Class problems #3

1. Show that for a coordinate transformation of the form

$$x^\mu \to x'^\mu = x^\mu + \varepsilon^\mu(x)$$

the corresponding transformation of $h_{\mu\nu}$ ($g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$ with $h_{\mu\nu} \ll 1$) is the following:

$$h_{\mu\nu}(x) \to h'_{\mu\nu}(x) = h_{\mu\nu}(x) - \partial_\mu \varepsilon_\nu - \partial_\nu \varepsilon_\mu,$$

assuming $\frac{\partial \varepsilon_\mu}{\partial x^\nu} \sim h_{\mu\nu}$.

2. For the Friedmann-Robertson-Walker metric calculate all non-zero elements of the connection $\Gamma^\lambda_{\mu\nu}$ and the Ricci tensor $R_{\mu\nu}$.

3. Show that for the Friedmann-Robertson-Walker metric

$$g_{tt} = 1, \qquad g_{ti} = 0, \qquad g_{ij} = -a^2(t)\tilde{g}_{ij}(x)$$

with $\tilde{g}_{rr} = (1 - kr^2)^{-1}$, $\tilde{g}_{\theta\theta} = r^2$, $\tilde{g}_{\varphi\varphi} = r^2 \sin^2\theta$:

- the *spatial* Ricci tensor R_{ij} is of the following form:

$$R_{ij} = \tilde{R}_{ij} - \tilde{g}_{ij}(a\ddot{a} + 2\dot{a}^2),$$

where $\tilde{R}_{ij}$ is the Ricci tensor calculated for the metric $\tilde{g}_{ij}$, and

- the *spatial* Ricci tensor $\tilde{R}_{ij}$ is given by:

$$\tilde{R}_{ij} = -2k\tilde{g}_{ij}\,.$$