## Class problems #5

The proper (physical) distance (defined along the surface of constant time dt = 0) to an object located at the comoving coordinate r at the moment t is:

$$D(t) = a(t) \int_0^r \frac{dr'}{(1 - kr'^2)^{1/2}}$$

- 1. Draw a recession velocity as a function of redshift assuming interpretation of the Hubble law in terms of the non-relativistic Doppler effect, relativistic Doppler effect and General relativity for a universe made of matter and cosmological constant, see fig 1 in [1] and sec. 3.1 in [2].
- 2. Clarify the Misconception #3 "Galaxies with recession velocities exceeding the speed of light exist but we cannot see them", see sec. 3.3 in [2].
- 3. For universes with the scale factor

(a) 
$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\alpha}$$

(b) 
$$a(t) \propto e^{Ht}$$
, (H is a constant)

find, draw and discuss the Hubble sphere  $D_{Hs}(t)$ , the null light cone  $D_{\gamma}(t)$ , the particle horizon  $D_{ph}(t)$  and the galaxy world line  $D_G(t)$ . For definitions see the lecture notes #3. Draw t as the vertical axis in the period from the Big Bang (t=0) till today  $(t_0)$ . Show that the crossing of the Hubble sphere with the past null cone takes place at the time  $t_{\times}$  such that  $\dot{D}_{\gamma}(t_{\times}) = 0$ . Find the time  $t_{\times}$  and the proper distance to the crossing point. The case (b) might be problematic.

- 4. Draw, discuss and explain fig. 2 of [1].
- 5. Draw, discuss and explain the lower panel of fig. 1 of [2].
- 6. Derive  $v_{\text{rec}}$  as a function of time t and the observed redshift z:

$$v_{\rm rec} = \frac{\dot{a}(t)}{a_0} \int_0^z \frac{dz'}{H(z')}.$$

7. Find the proper (physical) distance to the Hubble sphere for MD, RD and  $\Lambda$ -dominated universe.

## References

- [1] T. M. Davis and Ch. H. Lineweaver, "Superluminal Recession Velocities", AIP Conf. Proc. 555, 348 (2001).
- [2] T. M. Davis and Ch. H. Lineweaver, "Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe", 2004, Publications of the Astronomical Society of Australia, 21, 97-109;