Vacuum stability by vector dark matter

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- The Vector Dark Matter (VDM) model
- Vacuum stability
- Landau poles
- Experimental constraints
- Direct detection of dark matter
- Summary

The Vector Dark Matter (VDM) model


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The model:

- **extra** $U(1)_X$ gauge symmetry ($A^\mu_X$),

- a complex scalar field $S$, whose vev generates a mass for the $U(1)$'s vector field, $S = (0, 1, 1, 1)$ under $U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)_X$.

- SM fields neutral under $U(1)_X$,

- in order to ensure stability of the new vector boson a $\mathbb{Z}_2$ symmetry is assumed to forbid $U(1)$-kinetic mixing between $U(1)_X$ and $U(1)_Y$. The extra gauge boson $A^\mu$ and the scalar $S$ field transform under $\mathbb{Z}_2$ as follows

  $$A^\mu_X \rightarrow -A^\mu_X, \quad S \rightarrow S^*, \quad \text{where} \quad S = \phi e^{i\sigma}, \quad \text{so} \quad \phi \rightarrow \phi, \quad \sigma \rightarrow -\sigma.$$
The scalar potential

\[ V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2. \]

The vector bosons masses:

\[ M_W = \frac{1}{2} g v, \quad M_Z = \frac{1}{2} \sqrt{g^2 + g'^2 v} \quad \text{and} \quad M_Z' = g_x v_x, \]

where

\[ \langle H \rangle = \begin{pmatrix} 0 \\ v \sqrt{2} \end{pmatrix} \quad \text{and} \quad \langle S \rangle = \frac{v_x}{\sqrt{2}} \]

Positivity of the potential implies

\[ \lambda_H > 0, \quad \lambda_S > 0, \quad \kappa > -2 \sqrt{\lambda_H \lambda_S}. \]

The minimization conditions for scalar fields

\[ (2\lambda_H v^2 + \kappa v_x^2 - 2\mu_H^2) v = 0 \quad \text{and} \quad (\kappa v^2 + 2\lambda_S v_x^2 - 2\mu_S^2) v_x = 0 \]
For $\kappa^2 < 4\lambda_H\lambda_S$ the global minima are

$$v^2 = \frac{4\lambda_S\mu_H^2 - 2\kappa\mu_S^2}{4\lambda_H\lambda_S - \kappa^2} \quad \text{and} \quad v_x^2 = \frac{4\lambda_H\mu_S^2 - 2\kappa\mu_H^2}{4\lambda_H\lambda_S - \kappa^2}$$

Both scalar fields can be expanded around corresponding vev's as follows

$$S = \frac{1}{\sqrt{2}}(v_x + \phi_S + i\sigma_S) \quad , \quad H^0 = \frac{1}{\sqrt{2}}(v + \phi_H + i\sigma_H) \quad \text{where} \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}.$$  

The mass squared matrix $M^2$ for the fluctuations $(\phi_H, \phi_S)$ and their eigenvalues read

$$M^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa uv_x \\ \kappa uv_x & 2\lambda_S v_x^2 \end{pmatrix}, \quad M^2_{\pm} = \lambda_H v^2 + \lambda_S v_x^2 \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H\lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}$$

$$M^2_{\text{diag}} = \begin{pmatrix} M_{h_1}^2 \\ 0 \end{pmatrix}, \quad R = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \quad \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R^{-1} \begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix},$$
where $M_{h_1} = 125.7$ GeV is the mass of the observed Higgs particle. Then we obtain

$$
\sin 2\alpha = \frac{\text{sign}(\lambda_{SM} - \lambda_H) 2M_{12}^2}{\sqrt{(M_{11}^2 - M_{22}^2)^2 + 4(M_{12}^2)^2}}, \quad \cos 2\alpha = \frac{\text{sign}(\lambda_{SM} - \lambda_H)(M_{11}^2 - M_{22}^2)}{\sqrt{(M_{11}^2 - M_{22}^2)^2 + 4(M_{12}^2)^2}}.
$$

Note that since vev of $H$ is fixed at 246.22 GeV, with $\kappa = 0$ (no mass mixing) and $\lambda_H \neq \lambda_{SM}$ it is only $\phi_S$ which can have the observed Higgs mass of 125.7 GeV. Even though the mass matrix is diagonal in this case, however in order to satisfy our convention that $M_{h_1} = 125.7$ GeV a rotation by $\alpha = \pm \pi/2$ is required in such a case.

There are 5 real parameters in the potential: $\mu_H, \mu_S, \lambda_H, \lambda_S$ and $\kappa$. Adopting the minimization conditions $\mu_H, \mu_S$ could be replaced by $v$ and $v_x$. The SM vev is fixed at $v = 246.22$ GeV. Using the condition $M_{h_1} = 125.7$ GeV, $v_x^2$ could be eliminated in terms of $v^2, \lambda_H, \kappa, \lambda_S, \lambda_{SM} = M_{h_1}^2/(2v^2)$:

$$
v_x^2 = v^2 \frac{4\lambda_{SM}(\lambda_H - \lambda_{SM})}{4\lambda_S(\lambda_H - \lambda_{SM}) - \kappa^2}
$$

Eventually there are 4 independent parameters:

$$(\lambda_H, \kappa, \lambda_S, g_x),$$

where $g_x$ is the $U(1)_X$ coupling constant.
Figure 1: Contour plots for masses of the non-standard ($h_2$) Higgs particle in the plane $(\lambda_H, \kappa)$. In the bottom part of the plot ($\lambda_H < \lambda_{SM} = M_{h_1}^2/(2v^2) = 0.13$) the heavier Higgs is the currently observed one, while in the upper part ($\lambda_H > \lambda_{SM}$) the lighter state is the observed one. White regions in the upper and lower parts are disallowed by the positivity conditions for $v_x^2$ and $M_{h_2}^2$, respectively.

- Positivity of $v_x^2$ implies for $\lambda_H > \lambda_{SM}$ that $\lambda_H > \frac{\kappa^2}{4\lambda_S} + \lambda_{SM}$

- Positivity of $M_{h_2}^2$ implies for $\lambda_H < \lambda_{SM}$ that $\lambda_H > \frac{\kappa^2}{4\lambda_S}$
Figure 2: Contour plots for the vacuum expectation value of the extra scalar $\nu_x \equiv \sqrt{2} \langle S \rangle$ (left panel) and of the mixing angle $\alpha$ (right panel) in the plane $(\lambda_H, \kappa)$. 
Vacuum stability

\[ V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2 \]

2-loop running of parameters adopted

\[ \lambda_H(Q) > 0, \lambda_S(Q) > 0, \kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0 \]

**Figure 3:** Running of various parameters at 1- and 2-loop, in solid and dashed lines respectively. For this choice of parameters \( \lambda_H(Q) > 0 \) at 2-loop (right panel blue) but not at 1-loop. \( \lambda_S(Q) \) is always positive (right panel red), running of \( \kappa(Q) \) is very limited, however the third positivity condition \( \kappa(Q) + 2\sqrt{\lambda_H(Q)\lambda_S(Q)} > 0 \) is violated at higher scales even at 2-loops (right panel green).
The mass of the Higgs boson is known experimentally therefore within the SM the initial condition for running of $\lambda_H(Q)$ is fixed

$$\lambda_H(m_t) = M_{h_1}^2/(2v^2) = \lambda_{SM} = 0.13$$

For VDM this is not necessarily the case:

$$M_{h_1}^2 = \lambda_H v^2 + \lambda_S v^2_{x} \pm \sqrt{\lambda_S^2 v_x^4 - 2\lambda_H \lambda_S v^2 v_x^2 + \lambda_H^2 v^4 + \kappa^2 v^2 v_x^4}.$$ 

VDM:

- Larger initial values of $\lambda_H$ such that $\lambda_H(m_t) > \lambda_{SM}$ are allowed delaying the instability (by shifting up the scale at which $\lambda_H(Q) < 0$).

- Even if the initial $\lambda_H$ is smaller than its SM value, $\lambda_H(m_t) < \lambda_{SM}$, still there is a chance to lift the instability scale if appropriate initial value of the portal coupling $\kappa(m_t)$ is chosen.

$$\beta^{(1)}_{\lambda_H} = \beta^{SM}_{\lambda_H} (1) + \kappa^2$$
**Figure 4:** The stability frontier for the $H$ direction: these plots identify the renormalisation scale $t^* = \text{Log}_{10}(Q^*)$ at which $\lambda_H(Q^*) = 0$ and the vacuum becomes unstable, as a function of $(\lambda(m_t), \kappa(m_t))$. The horizontal solid black line corresponds to $\lambda_H(m_t) = \lambda_{SM} \approx 0.13$. 
Figure 5: The “in between” stability frontier: these plots identify the scale $Q^\ast = \log_{10}(Q^\ast)$ at which the positivity condition
\[ \kappa(Q) + 2 \sqrt{\lambda_H(Q) \lambda_S(Q)} > 0 \]
fails and the vacuum becomes unstable, as a function of $(\lambda(m_t), \kappa(m_t))$ for fixed choices of $(g_x(m_t), \lambda_S(m_t))$ specified above each panel. The horizontal solid black line corresponds to $\lambda_H(m_t) = \lambda_{SM} \simeq 0.13$. The gray area is excluded by the requirement that there is no Landau poles up to the Planck mass.
Figure 6: Contour plots of $\lambda_H(M_{Pl})$ in the plane of $(\lambda(m_t), \kappa(m_t))$ for fixed $g_x(m_t)$ and $\lambda_S(m_t)$ specified above each panel. The horizontal solid black line corresponds to $\lambda_H(m_t) = \lambda_{SM} \approx 0.13$. The plots allow one to identify regions (white) in which the $\lambda_H(Q)$ Landau pole is below the Planck scale.
Experimental constraints

• no invisible $h_1$ decays: $h_1 \rightarrow Z'Z'$, $h_1 \rightarrow h_2h_2$,

• LEP constraints for $e^+e^- \rightarrow Zh_2$ satisfied,

• LHC constraints on

$$\kappa_V \equiv \frac{g_{h_1VV}}{g_{h_1VV}^{SM}} \text{ with } 0.85 < \kappa_V < 1$$

• limits from electroweak precision data (S,T) satisfied at 95% CL

$$S = \frac{16\pi}{g^2} \frac{\cos^2 \theta_W}{g^2} \delta \Pi_{ZZ}^r(0), \quad T = \frac{4\pi}{e^2} \left( \frac{\delta \Pi_{WW}(0)}{M_W^2} - \frac{\delta \Pi_{ZZ}(0)}{M_Z^2} \right)$$

• DM abundance ($\Omega_{DM}h^2$) remains within the $5\sigma$ limit (micrOMEGAs and explicite calculation)
Figure 7: Combined plots of allowed and disallowed parameter space in the plane $(\lambda_H(m_t), \kappa(m_t))$ for $g_x(m_t) = g_1(m_t)$ and $\lambda_S(m_t) = \lambda_{SM}(m_t) = 0.13$. The thin red line denotes the frontier above which a Landau pole appears below $\lambda_H(M_{Pl})$. The thin blue line denotes the absolute stability frontier. Below the thin green line the positivity condition fails at some renormalisation scale (its wavy shape is a numerical artifact). The green area denotes LEP exclusions on Higgs-like scalars. In the outer red area positivity fails at the low scale, while in the orange area no physical solution of the vev $\nu_x$ exists. The blue area denotes an excess of the $h_1$ Higgs couplings to vector bosons $(\kappa_V)$. The remaining allowed region is in white. The green points are those for which also $\Omega_{DM} h^2$ constraint is fulfilled.
$g_x(m_t) = 0.25, \lambda_S(m_t) = 0.05$

$\lambda_H^2(M_{Pl}) > 2\pi$

$[\kappa + 2\sqrt{\lambda_H \lambda_S}](Q) > 0$

$\kappa > 0.85$

$\nu_x^2 < 0$

$\lambda_H(Q) > 0$

$\kappa > 0.85$

LEP excluded

$\lambda_H(M_{Pl}) > 2\pi$

$\lambda_H^2 > 4\lambda_H \lambda_S$

$\kappa > 0.85$

LEP excluded

$g_x(m_t) = 0.6, \lambda_S(m_t) = 0.2$

$\lambda_H^2(M_{Pl}) > 2\pi$

$[\kappa + 2\sqrt{\lambda_H \lambda_S}](Q) > 0$

$\kappa > 0.85$

$\nu_x^2 < 0$

$\lambda_H(Q) > 0$

$\kappa > 0.85$

LEP excluded

$\lambda_H^2 > 4\lambda_H \lambda_S$

$\kappa > 0.85$

LEP excluded
**Direct detection of dark matter**

\[
\sigma_{Z'N} = \frac{\mu^2}{4\pi} g_x^2 g_{hNN} \sin^2 2\alpha \left( \frac{1}{M_{h_1}^2} - \frac{1}{M_{h_2}^2} \right)^2
\]

- scan range: \(0.1 < g_x < 1, 0 < \lambda_H < 0.25\) and \(-0.5 < \kappa < 0.5\)

- \(\lambda_H < \lambda_{SM}\) (light dark matter): \(60\ \text{GeV} \lesssim M_{Z'} \lesssim 120\ \text{GeV}\),

- \(\lambda_H > \lambda_{SM}\) (heavy dark matter): \(63\ \text{GeV} \lesssim M_{Z'} \lesssim 1000\ \text{GeV}\).

**Figure 8:** The figure shows the DM-nucleon cross section, \(\sigma_{Z'N}\), as a function of the DM mass \(M_{Z'}\) for points which satisfy all other constraints for \(\lambda_H < \lambda_{SM}\). The singlet quartic coupling is fixed at \(\lambda_S = 0.2\). Colouring corresponds to the strength of the gauge coupling \(g_x\). The nearly horizontal lines are the experimental limits for \(\sigma_{Z'N}\) from XENON100, LUX (2103) and anticipated results for XENON 1T.
Figure 9: The left panel illustrates correlation between between $M_{h_2}$ and $M_{Z'}$, while the right one shows predictions for $\Omega_{DM} h^2$ as a function of $M_{Z'}$. The colouring corresponds to the cross section $\sigma_{Z'N}$. Above the right box resonances and channels which open as $M_{Z'}$ increases are shown. Coordinates in the parameter space $(\lambda_H, \kappa, \lambda_S)$ and corresponding $M_{h_2}$ and $v_x$ are shown above the right panel.
Figure 10: The figure shows the DM-nucleon cross section, $\sigma_{Z'N}$, as a function of the DM mass $M_{Z'}$ for points which satisfy all other constraints for $\lambda_H > \lambda_{SM}$. The singlet quartic coupling is fixed at $\lambda_S = 0.2$. Colouring corresponds to the strength of the gauge coupling $g_x$. The solid lines are the experimental limits for $\sigma_{Z'N}$ from XENON100, LUX (2013) and anticipated results for XENON 1T.
Figure 11: The left panel illustrates correlation between $M_{h_2}$ and $M_{Z'}$, while the right one shows predictions for $\Omega_{DM h_2^2}$ as a function of $M_{Z'}$. The colouring corresponds the the cross section $\sigma_{Z'N}$. Above the right box resonances and channels which open as $M_{Z'}$ increases are shown. Coordinates in the parameter space $(\lambda_H, \kappa, \lambda_S)$ and corresponding $M_{h_2}$ and $v_x$ are shown above the right panel.
Figure 12: The left panel illustrates correlation between between $M_{h_2}$ and $M_{Z'}$, while the right one shows predictions for $\Omega_{DM} h^2$ as a function of $M_{Z'}$. The colouring corresponds to the cross section $g_x$. Above the right box resonances and channels which open as $M_{Z'}$ increases are shown. Coordinates in the parameter space ($\lambda_H, \kappa, \lambda_S$) and corresponding $M_{h_2}$ and $v_x$ are shown above the right panel.
Summary

- VDM model has been presented: $Z'$ (DM), $h_2$ (extra Higgs)

- Vacuum stability was addressed: absolute stability

- Cosmological consequences were discussed, VDM easily consistent with $\Omega_{DM} h^2$ and $\sigma_{Z'N}$