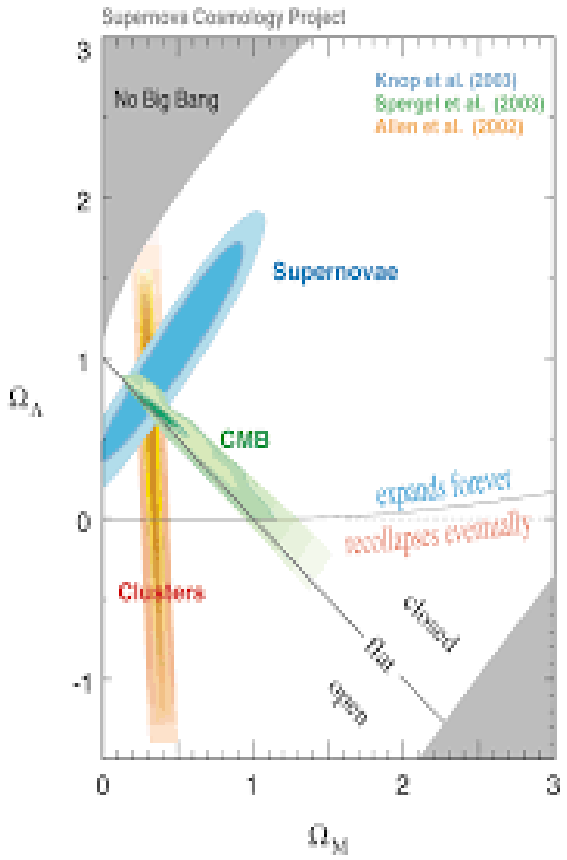

Figure 1: *Supernova Cosmology Project Knop et al. (2003)*

Class problems #5

1. Show, using the Friedmann, that sufficient conditions for exponential inflation are:

$$k = 0 \quad \text{and} \quad p = -\rho$$

or

$$k \neq 0 \quad \text{and} \quad p = -\frac{\rho}{3} - \frac{H^2}{4\pi G}\,,$$

with constant H. Note that p and ρ do not need to be constant.

2. Derive the non-relativistic analog of the Friedmann equation with non-zero cosmological constant Λ. Argue that positive Λ implies accelerating expansion of the universe.

3. Consider a universe with dust and k only, find and draw $a = a(t)$ for $k = \pm 1, 0$.

4. Discuss existence of polynomial ($a \propto t^\alpha$) solutions to the Friedmann equation for $k \neq 0$. Compare with the Milne universe: $\rho = p = \Lambda = 0$ and $k = -1$.

5. For $\Lambda > 0$ and $k = 1$ find $\Lambda = \Lambda_E$ such that $\dot{a}(t) = \ddot{a}(t) = 0$ for $a(t) = a_E$. This is the Einstein static universe that motivated him to introduce Λ. Find Λ_E, a_E and relation between them. Show that

   - for $\Lambda > \Lambda_E$, Λ eventually dominates and the universe expands forever,

- for $0 < \Lambda < \Lambda_E$, there exists a range of a which is forbidden.

Discuss possible universes.

6. Verify stability of the Einstein static universe, i.e. expand around $a = a_E$ and solve for the fluctuations.

7. Explain regions shown in fig. 1.