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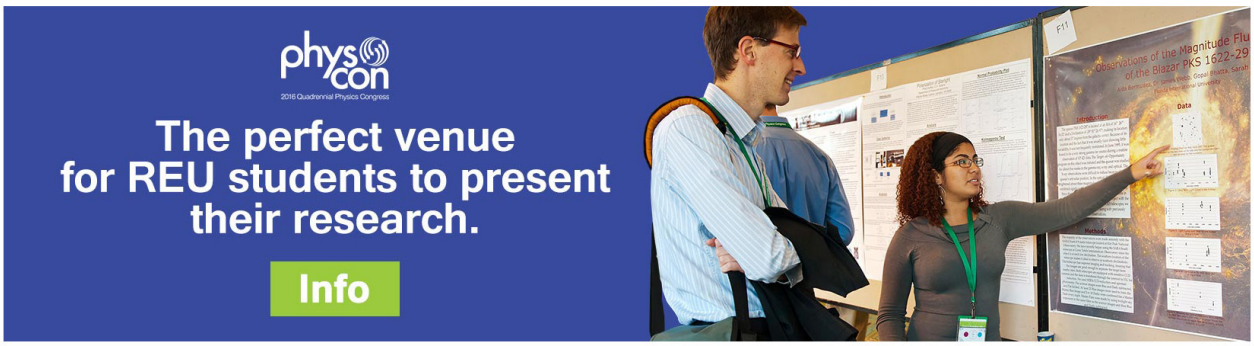

Comments on "Lost Horizons" by G. F. R. Ellis and T. Rothman [Am. J. Phys. 61 (10), 883–893 (1993)]

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I have enjoyed reading the lucid and well written article "Lost Horizons" by Ellis and Rothman. However, I would like to draw attention to one passage of this paper that seems to contain an incorrect statement.

In their Sec. VI (The Past Null Cone) E. and R. state that the reconvergence of light in our past light cone is "an example of gravitational lensing: light rays emitted by a distant enough galaxy in the past initially spread out, but are then refocussed by the matter in the universe and are reconverging by the time they reach us."

It seems to me that the reconvergence of light, i.e., the fact that the past null cone is shaped like an onion, is a kinematical effect which operates independent of the matter contents of the universe or of any gravitational lensing.

Imagine a light ray being emitted toward us by a test particle in the early universe. Calculating the proper velocity v_p of the individual photons relative to us as a function of time [differentiate Eq. (16) of the paper with respect to time], we obtain

$$v_p(t_e)=\frac{dl(t_e)}{dt_e}$$
$$=\frac{d}{dt_e}\left[R(t_e)\int_{t_e}^{t_0}\frac{c\,dt}{R(t)}\right]$$
$$=\dot{R}(t_e)\int_{t_e}^{t_0}\frac{c\,dt}{R(t)}-c=\dot{R}(t_e)r(t_e)-c=v_H(t_e)-c.$$

Here, $l(t_e)$ and $r(t_e)$ are the proper and comoving distance of the photon, respectively, R is the scale factor of the universe, a dot denotes differentiation with respect to t_e, and v_H is the local Hubble flow (at the instantaneous position of the photon) relative to us. [This equation is equivalent to E. and R.'s Eq. (25), derived from metric arguments.]

If the proper velocity v_p is positive at early times and negative at late times, we would say that the photon is receding from us at early times and returning to us at late times, i.e., we have a reconverging light cone structure as described by the authors.

Now, with the above equation it is easy to show that v_p changes its sign even in an open universe with vanishing mass content (the scale factor is here $R\propto t$), and the light rays initially emitted from another galaxy in our direction finally reconverge after an early phase of recession. Qualitatively, the same result holds in closed, flat, and open universes, only the time it takes for the light ray to reach the observer, and the exact trajectory, depend on the mass of the universe. Clearly, in a massless universe gravitational lensing or "refocussing" cannot be the right explanation. Thus the reconvergence of the light cone must be a kinematical effect that may be affected but not caused by the curvature of space.

A better explanation for the shape of the past null light cone involves the requirement by the theory of relativity of a constant speed of light in all local inertial frames, which Ellis and Rothman refer to in their paragraph VIII. Photons emitted toward us are swimming upstream against the Hubble flow, such as to attain at each location the same velocity c relative to the stream. While they are swept away from us in an early phase of expansion, if measured in proper coordinates they monotonically approach us in comoving coordinates all the time, and thus they enter regions with smaller and smaller Hubble velocity relative to us, until they finally pass a point where the Hubble velocity drops below the speed to light. Then $v_p=v_H-c$ reverses its sign, and the photons begin to approach us in proper coordinates with ever increasing velocity, reaching asymptotically the special relativistic speed of light $-c$ at our location. The zero of v_p marks the point where a photon has reached its maximum proper distance from us, and according to $v_p=v_H-c$, this occurs where $v_H=c$, independent of the cosmological model. In other words (using E. and R.'s nomenclature) the speed of light sphere (SLS) *always* intersects the past null cone at its maximum, and so it seems that contrary to the authors' assumption (page 888, top, left column) this is not just a coincidence.

One may ask whether the relation $v_p=v_H-c$ tells us something beyond being a simple consequence of GR–Friedmann models. Apparently (in a special relativistic sense) this condition could be considered as a kind of extended Machian principle, where the local motion of the matter in the universe (as given by the Hubble flow v_H) defines an absolute local frame relative to which photons have to conform to the universal value of the speed of light.

WRITING REVIEW PAPERS

It is rare to find a major contributor in a field who agrees to write [a review] article and then gets it done in 15 months. It's even rarer to find that wonder of wonders: a genuinely knowledgeable scholar with a deep interest in writing review articles for scholarly journals. There aren't very many. We live in a time in which almost everyone would rather write than read, would rather publish than think.

David Pines, Editor of *Reviews of Modern Physics*, in an interview in *APS News* 2(5), 8 (1993).

