Class problems #3

The proper (physical) distance (defined along the surface of constant time $dt = 0$) to an object located at the comoving coordinate r at the moment t is:

$$D(t) = a(t) \int_0^r \frac{dr'}{(1 - kr'^2)^{1/2}}$$

1. Draw a recession velocity as a function of redshift assuming interpretation of the Hubble law in terms of the non-relativistic Doppler effect, relativistic Doppler effect and General relativity for a universe made of matter and cosmological constant, see fig 1 in [1] and sec. 3.1 in [2].

2. Clarify the Misconception #3 "Galaxies with recession velocities exceeding the speed of light exist but we cannot see them", see sec. 3.3 in [2].

3. For universes with the scale factor

   (a)
   $$a(t) = a_0 \left(\frac{t}{t_0} \right)^{\alpha}$$

   (b)
   $$a(t) \propto e^{Ht}, \ (H \text{ is a constant})$$

   find, draw and discuss the Hubble sphere $D_{Hs}(t)$, the null light cone $D_\gamma(t)$, the particle horizon $D_{ph}(t)$ and the galaxy world line $D_G(t)$. For definitions see the lecture notes #3. Draw t as the vertical axis in the period from the Big Bang ($t = 0$) till today (t_0). Show that the crossing of the Hubble sphere with the past null cone takes place at the time $t_\times$ such that $\dot{D}_\gamma(t_\times) = 0$. Find the time $t_\times$ and the proper distance to the crossing point. The case (b) might be problematic.

4. Draw, discuss and explain fig. 2 of [1].

5. Draw, discuss and explain the lower panel of fig. 1 of [2].

6. Derive v_{rec} as a function of time t and the observed redshift z:
   $$v_{\text{rec}} = \frac{\dot{a}(t)}{a_0} \int_0^z \frac{dz'}{H(z')}.$$

7. Find the proper (physical) distance to the Hubble sphere for MD, RD and Λ-dominated universe.

References

[1] T. M. Davis and Ch. H. Lineweaver, "Superluminal Recession Velocities", AIP Conf. Proc. 555, 348 (2001).

[2] T. M. Davis and Ch. H. Lineweaver, "Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe", 2004, Publications of the Astronomical Society of Australia, 21, 97-109;