Homework problems #5

1. Find and draw solution $Y_\chi(x)$ of the Boltzmann equation

$$\frac{x}{Y_{\chi\,EQ}}\frac{dY_\chi}{dx} = -\frac{\Gamma}{H}\left[\left(\frac{Y_\chi}{Y_{\chi\,EQ}}\right)^2 - 1\right]$$

   assuming $g_{\star S}(x)$ =const. and $\Gamma = \Gamma_0 x^n$ for a universe dominated by a substance that satisfies the equation of state with the parameter w. Discuss various possible initial conditions.

2. Find the freeze-out "time" x_f for a decoupling of a hypothetical heavy Dirac stable neutrino of mass m. Assume that the $T \lesssim m \lesssim M_Z$ and the annihilation cross-section is constant

$$\sigma(x) = \sigma_0 \sim G_F^2 m^2.$$

3. Find z, T and t corresponding to the matter-radiation equality.

4. Find and discuss evolution equations for "temperature" and number density (i.e., "chemical potential") in an expanding universe by plugging the Maxwell-Boltzmann distribution into the collisionless Boltzmann equation.

5. Estimate the upper limit for the number of light neutrinos implied by a successful BBN.