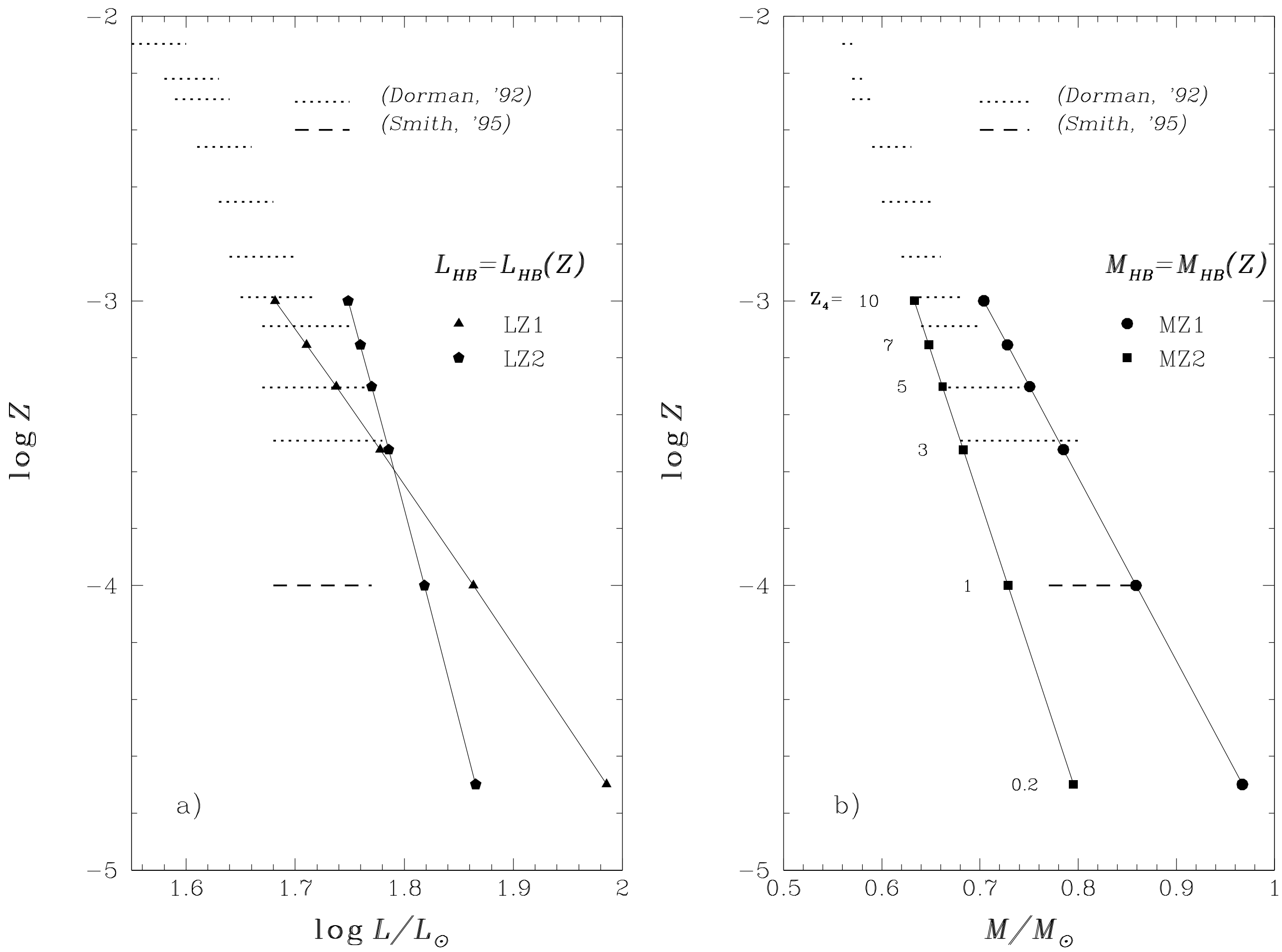

Figure 7: The *luminosity-metallicity* (a) and *mass-metallicity* (b) relations (oblique solid lines) for horizontal branch stars. Dashed horizontal lines are from evolutionary models. $Z_4 = Z \cdot 10^4$.

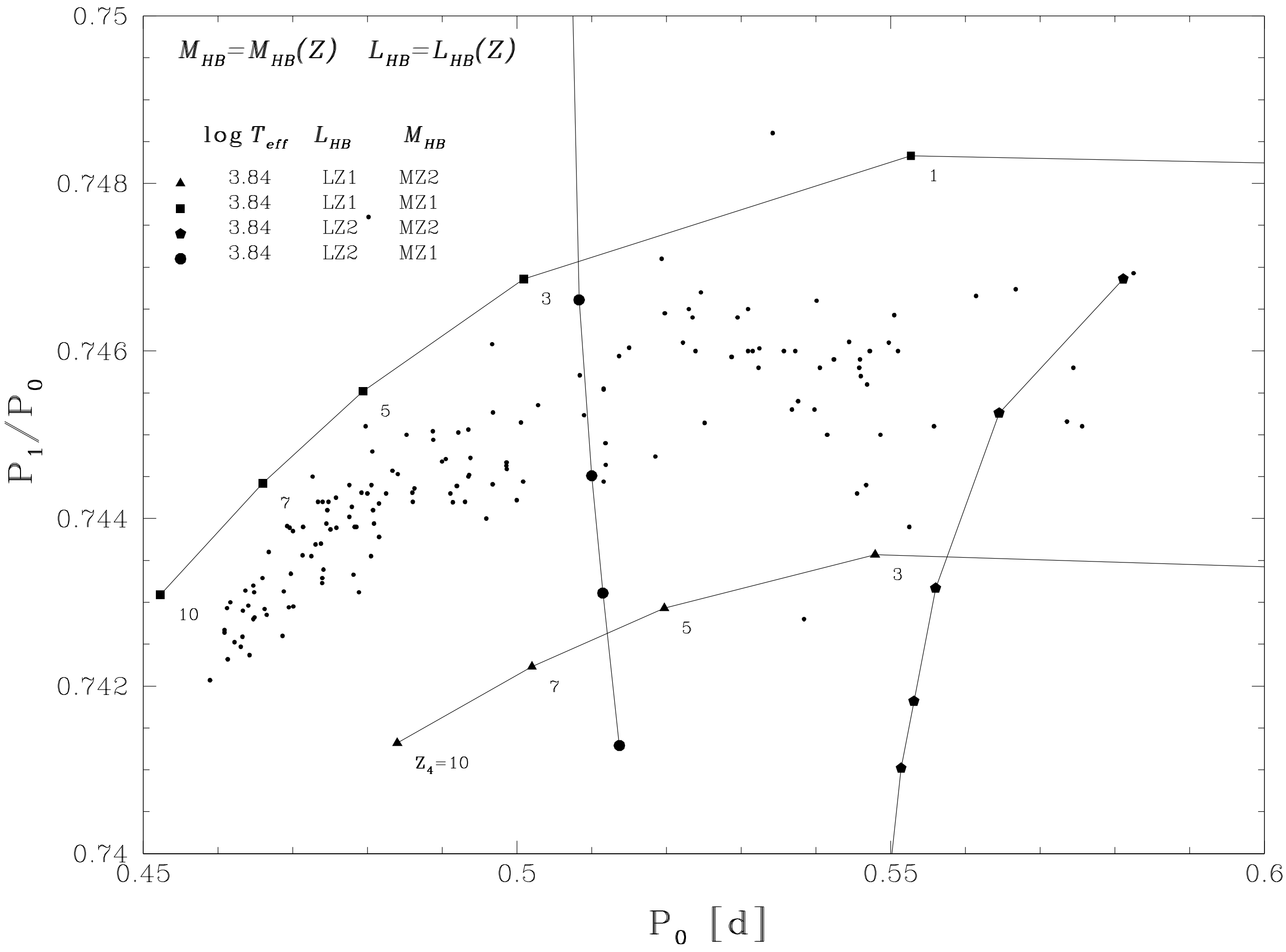

Figure 8: The $P_1/P_0 - P_0$ relations for models with adopted the $M(Z)$ and $L(Z)$ relations (see figure 7). The dots are observational data for RRd stars. The crucial role of the $L(Z)$ dependence in reproducing observations is visible. The shape of the observed band is well reproduced only with the LZ1 (Buonanno et al., 1990). $Z_4 = Z \cdot 10^4$.

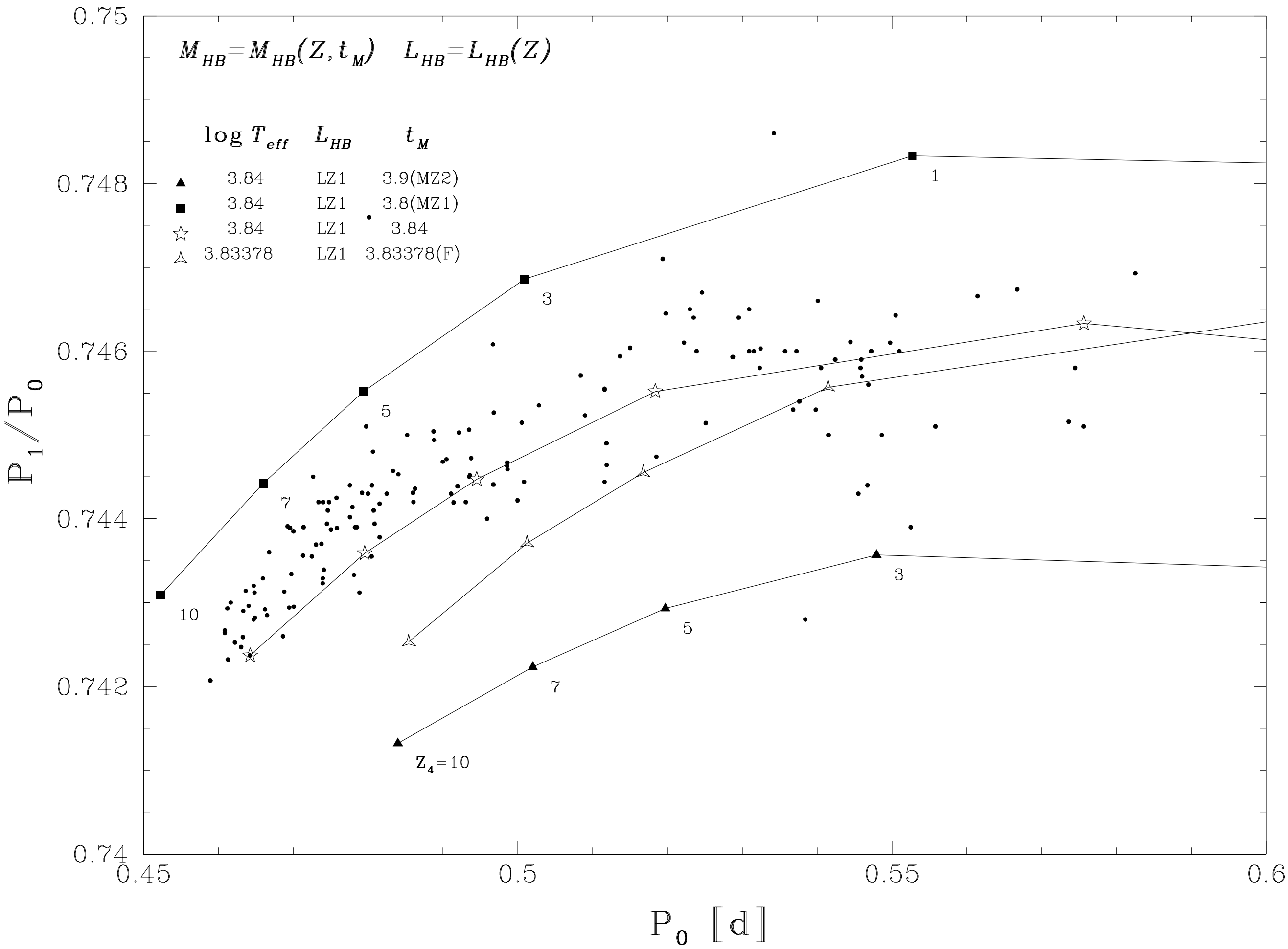

Figure 9: The $P_1/P_0 - P_0$ relations for models with the $M(Z)$ relations fitted to various model temperatures t_M. The plot shows that with this adjustment one may explain both the width and spread of the RRd band.

Conclusions

- The shape of the RRd band in the *Petersen diagram* is determined primarily by the *luminosity-metallicity* relation. With the empirical relation of Buonanno et al. (1990) the models reproduced the observed shape.
- The width of the band reflects the range of the metallicity parameter Z. In the Magellanic Clouds the double-mode RR Lyræ stars span the fundamental mode period range of $0.46 \div 0.58\ d$. According to our models it corresponds to the metallicity range $Z = 0.0001 \div 0.001$. However, these numbers are not trusted because of the uncertainties which we have discussed.
- There is a difference in the population of RRd between two Magellanic Clouds. In LMC we see (figure 1) many more objects at the short period end of the band that is objects with higher metallicities.

We believe that our work reveals usefulness of *Petersen diagrams* for diagnosis of the old stellar populations. The uncertainties in the calculated period ratios, which we identified, point to the need of further improvements in modeling of double-mode pulsation.

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