Homework problems #4

1. For a cosmological model $R(t) = R_0 \times (t/t_0)^\alpha$ $(0 < \alpha < 1)$, find and draw in the space (t, χ)

   - past null light cone,
   - Hubble sphere,
   - world-line (for $k = 0$) of an object that is at a coordinate χ_0 now,
   - particle horizon,

   where the coordinate χ is defined as follows

$$\chi = \int_0^r \frac{dr'}{(1 - kr'^2)^{1/2}} = \begin{cases} \arcsin r \\ r \\ \operatorname{arcsinh} r \end{cases} \quad \text{for } k = \begin{cases} +1 \\ 0 \\ -1 \end{cases}$$

2. For a cosmological model $R(t) = R_0 \times (t/t_0)^\alpha$ $(0 < \alpha < 1)$, find and draw in the space (η, χ)

   - past null light cone,
   - Hubble sphere,
   - world-line (for $k = 0$) of an object that is at a coordinate χ_0 now,
   - particle horizon

   where the conformal time η, is defined as follows

$$\eta(t) = \int_0^t \frac{dt'}{R(t')}$$

3. Evaluate the deceleration parameter as a function of the redshift, $q = q(z)$, neglect radiation but include a "matter" satisfying equation of state $p_X = w_X \rho_X$.

4. Find regions in the plane $\Omega_m, \Omega_\Lambda)$ corresponding to accelerating and decelerating universe (now).

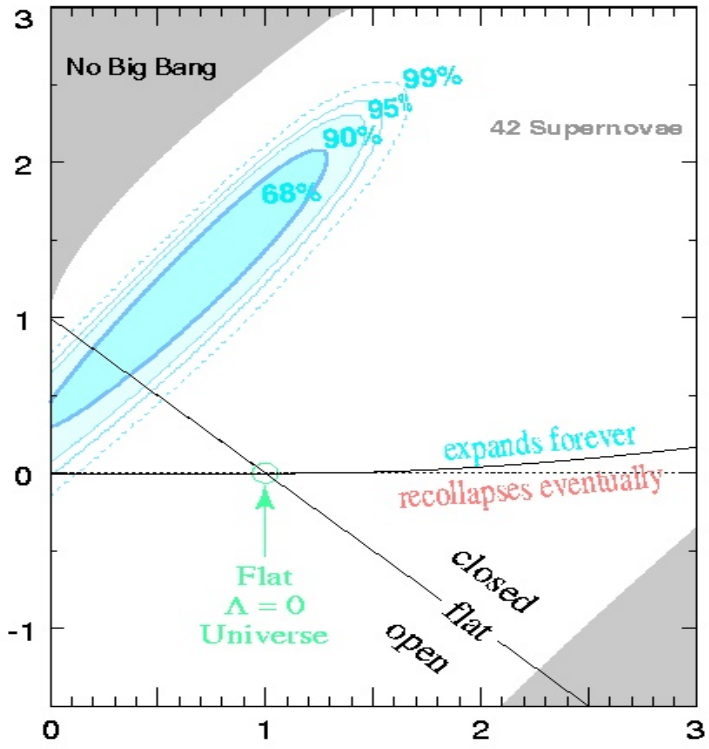
5. Assuming numbers accepted by the concordance model $\Omega_m^0 = 0.3$ and $\Omega_\Lambda^0 = 0.7$ find the redshift at which the observed presently acceleration began.

6. Explain the shape of the region "No Big Bang" in fig. 1

7. Calculate the fraction of matter-dominated universe that is visible, as a function of the redshift for $k = +1$ (for $k < 1$ the problem does not make sense), by the fraction one means $d_H(z)/(2\pi R(z))$, where $2\pi R(z)$ is the circumference of the universe.

8. Find the visible fraction of the dust universe at the moment of maximal expansion for $k = +1$.

Hint for the problems # 1 and 2 :
Check your results against fig.1 in T. M. Davis and Ch. H. Lineweaver, "Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe", 2004, Publications of the Astronomical Society of Australia, 21, 97-109. Note that in fig.1 a universe with $(\Omega_m^0, \Omega_\Lambda^0) = (0.3, 0.7)$ was adopted.



Rysunek 1: Confidence region for Ω_m vs. Ω_Λ plane, from SCP.