The Minimal Extension of the Standard Model
- the case for Dark Matter and neutrino physics -

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- The little hierarchy problem
- The model and the little hierarchy problem
- Dark Matter
- Neutrino physics
- Summary and comments

Figure 1: Red is the 90\% CL allowed range, from PDG 2008. $m_h < 161$ GeV at the 95\% CL.
The little hierarchy problem:

\[ m_h^2 = m_h^{(B)}^2 + \delta^{(SM)}m_h^2 + \ldots \]

\[ \delta^{(SM)}m_h^2 = \frac{\Lambda^2}{\pi^2 v^2} \left[ \frac{3}{2} m_t^2 - \frac{1}{8} (6m_W^2 + 3m_Z^2) - \frac{3}{8} m_h^2 \right] \]

\[ m_h = 130 \text{ GeV} \Rightarrow \delta^{(SM)}m_h^2 \simeq m_h^2 \text{ for } \Lambda \simeq 580 \text{ GeV} \]

- For \( \Lambda \gtrsim 580 \text{ GeV} \) there must be a cancellation between the tree-level Higgs mass

\[ m_h^{(B)}^2 \] and the 1-loop leading correction \( \delta^{(SM)}m_h^2 \):

\[ m_h^{(B)}^2 \sim \delta^{(SM)}m_h^2 \geq m_h^2 \]

\[ \downarrow \]

the perturbative expansion is breaking down.

- The SM cutoff is very low!
Solutions to the little hierarchy problem:

♠ Suppression of corrections growing with $\Lambda$ at the 1-loop level:

⇒ The Veltman condition, no $\Lambda^2$ terms at the 1-loop level:

\[
\frac{3}{2}m_t^2 - \frac{1}{8} (6m_W^2 + 3m_Z^2) - \frac{3}{8}m_h^2 = 0 \implies m_h \simeq 310 \text{ GeV}
\]

In general

\[
m_h^2 = m_h^{(B)}^2 + 2\Lambda^2 \sum_{n=0}^{\infty} f_n(\lambda, \ldots) \ln^n \left( \frac{\Lambda}{\mu} \right)
\]

where

\[
(n+1)f_{n+1} = \mu \frac{\partial}{\partial \mu} f_n = \beta_i \frac{\partial}{\partial \lambda_i} f_n
\]

with

\[
f_0 = \frac{1}{\pi^2 v^2} \left[ \frac{3}{2}m_t^2 - \frac{1}{8} (6m_W^2 + 3m_Z^2) - \frac{3}{8}m_h^2 \right]
\]

and

\[
f_n \propto \frac{1}{(16\pi^2)^{n+1}}
\]
Figure 2: Contour plots of $D_t$ corresponding to $D_t = 10 \ (10\%)$ and $100 \ (1\%)$ for $n \leq 2$, from Kolda & Murayama hep-ph/0003170.

$$D_t \equiv \frac{\delta^{(SM)} m_h^2}{m_h^2} = \frac{2 \Lambda^2}{m_h^2} \sum_{n=0}^{\infty} f_n(\lambda, \ldots) \ln^n \left( \frac{\Lambda}{\mu} \right)$$
To understand the region allowed by $D_t \leq 10,100$ in the SM:

- Assume $m_h$ is such that the Veltman condition is satisfied:

$$\frac{3}{2} m_t^2 - \frac{1}{8} \left( 6m_W^2 + 3m_Z^2 \right) - \frac{3}{8} m_h^2 = 0,$$

- then at the 1-loop level $\Lambda$ could be arbitrarily large, however

- higher loops limit $\Lambda$ since the Veltman condition implies no $\Lambda^2$ only at the 1-loop level, while higher loops grow with $\Lambda^2$.

$\Rightarrow$ SUSY

$$\delta^{(SUSY)} m_h^2 \sim m_t^2 \frac{3 \lambda_t^2}{8 \pi^2} \ln \left( \frac{\Lambda^2}{m_t^2} \right),$$

then for $\Lambda \sim 10^{16-18}$ GeV one gets $m_t^2 \ll 1$ TeV in order to have $\delta^{(SUSY)} m_h^2 \sim m_h^2$.

$\text{♠ Increase of the allowed value of } m_h$: the inert Higgs model by Barbieri, Hall, Rychkov, arXiv:hep-ph/0603188, (see also Ma) $\Rightarrow m_h \sim 400 - 600$ GeV, ($m_h^2$ terms in $T$ parameter canceled by $m_{H^\pm}, m_A, m_S$ contributions).
Our goal: to lift the cutoff to multi TeV range preserving $\delta^{(SM)} m_{h}^{2} \leq m_{h}^{2}$.

- Extra gauge singlet $\varphi$ with $\langle \varphi \rangle = 0$ (to prevent $H \leftrightarrow \varphi$ mixing from $\varphi^{2}|H|^{2}$).

- Symmetry $\mathbb{Z}_{2}$: $\varphi \rightarrow -\varphi$ (to eliminate $|H|^{2}\varphi$ couplings).

- Gauge singlet neutrinos: $\nu_{Ri}$ for $i = 1, 2, 3$.

\[
V(H, \varphi) = -\mu_{H}^{2}|H|^{2} + \lambda_{H}|H|^{4} + \mu_{\varphi}^{2}\varphi^{2} + \frac{1}{24}\lambda_{\varphi}\varphi^{4} + \lambda_{x}|H|^{2}\varphi^{2}
\]

with

\[
\langle H \rangle = \frac{v}{\sqrt{2}}, \quad \langle \varphi \rangle = 0 \quad \text{for} \quad \mu_{\varphi}^{2} > 0
\]

then

\[
m_{h}^{2} = 2\mu_{H}^{2} \quad \text{and} \quad m^{2} = 2\mu_{\varphi}^{2} + \lambda_{x}v^{2}
\]

- Positivity (stability) in the limit $h, \varphi \rightarrow \infty$: $\lambda_{H}\lambda_{\varphi} > 6\lambda_{x}^{2}$

- Unitarity in the limit $s \gg m_{h}^{2}, m^{2}$: $\lambda_{H} \leq \frac{4\pi}{3}$ (the SM requirement) and $\lambda_{\varphi} \leq 8\pi$, $\lambda_{x} < 4\pi$
\[ \delta(\varphi) m_h^2 = -\frac{\lambda_x}{8\pi^2} \left[ \Lambda^2 - m^2 \log \left( c + \frac{\Lambda^2}{m^2} \right) \right] \]

\[ |\delta m_h^2| = |\delta^{(SM)} m_h^2 + \delta(\varphi) m_h^2| = D_t m_h^2 \]

\[ \downarrow \]

\[ \lambda_x = \lambda_x(m, m_h, D_t, \Lambda) \]

Figure 3: Plot of \( \lambda_x \) corresponding to \( \delta m_h^2 > 0 \) as a function of \( m \) for \( D_t = 1, \Lambda = 56 \text{ TeV} \) (left panel) and \( \lambda_x \) as a function of \( \Lambda \) for \( D_t = 1, m = 20 \text{ TeV} \) (right panel). The various curves correspond to \( m_h = 130, 150, 170, 190, 210, 230 \text{ GeV} \) (starting with the uppermost curve). The solid (dashed) lines correspond to \( c = +1 \) (\( c = -1 \)). Note that \( \lambda_x < 4\pi \).
• When $m \ll \Lambda$, the $\lambda_x$ needed for the amelioration of the hierarchy problem is insensitive to $m$, $D_t$ or $\Lambda$:

$$
\lambda_x = \left\{ 4.8 - 3 \left( \frac{m_h}{v} \right)^2 + 2D_t \left[ \frac{2\pi}{(\Lambda/\text{TeV})} \right]^2 \right\} \left[ 1 - \frac{m^2}{\Lambda^2} \ln \left( \frac{m^2}{\Lambda^2} \right) \right] + O \left( \frac{m^4}{\Lambda^4} \right).
$$

• Since we consider $\lambda_x > 1$ higher order corrections could be important. In general

$$
\left| \delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2 + \Lambda^2 \sum_{n=1} f_n(\lambda_x, \ldots) \left[ \ln \left( \frac{\Lambda}{m_h} \right) \right]^n \right| = D_t m_h^2,
$$

where the coefficients $f_n(\lambda_x, \ldots)$ can be determined recursively (see Einhorn & Jones):

$$
f_n(\lambda_x, \ldots) \sim \left[ \frac{\lambda_x}{(16\pi^2)} \right]^{n+1}
$$

If $\Lambda = 100 \text{ TeV}$, $m_h = 120 - 250 \text{ GeV}$ and $m = 10 - 30 \text{ TeV}$ the relative next order correction remains in the range $4 - 12\%$. 

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Figure 4: Contour plots of the Barbieri-Giudice parameters $\Delta_{\Lambda}$ (left panel) and $\Delta_{m}$ (right panel) for $m_h = 150$ GeV and $\lambda_x = 3.68$. 

\[
\begin{align*}
\Delta_{\Lambda} & \equiv \frac{\Lambda}{m_h^2} \frac{\partial m_h^2}{\partial \Lambda} \\
\frac{\delta m_h^2}{m_h^2} &= \Delta_{\Lambda} \frac{\delta \Lambda}{\Lambda} \\
\Delta_{m} & \equiv \frac{m}{m_h^2} \frac{\partial m_h^2}{\partial m} \\
\frac{\delta m_h^2}{m_h^2} &= \Delta_{m} \frac{\delta m}{m}
\end{align*}
\]
<table>
<thead>
<tr>
<th>model</th>
<th>$\delta m^2_{h}$</th>
<th>$\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>$\Lambda^2 \left( \frac{3\lambda_t^2}{4\pi^2} + \cdots \right) + \frac{\Lambda^2}{2} f_1^{(SM)} \ln \left( \frac{\Lambda}{\mu} \right)$</td>
<td>see plots</td>
</tr>
<tr>
<td>SUSY</td>
<td>$m_t^2 \frac{3\lambda_t^2}{8\pi^2} \ln \left( \frac{\Lambda^2}{m_t^2} \right)$</td>
<td>$m_t \lesssim 1 \text{ TeV}$ for $\Lambda \sim 10^{16-18} \text{ GeV}$</td>
</tr>
<tr>
<td>SM + $\varphi$</td>
<td>$\Lambda^2 \left( \frac{3\lambda_t^2}{4\pi^2} + \cdots \right) - \frac{\lambda_x}{8\pi^2} \left[ \Lambda^2 - m^2 \ln(c + \frac{\Lambda^2}{m^2}) \right]$</td>
<td>For $D_t = 1$ $\Lambda \sim 60 \text{ TeV}, m \sim 20 \text{ TeV}$</td>
</tr>
</tbody>
</table>

For $D_t = 1$ (no fine-tuning) and $m_h = 130 \text{ GeV}$:

- **SM**: $\Lambda \sim 1 \text{ TeV}$, while

- **SM + $\varphi$**: $\Lambda \sim 60 \text{ TeV}$ for $\lambda_x = \lambda_x(m)$ (fine tuning!) with $m = 20 \text{ TeV}$,

- The range of $(m_h, \Lambda)$ space corresponding to a given $D_t$ is expected to be larger when $\varphi$ is added to the SM, if $\lambda_x = \lambda_x(m, m_h, D_t, \Lambda)$. 

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Dark Matter


It is possible to find parameters $\Lambda$, $\lambda_x$ and $m$ such that both the hierarchy is ameliorated to the prescribed level and such that $\Omega_{\phi} h^2$ is consistent with $\Omega_{DM}$.

$\phi\phi \to hh, W^+_L W^-_L, Z_L Z_L \Rightarrow \langle \sigma v \rangle = \frac{1}{8\pi} \frac{\lambda_x^2}{m^2}$

The Boltzmann equation $\Rightarrow x_f \left( \equiv \frac{m}{T_f} \right) \simeq \ln \left[ 0.038 \frac{m_{Pl} \langle \sigma v \rangle}{g^{1/2} x^{1/2}} \right]$

$\Omega_{\phi} h^2 \simeq 1.06 \cdot 10^9 \frac{x_f}{g^{1/2} m_{Pl} \langle \sigma v \rangle} \text{GeV}$
\[ x_f \simeq 30 \quad \Rightarrow \quad m \geq x_f T_c \simeq 8 \, \text{TeV} \]
\[ \Omega_\varphi = \Omega_{DM} \quad \Rightarrow \quad \lambda_x \sim \frac{1}{4} \frac{m}{\text{TeV}} \]
\[ |\delta m_h^2| = D_t m_h^2 \quad \Rightarrow \quad m = m(\Lambda) \]

Figure 5: Plot of \( m \) as a function of the cutoff \( \Lambda \) when \( D_t = 1 \) and \( \Omega_\varphi = \Omega_{DM} \) at the 1\( \sigma \) level: \( \Omega_\varphi h^2 = 0.114 \) (left panel) and \( \Omega_\varphi h^2 = 0.098 \) (right panel); for \( m_h = 130, 150, 170, 190, 210, 230 \, \text{GeV} \) (starting with the uppermost curve) and for \( c = +1 \) solid curves and \( c = -1 \) (dashed curves).
Neutrino physics

\[ \mathcal{L}_Y = -\bar{L}Y_l H l_R - \bar{L}Y_\nu \tilde{H}\nu_R - \frac{1}{2}(\nu_R)^c M\nu_R - \varphi(\nu_R)^c Y_\varphi \nu_R + \text{H.c.} \]

\[ \mathbb{Z}_2 : \quad H \rightarrow H, \varphi \rightarrow -\varphi, \quad L \rightarrow S_L L, \quad l_R \rightarrow S_{l_R} l_R, \quad \nu_R \rightarrow S_{\nu_R} \nu_R \]

The symmetry conditions \((S_i S_i^\dagger = S_i^\dagger S_i = 1)\):

\[ S_L^\dagger Y_l S_{l_R} = Y_l, \quad S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu, \quad S_{\nu_R}^T M S_{\nu_R} = +M, \quad S_{\nu_R}^T Y_\varphi S_{\nu_R} = -Y_\varphi \]

The implications of the symmetry:

\[ S_{\nu_R}^T M S_{\nu_R} = +M \quad \Rightarrow \quad S_{\nu_R} = \pm 1, \quad S_{\nu_R} = \pm \text{diag}(1, 1, -1) \]
\( S_{\nu R} = \pm 1 \implies Y_\varphi = 0 \) or \( S_{\nu R} = \pm \text{diag}(1, 1, -1) \implies Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \)

\[
S_L^\dagger Y_l S_{l_R} = Y_L \implies S_L = S_{l_R} = \text{diag}(s_1, s_2, s_3), \quad |s_i| = 1
\]

\[
S_L^\dagger Y_\nu S_{\nu R} = Y_\nu \implies 10 \text{ Dirac neutrino mass textures}
\]

For instance the solution corresponding to \( s_{1,2,3} = \pm 1 \):

\[
Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix}
\]
\[ \mathcal{L}_m = -(\bar{n}M_n n + \bar{N}M_N N) \]

with the see-saw mechanism explaining \( M_n \ll M_N \):

\[ M_N \sim M \quad \text{and} \quad M_n \sim (\nu Y_\nu) \frac{1}{M} (\nu Y_\nu)^T \]

where

\[ \nu_L = n_L + M_D \frac{1}{M} N_L \quad \text{and} \quad \nu_R = N_R - \frac{1}{M} M_D^T n_R \]

\[ Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix} \Rightarrow M_D = Y_\nu \frac{\nu}{\sqrt{2}} \Rightarrow M_n \]

To compare our results with the data, we use the following approximate lepton mixing matrix (tri-bimaximal lepton mixing) that corresponds to \( \theta_{13} = 0, \theta_{23} = \pi/4 \) and \( \theta_{12} = \arcsin(1/\sqrt{3}) \):

\[ U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \]
Writing the diagonal light neutrino mass matrix as

\[ m_{\text{light}} = \text{diag}(m_1, m_2, m_3) \]

we find

\[ M_n = U m_{\text{light}} U^T \]

\[ \Downarrow \]

\[ Y_\nu = \begin{pmatrix} a & b & 0 \\ \frac{-a}{2} & b & 0 \\ \frac{-a}{2} & b & 0 \end{pmatrix}, \quad m_1 = -3a^2 \frac{v^2}{M_1} \]
\[ m_2 = -6b^2 \frac{v^2}{M_2} \]
\[ m_3 = 0 \]

and

\[ Y_\nu = \begin{pmatrix} a & b & 0 \\ a & \frac{-b}{2} & 0 \\ a & \frac{-b}{2} & 0 \end{pmatrix}, \quad m_1 = -3b^2 \frac{v^2}{M_2} \]
\[ m_2 = -6a^2 \frac{v^2}{M_1} \]
\[ m_3 = 0 \]

Does \( Y_\varphi \neq 0 \) imply \( \varphi \to n_i n_j \) decays?

\[ Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix}, \quad Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \Rightarrow \varphi \to N_{1,2}^* N_3 \to n_{1,2,3} h \ N_3 \]

that can be kinematically forbidden by requiring \( M_3 > m \).
Does $\phi$ explain the PAMELA data?

**Figure 6:** Combined fit of different DM annihilation channels to the PAMELA positron and PAMELA anti-proton data, from Cirelli, Kadastik, Raidal and Strumia, arXiv:0809.2409.
Summary and comments

- The addition of a real scalar singlet $\varphi$ to the SM may ameliorate the little hierarchy problem (by lifting the cutoff $\Lambda$ to $50 - 100$ TeV range). Fine tuning remains.

- It also provides a realistic candidate for DM.

- Since $m \gtrsim 10$ TeV therefore $\varphi$ can properly describe the PAMELA results both for $e^+$ and $\bar{p}$.

- The $\mathbb{Z}_2$ symmetry implies a realistic texture for the neutrino mass matrix.

- $\varphi$ cannot be assumed to be responsible neither for inflation nor for dark energy.