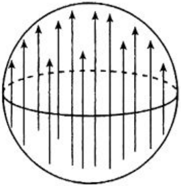
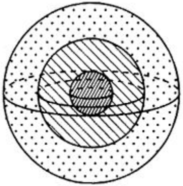
The Cosmological Principle

On large spatial scales ($\gtrsim 100$ Mpc), the Universe is homogeneous and isotropic

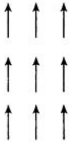
- If the Universe is isotropic then you will see no difference in the structure of the Universe looking in different directions.
- Homogeneity, when viewed on the largest scales, means that the average density of matter is about the same in all places in the Universe and the Universe is fairly smooth on large scales.



Homogeneous
Not isotropic



Isotropic
Not homogeneous



Homogeneous
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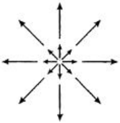


Isotropic
Not homogeneous

Notice that this is clearly not true for the Universe on small scales such as the size of the Earth, the size of the Solar System and even the size of the Galaxy. Terms such as “look the same” and “smooth in density” are applied only on very large scales. For cosmology, we only consider the isotropy and homogeneity of the Universe on scales greater than 100 Mpc.

The cosmological principle means that there is no ‘center’ to the Universe. This is an important point when we consider the origin of the Universe known as the Big Bang. Due to isotropy, there is no ‘place’ where the Big Bang occurred, there is no center point.

It could be formally shown that if the space is invariant under rotations with respect to any point, it will be invariant under translation, so homogeneous.

Olbers' Paradox and the Dark Night Sky

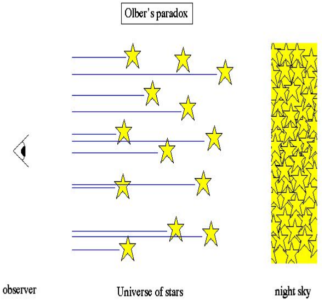
Another simple observation is that the visible night sky is dark. If the universe is infinite, eternal, and static, then the sky should be as bright as the surface of the Sun all of the time! Heinrich Olbers (lived 1758–1840) popularized this paradox in 1826, but he was not the first to come up with this conclusion. Thomas Digges wrote about it in 1576, Kepler stated it in 1610, and Edmund Halley and Jean Philippe de Cheseaux talked about it in the 1720's, but Olbers stated it very clearly, so he was given credit for it. This problem is called Olbers' Paradox.

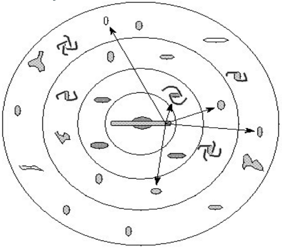
If the universe is uniformly filled with stars, then no matter which direction you look, your line of sight will eventually intersect a star (or other bright thing). Now it is known that stars are grouped into galaxies, but the paradox remains: your line of sight will eventually intersect a galaxy.

$$\Phi \propto \int dr\, r^2\, n\, I(r)$$

where n is the density of stars while $I(r)$ the energy flux observed at the distance r from a star. Since $I(r) \propto r^{-2}$, therefore

$$\Phi \propto \int dr\, r^2 n\, I(r) \propto n \int dr = \infty$$





Olbers' Paradox: No matter what direction you look, you will eventually see a bright object. Farther away objects are fainter, but there are more of them. So each shell has the **same** overall brightness. The night sky should be bright!

Possible Resolutions of Olbers's Paradox

- **Obscuration by dust**:
  Distant stars are blocked out by dust and appear fainter. Turns out this won't work because dust, if it absorbs energy will heat up and re-radiate the energy. This means that the Universe will still be filled with the same amount of radiation, the dust acts simply as a mediator.
- **Expansion of the Universe**:
  Redshift z ($1 + z \equiv \lambda_{\mathrm{obs}}/\lambda_{\mathrm{emit}}$) of photons implies λ_{obs} is larger than $\lambda_{\mathrm{emit}} \longrightarrow$ we observe lower energy photons than are produced by the distant stars. Distant objects in an expanding universe have apparent brightnesses which fall off faster than the inverse square law. This decreases the contributions from distant shells. The expanding universe effects partially explain Olbers's Paradox.

- Finite size and age of the Universe:
  A star (like the Sun) of radius R covers an area of size, $A = \pi R^2$. The fraction of the surface area of a sphere of radius r covered by such a star is then

$$f = \frac{\pi R^2}{4\pi r^2} = \left(\frac{R}{2r}\right)^2 .$$

  The total fraction of the shell covered by all of the stars in the shell is then the fraction due to one star $\times$ total number of stars

$$F = \left(\frac{R}{2r}\right)^2 \times (n \times 4\pi r^2 \times \Delta r) \simeq 1.7 \times 10^{-15} \times n \times \Delta r$$

  where n denotes the star number density and Δr is the shell width. In the last step I assumed the stellar density n as the number of stars per cubic parsec and the thickness of the shell measured in parsecs. ($R_\odot = 7 \cdot 10^8$ m, 1 pc $\simeq 3 \cdot 10^{16}$ m) These are convenient units because in our Galaxy, there is roughly 1 star per cubic parsec and the average separation between stars is of the order of 1 pc.

- Finite size and age of the Universe, *continuation*:
Since one shell of stars covers a fraction $1.7 \times 10^{-15} \times n \times \Delta r$ of the sky, therefore to make the night sky as bright as a star, we would like to make the stars cover most of the observable sky:

$$1.7 \times 10^{-15} \times n \times \Delta r \times N \simeq 1$$

where N is the number of shell needed. To calculate N, we note that there is roughly 1 star per cubic parsec in our Galaxy, choosing the shell thickness $\Delta r \simeq 1$ pc one finds

$$N \simeq 0.6 \times 10^{15}$$

Because each shell is ~ 1 pc thick therefore the Universe needs to be at least 0.6×10^{15} pc in radius. So the Universe must be at least 1.9×10^{15} ly in size in order to make the night sky as bright as the surface of the Sun. The current Universe is ~ 13.7 billion years old and has an observable size ("particle horizon") of ~ 45 billion light years ($\sim 1.5 \times 10^{10}$ pc). This is much less than needed to produce Olbers's Paradox. The fact that the Universe has a finite size and age is the principal explanation of Olbers's Paradox.

Hubble's law and the expanding Universe

The "redshift" z:

$$1 + z \equiv \frac{\lambda_{\rm obs}}{\lambda_{\rm emit}}$$

Hubble's law is a statement in physical cosmology which states that the redshift in light coming from distant galaxies is proportional to their distance (the redshift-distance relation):

$$z \propto r$$

If the redshift is interpreted as a non-relativistic Doppler effect:

$$1 + \frac{v}{c} = \frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} \qquad \longrightarrow \qquad \frac{v}{c} = z$$

then the redshift-distance relation yields a straightforward mathematical expression for Hubble's Law as follows:

$$\vec{v} = H_0 \vec{r}$$

where $\vec{v}$ is the recessional velocity, expressed in km/s. H_0 is the Hubble constant (in (km/s)/Mpc) and corresponds to the present (denoted by the subscript $_0$) value of $H(t)$ (often termed the Hubble parameter which is time dependent) in the Friedmann equations:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho,$$

where $H(t)$ is defined as

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

and the scale factor $a(t)$ is defined through the length element

$$d\tau^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right\}$$

where $k = \pm 1, 0$.

H_0 is the same throughout the universe for a given time t, and $\vec{r}$ is the distance (to be specified) from the galaxy to the observer, measured in Mpc.

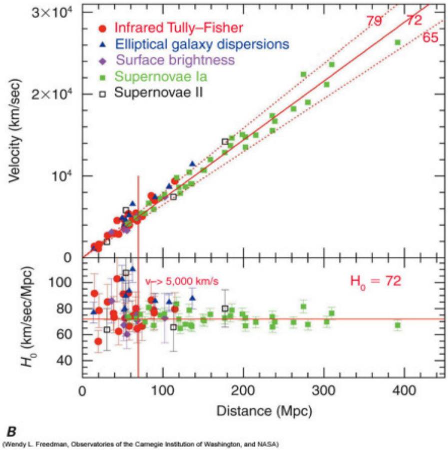


Figure 3: *The Hubble diagram for the redshift interpreted as a non-relativistic Doppler effect. The Hubble's law works very well up to distances of many hundred Mpc.*

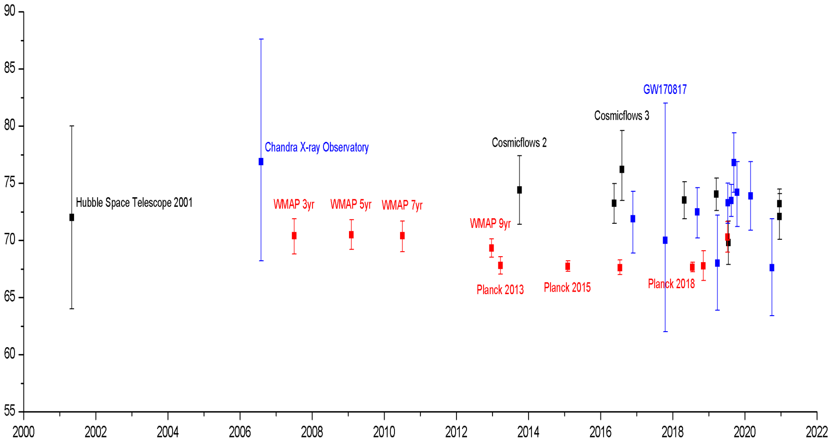


Figure 4: *Measured values of the Hubble constant, 2001–2020. Results in* **black** *represent calibrated distance ladder measurements ("late time" measurements) which tend to cluster around* 73 *km/ s/ Mpc;* **red** *represents early universe CMB measurements ("early time" measurements) with* ΛCDM *parameters which show good agreement on a figure near* 67 *km/ s/ Mpc, while blue are other techniques, whose uncertainties are not yet small enough to decide between the two.*

The law was first formulated by Edwin Hubble and Milton Humason in 1929 after nearly a decade of observations. It is considered the first observational basis for the expanding space paradigm and today serves as one of the most often cited pieces of evidence in support of the Big Bang. The most recent, "early time" measurements of the proportionality constant (from CMB) are

$$H_0 = 67.6 \times \begin{cases} +0.7 \\ -0.6 \end{cases} \text{ km/ s/ Mpc},$$

while from the Hubble Space Telescope and Planck mission one obtains (March 2019) so called "late time" measurements

$$H_0 = 74.03 \pm 1.42 \text{ km/ s/ Mpc}.$$

"Models" of the expanding Universe:

- The famous balloon analogy:
  To visualize the expanding universe one can compare 3d space with the 2d surface of an expanding balloon. This analogy was used by Arthur Eddington as early as 1933 in his book "The Expanding Universe".
  However one must remember that:
  - The 2d surface of the balloon is analogous to the 3d of space.
  - The 3d space in which the balloon is embedded is not analogous to any higher dimensional physical space.
  - The center of the balloon does not correspond to anything physical.
  - The universe may be finite in size and growing like the surface of an expanding balloon but it could also be infinite.
  - Galaxies move apart like points on the expanding balloon but the galaxies themselves do not expand because *they are gravitationally bound*.

- The raisin bread analogy:
  There is a very common misconception about the expansion. Many people envision this expansion as analogous to an explosion. In an explosion matter flies out to fill in space that is already there. This analogy is misleading. The raisin bread is the better analogy. The yeast dough is analogous to the space in the universe. The space in the universe, like the dough, is expanding causing the galaxies, or raisins, to move farther apart. The galaxies, or raisins, are not rushing out to fill in space, or dough, that is already there.

Questions:

- Is the expansion of the Universe consistent with the cosmological principle?
- Where is the center of the Universe?
- How large was the Universe at the Big Bang?

Cosmic microwave background radiation

The cosmic microwave background radiation (most often abbreviated CMB but occasionally CMBR, CBR or MBR, also referred to as relic radiation) is a form of electromagnetic radiation discovered in 1965 that fills the entire Universe. It has a thermal black-body spectrum at a temperature of 2.725 K. Thus the spectrum peaks in the microwave range at a frequency of 160.2 GHz, corresponding to a wavelength of 1.9 mm. Most cosmologists consider this radiation to be the best evidence for the Big Bang model of the universe.

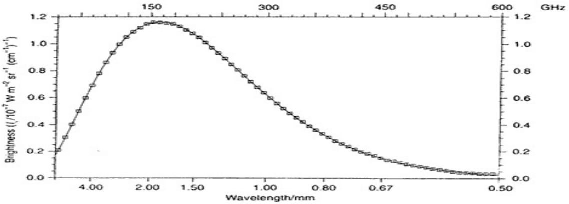


Fig. 2.1. The first published spectrum of the Cosmic Microwave Background Radiation as measured by the COBE satellite in the direction of the North Galactic Pole (Mather *et al.* 1990). Within the quoted errors, the spectrum is precisely that of a perfect black body at radiation temperature 2.735 ± 0.06 K. The more recent spectral measurements are discussed in the text. The units adopted for frequency on the ordinate are cm^{-1}. A useful conversion to more familiar units is 10^{-7} W $\mathrm{m}^{-2}\,\mathrm{sr}^{-1}\,(\mathrm{cm}^{-1})^{-1} = 3.34 \times 10^{-18}$ W $\mathrm{m}^{-2}\,\mathrm{Hz}^{-1}\,\mathrm{sr}^{-1} = 334$ MJ sr^{-1}.

The cosmic microwave background was predicted in 1948 by George Gamow and Ralph Alpher, and by Alpher and Robert Herman. Moreover, Alpher and Herman were able to estimate the temperature of the cosmic microwave background to be 5 K, though two years later, they re-estimated it at 28 K. In 1965, Arno Penzias and Robert Woodrow Wilson at the Crawford Hill location of Bell Telephone Laboratories had built a radiometer that they intended to use for radio astronomy and satellite communication experiments. Their instrument had an excess of radiation corresponding to a black-body of 3.5 K temperature which they could not account for. The spectral energy density

$$\varepsilon(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}$$

which has units of energy per unit volume per unit frequency (Joule per cubic meter per Hertz). The total energy density and the number density

$$\varepsilon_{\rm tot}(T) = \int_0^\infty \varepsilon(\nu, T) d\nu = \frac{8\pi^5 k^4}{15 h^3 c^3} T^4 \qquad n_{\rm tot}(T) = \int_0^\infty n(\nu) d\nu = \frac{16\zeta(3)\pi k^3}{c^3 h^3} T^3$$

where $n(\nu) \equiv \varepsilon(\nu)/(h\nu)$.

The average photon energy reads:

$$\langle \varepsilon \rangle = \frac{\varepsilon_{\rm tot}(T)}{n_{\rm tot}(T)} = \frac{\pi^4}{30\zeta(3)} kT \simeq 2.7kT$$

The maximum of the spectral energy density:

$$\nu_{\rm max} \simeq 2.8\ \frac{kT}{h} \qquad E_{\rm max} = h\nu_{\rm max} \simeq 2.8\ kT$$

Dark matter (DM): galactic rotation curves and the Bullet Cluster

The first who suggested the existence of DM were Dutch astronomers Jacobus Kapteyn (1922) and Jan Oort (radio astronomy pioneer) (1932). In 1933, Swiss astrophysicist Fritz Zwicky, who studied galactic clusters while working at the Caltech also hypothesized the presence of extra invisible mass that he called “dunkle Materie”, i.e. “dark matter”.

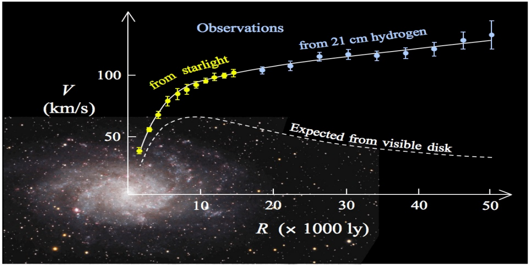


Figure 6: *Rotation curve of the typical spiral galaxy M 33 (yellow and blue points with error-bars) from: E. Corbelli, P. Salucci (2000). "The extended rotation curve and the dark matter halo of M33". Monthly Notices of the Royal Astronomical Society. 311 (2): 441–447.*

For nearly 40 years after Zwicky's initial observations, no other observations indicated that there might be a mass deficit to explain velocities of distant stars in galaxies. Then, in the late 1960s and early 1970s, Vera Rubin, a young American astronomer presented findings based on a new sensitive spectrograph that could measure the velocity curve of edge-on spiral galaxies to a greater degree of accuracy than had ever before been achieved. Together with Kent Ford, Rubin announced at a 1975 meeting of the American Astronomical Society the astonishing discovery that most stars in spiral galaxies orbit at roughly the same speed. This result suggests that:

- either at least 50% of the mass of galaxies is contained in the relatively dark galactic halo,
- or Newtonian dynamics does not apply universally, see MOND.

Dark matter theories suggests that each galaxy contains a halo of yet unidentified type of matter that provides an overall mass distribution different from the observed distribution of visible matter. This dark matter modifies gravity so as to cause the flat rotational curves.

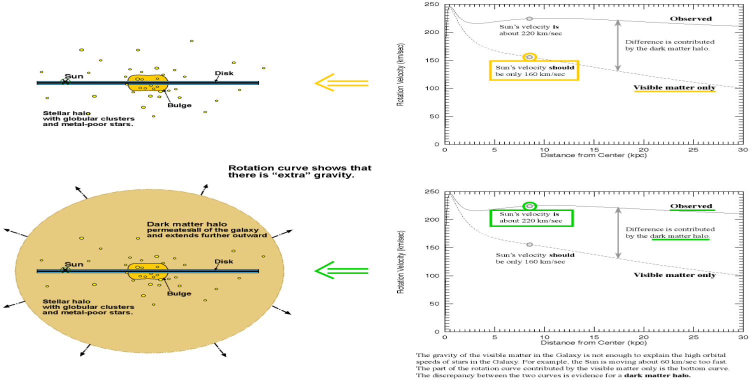


Figure 7: *Dark matter halo and rotational curves, from http://www.astronomynotes.com/ismnotes/s7.htm*

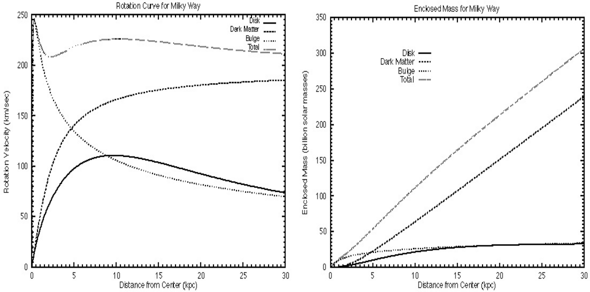


Figure 8: *Rotational curves and dark-matter distribution, from http://www.astronomynotes.com/ismnotes/s7.htm*

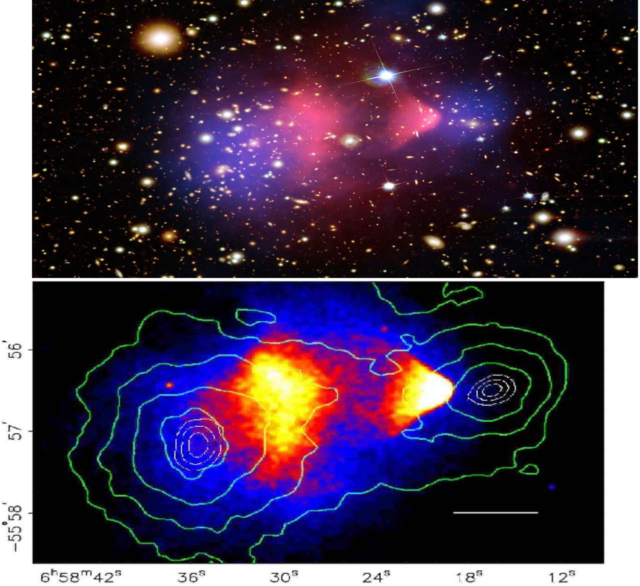


Figure 9: *The Bullet Cluster, X-ray: NASA/CXC/CfA/ M.Markevitch et al.; Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/ D.Clowe et al. Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.;*

The Bullet Cluster (1E 0657-558) consists of two colliding clusters of galaxies.

Upper panel: The blue color shows the distribution of dark matter, which passed through the collision without slowing down. The purple color shows the hot X-ray emitting gas. The stars present in galaxies are observable in visible light.

Lower panel: The bright colours show the hot baryon gas, the green lines are contours of constant gravitational potential.

The hot gas of the two colliding components, seen in X-rays, represents most of the baryonic, i.e. ordinary, matter in the cluster pair. The gases interact electromagnetically, causing the gases of both clusters to slow. The extra component, the dark matter, was detected indirectly by the gravitational lensing of background objects. In theories without dark matter, such as Modified Newtonian Dynamics (MOND), the lensing would be expected to follow the baryonic matter; i.e. the X-ray gas. However, the lensing turns out to be stronger in two separated regions.

MOND (Modified Newtonian Dynamics)

MOND has been proposed by Mordehai Milgrom in 1983 as a way to model observed flat rotational curves. Milgrom noted that Newton's law for gravitational force has been verified only where gravitational acceleration is large, and suggested that for extremely low accelerations the theory may not hold. MOND theory assumes that acceleration is not linearly proportional to force at low values. For centripetal acceleration one obtains

$$G_N \frac{Mm}{r^2} = ma \qquad \rightsquigarrow \qquad G_N \frac{Mm}{r^2} = m\mu\left(\frac{a}{a_0}\right) a$$

where

$$\mu(x) = \begin{cases} 1 & \text{for} \quad x \gg 1 \\ x & \text{for} \quad x \ll 1 \end{cases}$$

Then assuming $a \ll a_0$

$$G_N \frac{Mm}{r^2} = ma = m\frac{v^2}{r} \qquad \rightsquigarrow \qquad G_N \frac{M}{r^2} = \frac{a}{a_0} a = \frac{1}{a_0}\left(\frac{v^2}{r}\right)^2$$

$$\Downarrow$$

$$v = \sqrt[4]{G_N M a_0}$$

Typical value of a_0: $a_0 \sim 1.2 \times 10^{-8} \mathrm{cm\ s}^{-2}$.

Tensor-Vector-Scalar gravity (TeVeS) proposed by Jacob Bekenstein in 2004 is a relativistic theory that is equivalent to MOND in the non-relativistic limit, which explains the galaxy rotation problem without invoking dark matter. The break-through of TeVeS over MOND is that it can also explain the phenomenon of gravitational lensing, a cosmic phenomenon in which nearby matter bends light, which has been confirmed many times. However, TeVeS faces problems when confronted with data on the anisotropy of the CMB.

Natural units

The Planck length (quantum gravity is relevant below):

$$l_{\rm Pl} \equiv \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \simeq 1.6 \times 10^{-35} \text{ m},$$

where G is the gravitational constant.

The Planck mass (quantum gravity relevant for energies above $M_{\rm Pl}$) :

$$M_{\rm Pl} \equiv \left(\frac{\hbar c}{G_N}\right)^{1/2} \simeq 1.2 \times 10^{19} \text{ GeV}/c^2$$

$$\begin{aligned}
\hbar \equiv \frac{h}{2\pi} &= 6.58 \times 10^{-16} \text{ eV s} = 1.05 \times 10^{-34} \text{ J s} \\
c &= 2.99 \times 10^{8} \text{ m s}^{-1} \\
k_B &= 8.62 \times 10^{-5} \text{ eV K}^{-1} = 1.38 \times 10^{-23} \text{ J K}^{-1}
\end{aligned}$$

We will be adopting the following units:

$$\hbar = c = k_B = 1$$

$$\Downarrow$$

$$[E] = [M] = [T] = \text{GeV}, \qquad [l] = [t] = \text{GeV}^{-1}$$

$$\begin{aligned} \text{s} &= 1.51 \times 10^{24}\ \text{GeV}^{-1} \\ \text{m} &= 5.08 \times 10^{15}\ \text{GeV}^{-1} \\ \text{kg} &= 5.57 \times 10^{26}\ \text{GeV} \\ \text{K} &= 8.62 \times 10^{-14}\ \text{GeV} \end{aligned}$$

Then

$$G_N = 6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2} = 6.89 \times 10^{-39}\ \text{GeV}^{-2}$$

Summary

- Cosmological Principle
- Hubble law and the expanding Universe
- Cosmic microwave background radiation
- Dark Matter