SM

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symmetry group: $SU(2)_L \times U(1)_Y$

leptons

$SU_L(2)$:

$$\Psi_l^L(x) \to e^{ig\frac{\tau_j}{2}\omega_j(x)}\, \Psi_l^L(x) \qquad , \quad \Psi_l^L(x) = \begin{pmatrix} \psi^L_{\nu_l}(x) \\ \psi^L_l(x) \end{pmatrix}$$

not the same

$$\bar{\Psi}_l^L(x) \to \bar{\Psi}_l^L(x)\, e^{-ig\frac{\tau_j}{2}\omega_j(x)}$$

$j = 1,2,3$

$$\psi_l^R(x) \to \psi_l^R(x) \qquad \psi^R_{\nu_l}(x) \to \psi^R_{\nu_l}(x)$$

$$\bar{\psi}_l^R(x) \to \bar{\psi}_l^R(x) \qquad \bar{\psi}^R_{\nu_l}(x) \to \psi^R_{\nu_l}(x)$$

$U_Y(1)$:

$$\psi(x) \to e^{ig'Yf(x)}\, \psi(x)$$

$$\bar{\psi}(x) \to \bar{\psi}(x)\, e^{-ig'Yf(x)}$$

$Y = -\frac{1}{2}, -1, 0$ for $\Psi_l^L, \psi_l^R, \psi^R_{\nu_l}$

kinetic Lagrangian $\mathcal{L}^{(l)} = \bar{\Psi}^L_l\, i\not{D}\, \Psi^L_l + \bar{\psi}^R_l\, i\not{D}\, \psi^R_l + \bar{\psi}^R_{\nu_l}\, i\not{D}\, \psi^R_{\nu_l}$

with

$$D^\mu \Psi^L_l(x) = \left[\partial^\mu + ig\frac{\tau_j}{2}W^\mu_j(x) - i\frac{g'}{2}B^\mu(x)\right]\Psi^L_l(x) \qquad Y(\Psi^L_l) = -\frac{1}{2}$$

$$D^\mu \psi^R_l(x) = \left[\partial^\mu - ig'B^\mu(x)\right]\psi^R_l(x) \qquad Y(\psi^R_l) = -1$$

$$D^\mu \psi^R_{\nu_l}(x) = \partial^\mu \psi^R_{\nu_l}(x) \qquad Y(\psi^R_{\nu_l}) = 0$$

$W^\mu_j(x)$ and $B^\mu(x)$ are gauge fields corresponding to $SU(2)$ and $U(1)$.

- Problem: why only "diagonal" terms $\bar{\psi} i\not{D}\psi$ appear in $\mathcal{L}_L$?

$$\mathcal{L}^{(l)} = \mathcal{L}^{(l)}_0 + \mathcal{L}^{(l)}_1$$

$\mathcal{L}^{(l)}_0$ – free lepton Lagrangian

$$\mathcal{L}^{(l)}_1 = -g J^\mu_i(x)\, W_{i\mu}(x) - g' J^\mu_Y(x)\, B_\mu(x)$$

- Derive $J^\mu_i(x)$ and $J^\mu_Y(x)$, are they conserved?

$$J^\mu_i = \bar{\Psi}^L_l \frac{\tau_i}{2}\gamma^\mu \Psi^L_l \qquad , \qquad J^\mu_Y = -\frac{1}{2}\bar{\Psi}^L_l\gamma^\mu\Psi^L_l - \bar{\psi}^R_l\gamma^\mu\psi^R_l$$

$$= \bar{\Psi}^L_l\gamma^\mu Y\Psi^L_l + \bar{\psi}^R_l\gamma^\mu Y\psi^R_l =$$

$$= -\frac{1}{2}\bar{\psi}^L_{\nu_l}\gamma^\mu\psi^L_{\nu_l} - \frac{1}{2}\bar{\psi}^L_l\gamma^\mu\psi^L_l - \bar{\psi}^R_l\gamma^\mu\psi^R_l + \frac{1}{2}\bar{\psi}^L_l\gamma^\mu\psi^L_l =$$

$$= +\frac{s^\mu(x)}{e} - J^\mu_3 \quad \text{where } s_\mu(x) = -e\,\bar{\psi}_l(x)\gamma_\mu\psi_l(x) \; , \quad \left(\psi_l = \psi^L_l + \psi^R_l\right)$$

is the electromagnetic current.

$$J^\mu_Y = \frac{s^\mu}{e} - J^\mu_3$$

let's define

$$W_\mu(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) - iW_{2\mu}(x)\right], \quad W^+_\mu(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) + iW_{2\mu}(x)\right]$$

and

$$W_{3\mu}(x) \equiv \cos\Theta_W Z_\mu(x) + \sin\Theta_W A_\mu(x)$$

$$B_\mu(x) \equiv -\sin\Theta_W Z_\mu(x) + \cos\Theta_W A_\mu(x)$$

θ_W is the Weinberg angle that will be defined later.

neutral currents:

Then

$$-g J_3^\mu W_{3\mu} - g' J_Y^\mu B_\mu = -g J_3^\mu (c_w Z_\mu + s_w A_\mu) - g'\left(\frac{S^\mu}{e} - J_3^\mu\right)(-s_w Z_\mu + c_w A_\mu) =$$

$$= -\frac{g'}{e} S^\mu(-s_w Z_\mu + c_w A_\mu) +$$
$$- J_3^\mu\left[(g c_w + g' s_w) Z_\mu + \underbrace{(g s_w - g' c_w)}_{} A_\mu\right] =$$

We want to identify A_μ with the QED vector potential, so

$$-\frac{g'}{e} c_w = -1 \quad \text{and} \quad g s_w - g' c_w = 0$$

$$\boxed{c_w = \frac{e}{g'} \, , \quad s_w = \frac{e}{g}}$$

$$= -S^\mu A_\mu + Z_\mu\left[+\frac{1}{c_w} s_w S^\mu - \underbrace{\left(g c_w + \frac{e}{c_w} s_w\right)}_{} J_3^\mu\right] = -A_\mu S^\mu - \frac{g}{c_w} Z\left[-\frac{s_w^2}{e} S^\mu + J_3^\mu\right] =$$

$$\frac{g}{c_w}\frac{s_w}{g} = \frac{g}{c_w} s_w \frac{s_w}{e} \qquad \hookrightarrow \frac{g}{c_w}\left(c_w^2 + \frac{e}{g} s_w\right) = \frac{g}{c_w}$$

$$= -S^\mu A_\mu - \frac{g}{c_w} \sum_{f = l, \nu_l} \left(g_L^f \bar{\psi}_f^L \gamma^\mu \psi_f^L + g_R^f \bar{\psi}_f^R \gamma^\mu \psi_f^R\right) Z_\mu$$

$$S^\mu = -e \bar{\psi}_l^L \gamma^\mu \psi_l^L - e \bar{\psi}_l^R \gamma^\mu \psi^R$$

$$J_3^\mu = \frac{1}{2}\left(\bar{\psi}_{\nu_l}^L \gamma^\mu \psi_{\nu_l}^L - \bar{\psi}_l^L \gamma^\mu \psi_l\right)$$

$$\boxed{\begin{array}{ll} g_L^{\nu_l} = +\frac{1}{2} & g_R^{\nu_l} = 0 \\ g_L^{l} = s_w^2 - \frac{1}{2} & g_R^{l} = s_w^2 \end{array}}$$

charged currents

$$W_\mu^+(x) \equiv \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) - i W_{2\mu}(x)\right], \quad W_\mu^-(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) + i W_{2\mu}(x)\right] \Rightarrow \begin{cases} W_{1\mu} = \frac{1}{\sqrt{2}}(W_\mu^- + W_\mu^+) \\ W_{2\mu} = \frac{-i}{\sqrt{2}}(W_\mu^- - W_\mu^+) \end{cases}$$

$$\mathcal{L}_1^{(l)} \rightarrow -g \sum_{j=1,2} J_j^\mu W_\mu^j = -g \frac{1}{\sqrt{2}}\left[J_1^\mu (W_\mu^+ + W_\mu^-) + i J_2^\mu (W_\mu^+ - W_\mu^-)\right] =$$

$$= -\frac{g}{\sqrt{2}}\left[\underbrace{(J_1^\mu + i J_2^\mu)}_{\substack{\shortparallel \\ J^{\mu +}}} W_\mu^+ + \underbrace{(J_1^\mu - i J_2^\mu)}_{\substack{\shortparallel \\ J^{\mu -}}} W_\mu^-\right]$$

$$+ -\frac{g}{\sqrt{2}}\Big[\underbrace{(J_1^\mu + iJ_2^\mu)}_{\frac{J^{\mu+}}{2}} W_\mu + \underbrace{(J_1^\mu - iJ_2^\mu)}_{\frac{J^\mu}{2}} W_\mu\Big]$$

– Find an explicite form of J^μ

$$J_i^\mu = \overline{\Psi}_L^L \gamma^\mu \frac{\tau_i}{2} \Psi_L^L \rightarrow J_1^\mu = \frac{1}{2}\left(\overline{\psi}_{\nu_L}^L, \overline{\psi}_L^L\right)\gamma^\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \psi_{\nu_L}^L \\ \psi_L^L \end{pmatrix} =$$

$$= \frac{1}{2}\left(\overline{\psi}_{\nu_L}^L \gamma^\mu \psi_L^L + \overline{\psi}_L^L \gamma^\mu \psi_{\nu_L}^L\right)$$

$$J_2^\mu = \frac{i}{2}\left(\overline{\psi}_{\nu_L}^L, \overline{\psi}_L^L\right)\gamma^\mu \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \psi_{\nu_L}^L \\ \psi_L^L \end{pmatrix} =$$

$$= \frac{i}{2}\left(-\overline{\psi}_{\nu_L}^L \gamma^\mu \psi_L^L + \overline{\psi}_L^L \gamma^\mu \psi_{\nu_L}^L\right)$$

$$J_{(1)}^\mu = 2(J_1^\mu - iJ_2^\mu) = 2\,\overline{\psi}_L^L \gamma^\mu \psi_{\nu_L}^L = \overline{\psi}_L \gamma^\mu (1-\gamma_5)\psi_{\nu_L}$$

$$J_{(1)}^{\mu+} = 2(J_1^\mu + iJ_2^\mu) = \quad = \overline{\psi}_{\nu_L}\gamma^\mu(1-\gamma_5)\psi_L$$

$$\mathcal{L}_1^{(1)} \rightarrow -\frac{g}{2\sqrt{2}}\left(J_{(1)}^{\mu+} W_\mu^+ + J_{(1)}^\mu W_\mu^-\right)$$

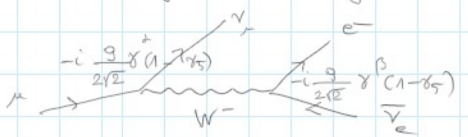
$$\boxed{\begin{aligned} \mathcal{L}_1^{(1)} &= -A_\mu s^\mu - \frac{g}{c_W} Z_\mu\left(-\frac{s_W^2}{e}s^\mu + J_3^\mu\right) - \frac{g}{2\sqrt{2}}\left(J^{\mu+}W_\mu^+ + J^\mu W_\mu^-\right) \\ \text{for} \quad s^\mu &= -e\,\overline{\psi}_L \gamma^\mu \psi_L \\ J_3^\mu &= \frac{1}{2}\left[\overline{\psi}_{\nu_L}\gamma^\mu \frac{1}{2}(1-\gamma_5)\psi_{\nu_L} - \overline{\psi}_L \gamma^\mu \frac{1}{2}(1-\gamma_5)\psi_L\right] \\ J^\mu &= \overline{\psi}_L \gamma^\mu (1-\gamma_5)\psi_{\nu_L} \end{aligned}}$$

– Draw Feynman diagrams for

$\nu_e e^- \rightarrow \nu_e e^-$ scattering

$\Rightarrow \quad s_W^2 \simeq 0.22$

– Draw a Feynman diagram for the muon decay and



determine the S-matrix element knowing that the W-propagator reads

$$D_{\mu\nu}(q) = \frac{-i}{q^2 - m_W^2}\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)$$

$$S = \bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu \; \bar{e}\gamma^\beta(1-\gamma_5)\nu_e \frac{-i\left(\eta_{\alpha\beta} - \frac{q_\alpha q_\beta}{m_W^2}\right)}{q^2 - m_W^2} \cdot \left(\frac{-ig}{2\sqrt{2}}\right)^2 \simeq \frac{-ig^2}{8m_W^2} \bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu \; \bar{e}\gamma_\alpha(1-\gamma_5)\nu_e$$

IVB hypothesis $\Rightarrow$ $\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} J^\mu_{(L)} J^+_{(L)\mu}$ $\longrightarrow$ $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$

Quarks

$SU(2)_L$: $Q^L_L(x) \to Q^{L\prime}_L(x) = e^{ig\frac{\tau_i}{2}\alpha_i(x)} Q^L_L(x)$ $\qquad Q^L_L = \begin{pmatrix} u^L_L \\ d^L_L \end{pmatrix}$, $L = 1,2,3$

$U(1)_Y$ $\quad Q^L_L(x) \to Q^{L\prime}_L(x) = e^{ig' Y f(x)} Q^L_L(x)$ $\qquad Y(Q^L_L) = \frac{1}{6}$ $\quad T_3(u^L_L) = -T_3(d^L_L) = \frac{1}{2}$

$u^R_L(x) \to u^{R\prime}_L(x) = e^{ig'Yf(x)} u^R_L(x)$ $\qquad Y(u^R_L) = \frac{2}{3}$ $\quad T_3(u^R_L) = 0$

$d^R_L(x) \to d^{R\prime}_L(x) = e^{ig'Yf(x)} d^R_L(x)$ $\qquad Y(d^R_L) = -\frac{1}{3}$ $\quad T_3(d^R_L) = 0$

$\Rightarrow Q = T_3 + Y$

$$\mathcal{L}^{(q)} = \overline{Q^L_L} i \not{D} Q^L_L + \overline{u^R_L} i\not{D} u^R_L + \overline{d^R_L} i \not{D} d^R_L$$

$$D_\mu \psi = \left(\partial_\mu + ig\frac{\tau_i}{2} W_{i\mu} + ig' Y B_\mu\right)\psi \quad , \quad \text{for } \psi = Q^L_L, u^R_L, d^R_L$$

charged current

$$\mathcal{L}^{(q)}_1 \to -\frac{g}{2\sqrt{2}}\left(J^{\mu +}_{(q)} W^+_\mu + J^\mu_{(q)} W_\mu\right) \quad \text{for} \quad J^\mu_{(q)} = \bar{d}\gamma^\mu(1-\gamma_5)u$$

$$J^{\mu +}_{(q)} = \bar{u}\gamma^\mu(1-\gamma_5)d$$

neutral currents

$$-g J^\mu_3 W_{3\mu} - g' J^\mu_Y B_\mu = -g J^\mu_3 (c_W Z_\mu + s_W A_\mu) - g' J^\mu_Y(-s_W Z_\mu + c_W A_\mu) = -S^\mu_{(q)} A_\mu - \frac{g}{c_W} J^\mu_0 Z_\mu$$

$$= c_W^2 J^\mu_3 - g' s_W \frac{c_W}{g} J^\mu_Y = c_W^2 J^\mu_3 - s_W^2 J^\mu_Y = c_W^2 J^\mu_3 - s_W^2\left(\frac{1}{e} S^\mu_{(q)} - J^\mu_3\right) = J^\mu_3 - \frac{s_W^2}{e} S^\mu_{(q)}$$

electromagnetic current $\quad S^\mu_{(q)} = g s_W J^\mu_3 + g' c_W J^\mu_Y = e\left(J^\mu_3 + J^\mu_Y\right)$ $\qquad Q = T_3 + Y$

$$\boxed{c_W = \frac{e}{g'} \; , \quad s_W = \frac{e}{g}}$$

$$J^\mu_Y = \sum_\psi \bar{\psi}\gamma^\mu Y(\psi)\psi \quad \text{for} \quad \psi = u^L_L, d^L_L, u^R_L, d^R_L$$

$$J^\mu_3 = \frac{1}{2}\left(\bar{u}^L_L \gamma^\mu u^L_L - \bar{d}^L_L \gamma^\mu d^L_L\right) = \sum_\psi \bar{\psi}\gamma^\mu T_3(\psi)\psi \quad \text{for} \quad \psi = u^L_L, d^L_L$$

$$S^\mu = e\sum \bar{\psi}\gamma^\mu (Y + T_3)\psi = e\sum \bar{\psi}\gamma^\mu Q(\psi)\psi =$$

$$J_3^\mu = \frac{1}{2}\left(\bar{u}^L_L \gamma^\mu u^L_L - \bar{d}^L_L \gamma^\mu d^L_L\right) = \sum_\psi \bar{\psi}\gamma^\mu T_3(\psi)\psi \quad \text{for} \quad \psi = u^L_L,\ d^L_L$$

$$S^\mu_q = e\sum_\psi \bar{\psi}\gamma^\mu \underbrace{(Y+T_3)}_{Q}\psi = e\sum \bar{\psi}\gamma^\mu Q(\psi)\psi =$$

$$= e\left(Q(u)\,\bar{u}\gamma^\mu u + Q(d)\,\bar{d}\gamma^\mu d\right)$$

$$Q(u) = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$Q(d) = -\frac{1}{2} + \frac{1}{6} = -\frac{2}{6} = -\frac{1}{3}$$

$$Q(l) = -\frac{1}{2} - \frac{1}{2} = -1$$

$$= -A_\mu S^\mu_{(q)} - \frac{g}{c_W} Z_\mu \underbrace{\left(+J_{(q)3}^\mu - \frac{s_W^2}{e} S^\mu_{(q)}\right)}_{J_0^\mu} =$$

$$J_0^\mu = \sum_{f=u,d}\left[g_L^f\, \bar{f}^L\gamma^\mu f^L + g_R^f\, \bar{f}^R\gamma^\mu f^R\right]$$

$$\boxed{\begin{array}{ll} g_L^u = \frac{1}{2} - \frac{2}{3}s_W^2 & g_R^u = -\frac{2}{3}s_W^2 \\ g_L^d = -\frac{1}{2} + \frac{1}{3}s_W^2 & g_R^d = \frac{1}{3}s_W^2 \end{array}}$$

$$\Rightarrow g^f_{L,R} = T_3(f^{L,R}) - s_W^2\,\frac{Q(f)}{e}$$

$$\mathcal{L}_1^{(q)} = -A_\mu S^\mu - \frac{g}{c_W} Z_\mu\left(-\frac{s_W^2}{e}S^\mu + J_3^\mu\right) - \frac{g}{2\sqrt{2}}\left(J^{\mu +} W^+_\mu + J^\mu W_\mu\right)$$

for

$$S^\mu = e\sum_f Q_f\, \bar{f}\gamma^\mu f$$

$$J_3^\mu = \frac{1}{2}\sum_{u,d}\left[\bar{u}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)u - \bar{d}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)d\right]$$

$$J^\mu = \bar{d}\gamma^\mu(1-\gamma_5)u$$

gauge bosons

$SU(2)\times U(1)$

$$\mathcal{L}_G = -\frac{1}{4}G_{i\mu\nu}G_i^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$G_i^{\mu\nu} \equiv \underbrace{\partial^\mu W^{i\nu} - \partial^\nu W^{i\mu}}_{\equiv W_i^{\mu\nu}} - g\,\varepsilon_{ijk}W_j^\mu W_k^\nu$$

$$B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu$$

$$B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu$$

$$\mathcal{L}_G = \underbrace{-\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{i\,\mu\nu} W_i^{\mu\nu}}_{\mathcal{L}_{G0}} + g\, \varepsilon_{ijk} W_{i\mu} W_{j\nu} \partial^\mu W_k^\nu - \frac{1}{4} g^2 \varepsilon_{ijk} \varepsilon_{ilm} W_j^\mu W_k^\nu W_{l\mu} W_{m\nu}$$

in terms of A_μ, Z_μ and $W_\mu^\pm$ we get

$$\mathcal{L}_{G0} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F^+_{W\,\mu\nu} F_W^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu}$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$$

$\leftarrow$ non-abelian theory

gauge symmetry breaking via the Higgs mechanism

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi\rangle} U(1)_{em}$$

$$\Downarrow$$

3 gauge boson become massive

1 remain massless

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad Y(\phi) = \frac{1}{2}$$

$$\phi \xrightarrow{SU(2)} \phi' = e^{ig\frac{T_j}{2}\omega_j(x)} \phi$$

$$\phi \xrightarrow{U(1)} \phi' = e^{ig' Y f(x)} \phi$$

$$\mathcal{L}_\phi = (D_\mu \phi)^+ D^\mu \phi - V(\phi)$$

with $$D_\mu \phi = \left(\partial_\mu + ig \frac{T_j}{2} W_{j\mu} + ig' \frac{1}{2} B_\mu\right) \phi$$

$$V(\phi) = -\mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2$$

The most general $SU(2) \times U(1)$ invariant Yukawa interactions

$$\mathcal{L}_Y = \Gamma^{(l)}_{lk} \overline{\Psi^L_l} \phi \Psi^R_k + \Gamma^{(u)}_{lk} \overline{Q^L_l} \tilde{\phi} u^R_k + \Gamma^{(d)}_{lk} \overline{Q^L_l} \phi d^R_k + \text{H.c.}$$

where $\tilde{\phi} \equiv i\tau_2 \phi^*$

– Problem: find the transformation rule for $\tilde{\phi}$

$$\phi \to \phi' = e^{ig\frac{\tau_a}{2}\omega_a} \phi$$

$$\phi \to \phi' = e^{ig\frac{\tau_a}{2}\omega_a}\phi$$

$$\tilde{\phi} \to i\tau_2(\phi')^* = i\tau_2\, e^{-ig\frac{\tau_a^*}{2}\omega_a}\phi^* = i\tau_2\, e^{-ig\frac{\tau_a^*}{2}\omega_a}\, i\tau_2 \underbrace{(i\tau_2\phi^*)}_{\tilde{\phi}}$$

$$e^{i\alpha\,\vec{n}\cdot\vec{\tau}} = \mathbb{1}\cos\alpha + i(\vec{n}\cdot\vec{\tau})\sin\alpha \quad , \quad |\vec{n}| = 1$$

$$e^{-ig\frac{\tau_a}{2}\omega_a} = e^{i\left(-g\frac{\omega_a}{2}\right)\tau_a}$$

$$-g\frac{\omega_a}{2} = \alpha\cdot n_a$$

$$\tau_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \tau_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \quad \tau_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$i\tau_2\left(e^{i\alpha\,\vec{n}\cdot\vec{\tau}}\right)^* i\tau_2 = i\tau_2\left[\mathbb{1}\cos\alpha + i(n_a\tau_a^*)\sin\alpha\right] i\tau_2 =$$

$$\left.\begin{array}{l} [\tau_a, \tau_b] = 2i\varepsilon_{abc}\tau_c \\ [\tau_1, \tau_2] = 2i\tau_3 \\ [\tau_3, \tau_2] = -2i\tau_1 \end{array}\right\}$$

$$\tau_2\tau_1^*\tau_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\cdot\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} =$$

$$= \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix} = \begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix} = -\tau_1$$

$$\tau_2\tau_2^*\tau_2 = -\tau_2$$

$$\tau_2\tau_3^*\tau_2 = \tau_2\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix} = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix} = -\tau_3$$

$$\boxed{(i\tau_2)\,\tau_a^*\,(i\tau_2) = \tau_a}$$

$$(i\tau_2)\left(e^{-ig\frac{\tau_a}{2}\omega_a}\right)^*(i\tau_2) = e^{-ig\frac{\tau_a}{2}\omega_a} \Rightarrow \tilde{\phi} \to \tilde{\phi}' = e^{-i\frac{\tau_a}{2}\omega_a}\tilde{\phi} \qquad \square$$

Particle spectra and interactions in the unitary gauge

It could be shown that any $\phi = \begin{pmatrix}\phi^+\\ \phi_0\end{pmatrix}$ could be parametrized as

$$\phi(x) = U^{-1}(\xi)\begin{pmatrix}0\\ \frac{1}{\sqrt{2}}(v+\eta(x))\end{pmatrix}$$

for $U(\xi) = e^{i\xi_a(x)\cdot\frac{\tau_a}{v}}$

One can perform a gauge SU(2) transformation $U(\xi)$

$$\phi \to U(\xi)\,\phi = U(\xi)\,U^{-1}(\xi)\begin{pmatrix}0\\ v+\eta(x)\end{pmatrix}\begin{pmatrix}0\\ \end{pmatrix} \quad \phi' = v+\eta(x)$$

One can perform a gauge SU(2) transformation $U(\xi)$

$$\phi \to U(\xi)\,\phi = U(\xi)\,U^{-1}(\xi)\begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}, \qquad \phi' = \frac{v+\eta(x)}{\sqrt{2}}\cdot\chi$$

$$\Psi^L_l \to \Psi^{L'}_l = U\,\Psi^L_l \qquad\qquad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\vdots$

Then we can find the minimum of

$$V(\phi) = -\mu^2 \phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

in term of v:

$$V(v) = -\mu^2 \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} + \lambda\left[\begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}\right]^2 = -\frac{\mu^2}{2}v^2 + \frac{\lambda}{4}v^4$$

$$\frac{\partial V}{\partial v} = -\mu^2 v + \lambda v^3 = v(-\mu^2 + \lambda v^2) = 0 \qquad v^2 = \frac{\mu^2}{\lambda}$$

$$(D_\mu\phi)^\dagger(D^\mu\phi) = (D_\mu\phi')^\dagger(D^\mu\phi') = \left\{\left(\partial_\mu - \frac{i}{2}g\tau_a W_{a\mu} - ig'\frac{1}{2}B_\mu\right)\left[\frac{v+\eta(x)}{\sqrt{2}}\right]\chi\right\}^\dagger \cdot$$

$$\cdot\left\{\left(\partial^\mu - \frac{i}{2}g\tau_b W_b^\mu - ig'\frac{1}{2}B^\mu\right)\left[\frac{v+\eta(x)}{\sqrt{2}}\right]\chi\right\}$$

$$V(\phi') = -\mu^2\frac{1}{2}(v+\eta)^2 + \lambda\frac{1}{4}(v+\eta)^4 = -\mu^2\frac{v^2}{2}$$

$$+\,\eta\underbrace{(-\mu^2 v + \lambda v^3)}_{=0} +$$

$$(v+\eta)^4 = (v^2 + 2v\eta + \eta^2)^2 =$$

$$= v^4 + 4v^2\eta^2 + \eta^4 + 2_{\!} v^3\eta + 2v^2\eta^2 + 4v\eta^3 =$$

$$= v^4 + 4v^3\eta + 6v^2\eta^2 + 4v\eta^3 + \eta^4$$

$$+\,\eta^2\underbrace{\left(-\frac{1}{2}\mu^2 + \frac{6}{4}\lambda v^2\right)}_{= -\frac{1}{2}\lambda v^2 + \frac{3}{2}\lambda v^2 = \lambda v^2} +$$

$$+\,\eta^3\left(4v\frac{1}{4}\lambda\right) +$$

$$+\,\eta^4\frac{1}{4}\lambda$$

$$\boxed{V(\eta) = \frac{1}{2}(2\lambda v^2)\eta^2 + \lambda v\eta^3 + \frac{\lambda}{4}\eta^4}$$

$$m_\eta^2 = 2\lambda v^2 = 2\mu^2$$

The most general SU(2)×U(1) invariant Yukawa interactions

$$\mathcal{L}_Y = \Gamma^{(l)}_{lk}\,\overline{\Psi^L_l}\,\phi\,\psi^R_k + \Gamma^{(u)}_{lk}\,\overline{Q^L_l}\,\tilde{\phi}\,u^R_k + \Gamma^{(d)}_{lk}\,\overline{Q^L_l}\,\phi\,d^R_k + \text{H.c.} =$$

$$= \frac{\eta(x)+v}{\sqrt{2}}\cdot\left[\Gamma^{(l)}_{lk}\,\overline{\psi^L_l}\,\psi^R_k + \Gamma^{(u)}_{lk}\,\overline{u^L_l}\,u^R_k + \Gamma^{(d)}_{lk}\,\overline{d^L_l}\,d^R_k + \text{H.c.}\right]$$

Mass spectrum

- scalar mass: $m_\eta = \sqrt{2\lambda}\, v = \sqrt{2}\,\mu$, $\quad m_\eta = \begin{cases} 126.0 \pm 0.4 \pm 0.4 \text{ GeV} & \text{(ATLAS)} \\ 125.3 \pm 0.4 \pm 0.5 \text{ GeV} & \text{(CMS)} \end{cases}$

- fermion masses: $\left(M^{(l)}\right)_{lk} = -\frac{v}{\sqrt{2}} \Gamma^{(l)}_{lk}$, $\left(M^{(u)}\right)_{lk} = -\frac{v}{\sqrt{2}} \Gamma^{(u)}_{lk}$, $\left(M^{(d)}\right)_{lk} = -\frac{v}{\sqrt{2}} \Gamma^{(d)}_{lk}$

- vector boson masses:

$$(D_\mu \phi)^+ (D^\mu \phi) \longrightarrow \frac{v^2}{2} \chi^+ \left(\frac{g}{2} \vec{\tau} \cdot \vec{W}_\mu + \frac{g'}{2} B_\mu \right) \left(\frac{g}{2} \vec{\tau} \cdot \vec{W}^\mu + \frac{g'}{2} B^\mu \right) \chi \quad \Big\} \text{ in the unitary gauge}$$

$$\chi^+ \vec{\tau} \cdot \vec{W}_\mu \, \vec{\tau} \cdot \vec{W}^\mu \chi = \chi^+ \tau_a \tau_b \chi \, W_{a\mu} W_b^\mu = W_{a\mu} W_a^\mu$$

$$\tau_a \tau_b = \frac{1}{2} \{\tau_a, \tau_b\} + \frac{1}{2} [\tau_a, \tau_b] = \delta_{ab} \mathbb{1} + i \varepsilon_{abc} \tau_c$$

$$\varepsilon_{abc} W_{a\mu} W_b^\mu = \varepsilon_{bac} W_{b\mu} W_a^\mu =$$

$$\chi^+ \tau_{1,2} \chi = (0,1) \begin{pmatrix} 0 & \cdot \\ \cdot & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (0\ 1) \begin{pmatrix} \cdot \\ 0 \end{pmatrix} = 0 \qquad = -\varepsilon_{abc} W_b^\mu W_{a\mu} = 0$$

$$\chi^+ \tau_3 \chi = (0,1) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -1$$

$$(D_\mu \phi)^+ D^\mu \phi \longrightarrow \frac{v^2}{2} \left\{ g^2 \frac{1}{4} W_{a\mu} W_a^\mu + g g' W_{3\mu} B^\mu \frac{1}{4} \cdot 2 \cdot (-) + g'^2 \frac{1}{4} B_\mu B^\mu \right\} =$$

$$= \frac{v^2}{8} \left\{ g^2 \left(W_{1\mu} W_1^\mu + W_{2\mu} W^{2\mu} \right) + \left(g W_{3\mu} - g' B_\mu \right) \left(g W_3^\mu - g' B^\mu \right) \right\} =$$

$$= M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

<u>charged bosons:</u>

$$W_\mu^-(x) = \frac{1}{\sqrt{2}} \left[W_{1\mu}(x) + i W_{2\mu}(x) \right], \quad W_\mu^+(x) = \frac{1}{\sqrt{2}} \left[W_{1\mu}(x) - i W_{2\mu}(x) \right] \Rightarrow \begin{cases} W_{1\mu} = \frac{1}{\sqrt{2}} \left(W_\mu^- + W_\mu^+ \right) \\ W_{2\mu} = \frac{-i}{\sqrt{2}} \left(W_\mu^- - W_\mu^+ \right) \end{cases}$$

$$W_\mu^+ W^{-\mu} = \frac{1}{2} \left(W_1 \cdot W_1 - i W_1 \cdot W_2 + i W_2 \cdot W_1 + W_2 W_2 \right) =$$

$$= \frac{1}{2} \left(W_{1\mu} \cdot W_1^\mu + W_{2\mu} W_2^\mu \right) \Longrightarrow M_W^2 = \frac{v^2}{4} g^2 \qquad m_W \cong 80.4 \text{ GeV}$$

<u>neutral bosons:</u>

$$\frac{1}{2} M_Z^2 Z_\mu Z^\mu = \frac{v^2}{8}\left(g W_{3\mu} - g' B_\mu\right)\left(g W^{3\mu} - g' B^\mu\right) = \frac{v^2}{8}\left(W_{3\mu}, B_\mu\right)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} =$$

$$\det(\;) = g^2 g'^2 - (gg')^2 = 0$$

$$= \frac{1}{2}\left(Z_\mu, A_\mu\right)\begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_3 \\ B \end{pmatrix} \qquad M_Z^2 = \frac{v^2}{4}\left(g^2 + g'^2\right) \qquad m_Z = 91{,}2 \text{ GeV}$$

$$\operatorname{tg}\theta_W = \frac{g'}{g}$$

check :

$$\begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} Z \\ A \end{pmatrix} \qquad s_W = \frac{g'}{\sqrt{g^2+g'^2}}, \quad c_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\begin{pmatrix} g & -gg' \\ -gg' & g'^2 \end{pmatrix} \to \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}\underbrace{\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}\begin{pmatrix} g^2 c_W + gg' s_W ; & g^2 s_W - gg' c_W \\ -gg' c_W - g'^2 s_W ; & -gg' s_W + g'^2 c_W \end{pmatrix} =$$

$$= \begin{pmatrix} \underbrace{g^2 c_W^2 + gg' c_W s_W + gg' s_W c_W + g'^2 s_W^2} ; & g^2 s_W c_W - gg' c_W^2 + gg' s_W^2 - g'^2 s_W c_W \\ & ; \underbrace{g^2 s_W^2 - gg' s_W c_W - gg' c_W s_W + g'^2 c_W^2} \end{pmatrix}$$

$$= \frac{1}{g^2+g'^2}\left(g^2 gg' - gg' g^2 + gg' g'^2 - g'^2 gg\right) = 0 \qquad \frac{1}{g^2+g'^2}\left(g^2 g'^2 - 2(gg')^2 + g'^2 g^2\right) = 0$$

$$(1,1) = \frac{1}{g^2+g'^2}\left(g^4 + 2(gg')^2 + g'^4\right) = g^2 + g'^2 \qquad \square$$

<u>the ρ parameter :</u>

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{\frac{1}{4} v^2 g^2}{\frac{1}{4} v^2 (g^2 + g'^2) \, \frac{g^2}{g^2+g'^2}} = 1$$

The fermion mixing

The most general $SU(2)\times U(1)$ invariant Yukawa interactions

$$\mathcal{L}_Y = \Gamma^{(l)}_{lk} \overline{\Psi^L_l} \phi \psi^R_k + \Gamma^{(u)}_{lk} \overline{Q^L_l} \tilde{\phi} u^R_k + \Gamma^{(d)}_{lk} \overline{Q^L_l} \phi d^R_k + \text{H.c.} =$$

$$= \frac{\eta(x) + v}{\sqrt{2}} \cdot \left[\Gamma^{(l)}_{lk} \overline{\psi^L_l} \psi^R_k + \Gamma^{(u)}_{lk} \overline{u^L_l} u^R_k + \Gamma^{(d)}_{lk} \overline{d^L_l} d^R_k + \text{H.c.}\right]$$

$$\mathcal{L}_Y = \Gamma_{lk} \bar{\Psi}_l \Phi \Psi_k + \Gamma_{lk} \bar{Q}_l \Phi u_k + \Gamma_{lk} \bar{Q}_l \Phi d_k + H.c. =$$

$$= \frac{\eta(x)+v}{\sqrt{2}} \cdot \left[\Gamma^{(l)}_{lk} \overline{\psi^L_l} \psi^R_k + \Gamma^{(u)}_{lk} \overline{u^L_l} u^R_k + \Gamma^{(d)}_{lk} \overline{d^L_l} d^R_k + H.c. \right]$$

fermion masses: $(M_l)_{lk} = -\frac{v}{\sqrt{2}} \Gamma^{(l)}_{lk}$, $(M_u)_{lk} = -\frac{v}{\sqrt{2}} \Gamma^{(u)}_{lk}$, $(M_d)_{lk} = -\frac{v}{\sqrt{2}} \Gamma^{(d)}_{lk}$

Γ's are generic complex $n \times n$ matrices

Any complex matrix M could be decomposed as $M = H \cdot V$, where H is Hermitian while V is unitary

M could be diagonalized by biunitary transformations:

$$M_d = S^+ M R$$

where S and R are unitary matrices such that $S^+ H S = H_d$ is diagonal and $R = V^+ S$ is unitary.

$$M_d = S^+ H V R = \underbrace{S^+ H S}_{H_d} \underbrace{S^+ V R}_{1\!\!1} = H_d$$

$$\frac{\eta(x)+v}{\sqrt{2}} \cdot \left[\Gamma^{(l)}_{lk} \overline{\psi^L_l} \psi^R_k + \Gamma^{(u)}_{lk} \overline{u^L_l} u^R_k + \Gamma^{(d)}_{lk} \overline{d^L_l} d^R_k + H.c. \right]$$

i) $u^L_l, u^R_l, d^R_l \to u'^L_l, u'^R_l, d'^R_l$

quark mass matrices: $\mathcal{L}_m = - \overline{u'^L} M_u u'^R - \overline{d'^L} M_d d'^R + H.c.$

$$u'^L = S_u u^L, \quad u'^R = R_u u^R$$

$$d'^L = S_d d^L, \quad d'^R = R_d d^R$$

then

$$\mathcal{L}_m = - \overline{u^L} S_u^+ M_u R_u u^R - \overline{d^L} S_d^+ M_d R_d d^R + H.c. =$$

$$= - \overline{u^L} M^{diag} u^R - \overline{d^L} M_d^{diag} d^R + H.c.$$

for $S_{u/d}$, $R_{u/d}$ such that $S_u^+ M_u R_u = M_u^{diag}$ and $S_d^+ M_d R_d = M_d^{diag}$

are diagonal

- $M_{u/d}^{diag}$ may be assumed to be positive therefore we get

$$\mathcal{L}_m = -\bar{u}\, M_u^{diag}\, u - \bar{d}\, M_d^{diag}\, d$$

for $u \equiv u^L + u^R$ and $d \equiv d^L + d^R$

- if an entry of $M_{u/d}^{diag}$ was negative then the following chiral transformation could be adopted to change the sign:

$$\bar{f}_L f_R \to \overline{(-i f_L)}\,(i f_R) = -\bar{f}_L f_R ,$$

so $f_{L/R} \to \mp i f_{L/R}$

$m_u \cong 2,3$ MeV $m_c \cong 1,28$ GeV $m_t \cong 173,2$ GeV

$m_d \cong 4,8$ MeV $m_s \cong 95$ MeV $m_b \cong 4,18$ GeV

ii) similarly for leptons

$$\psi^L = P\, l^L \qquad \psi^R = T\, l^R$$

$$\mathcal{L}_m = -\overline{l^L}\, \underbrace{P^+ M_L T}_{M_L^{diag}}\, l^R + h.c. = -\bar{l}\, M_L^{diag}\, l$$

$m_e \cong 0.511$ MeV $m_\mu \cong 0,106$ GeV $m_\tau \cong 1,7$ GeV

- Are there any further consequences of the above basis transformations?

- Are there any further consequences of the above basis transformations?

$$\boxed{\begin{aligned}\mathcal{L}_1^{(q)} &= -A_\mu s^\mu - \frac{g}{c_w} Z_\mu\left(-\frac{s_w^2}{e}s^\mu + J_3^\mu\right) - \frac{g}{2\sqrt{2}}\left(J^{\mu +}W_\mu^+ + J^\mu W_\mu\right)\\ \text{for}\quad s^\mu &= e\sum_f Q_f \bar{f}\gamma^\mu f\\ J_3^\mu &= \frac{1}{2}\sum_{u,d}\left[\bar{u}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)u - \bar{d}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)d\right]\\ J^\mu &= \bar{d}\gamma^\mu(1-\gamma_5)u\end{aligned}}$$

$$J^\mu = \bar{d}'_L\gamma^\mu u'_L = \bar{d}_L S_d^+\gamma^\mu S_u u_L = \bar{d}_L U_{CKM}\gamma^\mu u_L$$

$$u'^L = S_u u^L, \quad u'^R = R_u u^R$$

$$d'^L = S_d d^L, \quad d'^R = R_d d^R$$

$$U_{CKM} = S_d^+ S_u \qquad U_{CKM}^+ = S_u^+ S_d$$

$$U_{CKM}U_{CKM}^+ = S_d^+ S_u S_u^+ S_d = \mathbb{1}$$

Cabibbo – Kobayashi – Maskawa matrix

- show that for $n \geq 3$ U_{CKM} matrix contains physical non-zero phases

  - $n\times n$ unitary matrix $\quad UU^+ = \mathbb{1}$

$$U_{ij}(U^+)_{jk} = \delta_{ik}$$

$$U_{ij}U^*_{kj} = \delta_{ik} \qquad \text{for } i=k \quad U_{ij}U^*_{ij} = 1$$

$$UU^+ = \mathbb{1} \Rightarrow \frac{n^2-n}{2} \text{ complex conditions}$$

$$n + \frac{n^2-n}{2}\,2 \text{ real conditions}$$

$$\boxed{\text{unitary matrix has } 2n^2 - \left(n + \frac{n^2-n}{2}\,2\right) = 2n^2 - n^2 = n^2 \text{ real parameters}}$$

  - $n\times n$ orthogonal matrix $\quad OO^T = \mathbb{1}, \quad n^2 - \left(n + \frac{n^2-n}{2}\right) = n^2 - \frac{n}{2} - \frac{n^2}{2} = \frac{n^2-n}{2}$

parameter

$\Downarrow$

$n \times n$ unitary matrix has $n^2 - \frac{n^2 - n}{2} = \frac{n^2+n}{2}$ phases

$$\bar{d}_L U_{CKM} \gamma^\mu u_L \longrightarrow \bar{d}_L \begin{pmatrix} e^{-i\alpha_1} & & & \\ & e^{-i\alpha_2} & & \\ & & \ddots & \\ & & & e^{-i\alpha_n} \end{pmatrix} U_{CKM} \begin{pmatrix} e^{i\delta_1} & & & \\ & e^{i\delta_2} & & \\ & & \ddots & \\ & & & e^{i\delta_n} \end{pmatrix} \gamma^\mu u_L$$

$u_{iL} \to e^{i\delta_i} u_{iL}$

$d_{jL} \to e^{i\alpha_j} d_{jL}$

$2n-1$ phases of U_{CKM} could be absorbed (eliminated) by the above phase redefinitions

$\Downarrow$

# of physical phases in the CKM matrix is

$$\frac{n^2+n}{2} - (2n-1) = \frac{n^2+n-4n+2}{2} = \frac{n^2-3n+2}{2} = \frac{(n-2)(n-1)}{2}$$

Conclusion : at least 3 generations of quarks are needed for CPV $\square$

$$\frac{-g}{\sqrt{2}}(\overline{u_L}, \overline{c_L}, \overline{t_L})\gamma^\mu W_\mu^+ V_{\rm CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}, \qquad V_{\rm CKM} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{11.2}$$

This Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2] is a 3×3 unitary matrix. It can be parameterized by three mixing angles and the CP-violating KM phase [2]. Of the many possible conventions, a standard choice has become [3]

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{11.3}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and δ is the phase responsible for all CP-violating phenomena in flavor-changing processes in the SM. The angles θ_{ij} can be chosen to lie in the first quadrant, so $s_{ij}, c_{ij} \geq 0$.

Wolfenstein parametrization :

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,.$$

Absolut values of the CKM matrix elements:

$$V_{(} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix},$$

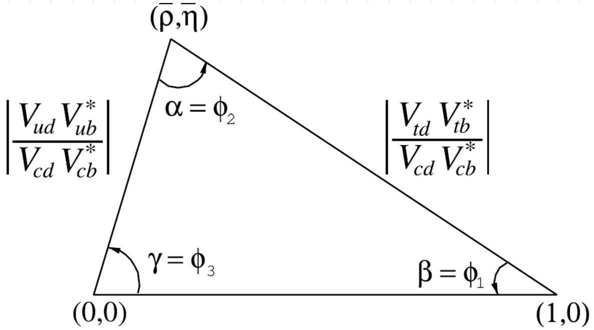


Figure 11.1: Sketch of the unitarity triangle.

The CKM matrix elements are fundamental parameters of the SM, so their precise determination is important. The unitarity of the CKM matrix imposes $\sum_i V_{ij}V_{ik}^* = \delta_{jk}$ and $\sum_j V_{ij}V_{kj}^* = \delta_{ik}$. The six vanishing combinations can be represented as triangles in a complex plane, of which the ones obtained by taking scalar products of neighboring rows or columns are nearly degenerate. The areas of all triangles are the same, half of the Jarlskog invariant, J [7], which is a phase-convention-independent measure of CP violation, defined by $\mathrm{Im}[V_{ij}V_{kl}V_{il}^*V_{kj}^*] = J\sum_{m,n}\varepsilon_{ikm}\varepsilon_{jln}$.

The most commonly used unitarity triangle arises from

$$V_{ud}\,V_{ub}^* + V_{cd}\,V_{cb}^* + V_{td}\,V_{tb}^* = 0\,, \tag{11.6}$$

by dividing each side by the best-known one, $V_{cd}V_{cb}^*$ (see Fig. 1). Its vertices are exactly $(0,0)$, $(1,0)$, and, due to the definition in Eq. (11.4), $(\bar\rho,\bar\eta)$. An important goal of flavor physics is to overconstrain the CKM elements, and many measurements can be conveniently displayed and compared in the $\bar\rho,\bar\eta$ plane.

- Find the shape of U_{CKM} in a world with equal down-type quarks

- Find the shape of U_{CKM} in a world with equal down-type quarks

$$J^\mu = \bar{d}_L \gamma^\mu U_{CKM} u_L \qquad \mathcal{L}_m = -m_d \underbrace{\bar{d}\, \mathbb{1}\, d}_{\bar{d}_L d_R + \bar{d}_R d_L} - \bar{u} M_u u$$

redefine d_L as follows

$$\bar{d}_L U_{CKM} = \bar{\tilde{d}}_L$$

$$U_{CKM}^+ d_L = \tilde{d}_L$$

$$d_L = U_{CKM} \tilde{d}_L \longrightarrow \mathcal{L}_m = -m_d \left(\bar{\tilde{d}}_L U_{CKM}^+ U_{CKM} \tilde{d}_R + \bar{\tilde{d}}_R U_{CKM}^+ U_{CKM} \tilde{d}_L \right) - \bar{u} M_u u$$

and $d_R = U_{CKM} \tilde{d}_R$

$$= -m_d \bar{\tilde{d}} \tilde{d} - \bar{u} M_u u$$

$$\Downarrow$$

$$U_{CKM} = \mathbb{1}$$

- What would happened to U_{CKM} if $M_u = \omega$?

answer : $U_{CKM} = \mathbb{1}$

- What happens to the neutral currents under the redefinition

$$u'^L = S_u u^L \,, \quad u'^R = R_u u^R$$

$$d'^L = S_d d^L \,, \quad d'^R = R_d d^R \qquad ?$$

$$\boxed{\begin{aligned} \mathcal{L}_1^{(q)} &= -A_\mu s^\mu - \frac{g}{c_W} Z_\mu\left(-\frac{s_W^2}{e}s^\mu + J_3^\mu\right) - \frac{g}{2\sqrt{2}}\left(J^{\mu +}W_\mu^+ + J^\mu W_\mu\right) \\ \text{for} \quad s^\mu &= e\sum_f Q_f \bar{f}\gamma^\mu f \\ J_3^\mu &= \frac{1}{2}\sum_{u,d}\left[\bar{u}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)u - \bar{d}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)d\right] \\ J^\mu &= \bar{d}\gamma^\mu(1-\gamma_5)u \end{aligned}}$$

$$s^\mu = \sum_{\substack{f=u,c,t\\ d,s,b\\ e,\mu,\tau}} Q_f \bar{f}\gamma^\mu f \qquad Q_{u,c,t} = \frac{2}{3} \qquad Q_{d,s,b} = -\frac{1}{3} \qquad Q_{e,\mu,\tau} = -1$$

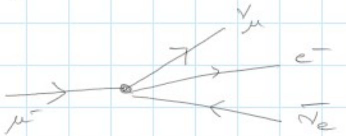
- μ – decay

V–A theory :

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} J_\mu^+ J^\mu + \text{H.c.}$$

$$J_\mu = J_{\ell_\mu} + J_{h_\mu}$$

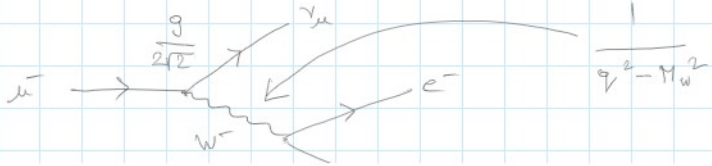
$$J_{\ell_\mu} = \bar{\nu}_e\gamma_\mu(1-\gamma_5)e + \bar{\nu}_\mu\gamma_\mu(1-\gamma_5)\mu$$



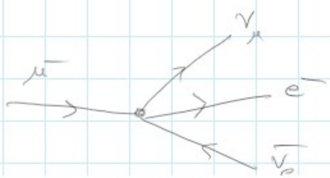
$$\tau_\mu \Longrightarrow G_F \simeq \frac{10^{-5}}{m_p^2}$$

IVB theory :

$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}}\left(J_\mu W^\mu + \text{H.c.}\right)$$

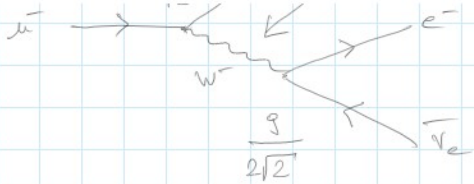


$\Longrightarrow$

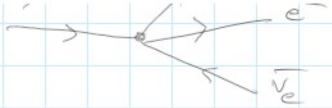


$q^2 \ll M_W^2$

$q^2 - M_W^2$



$\Longrightarrow$



$q^2 \ll M_W^2$

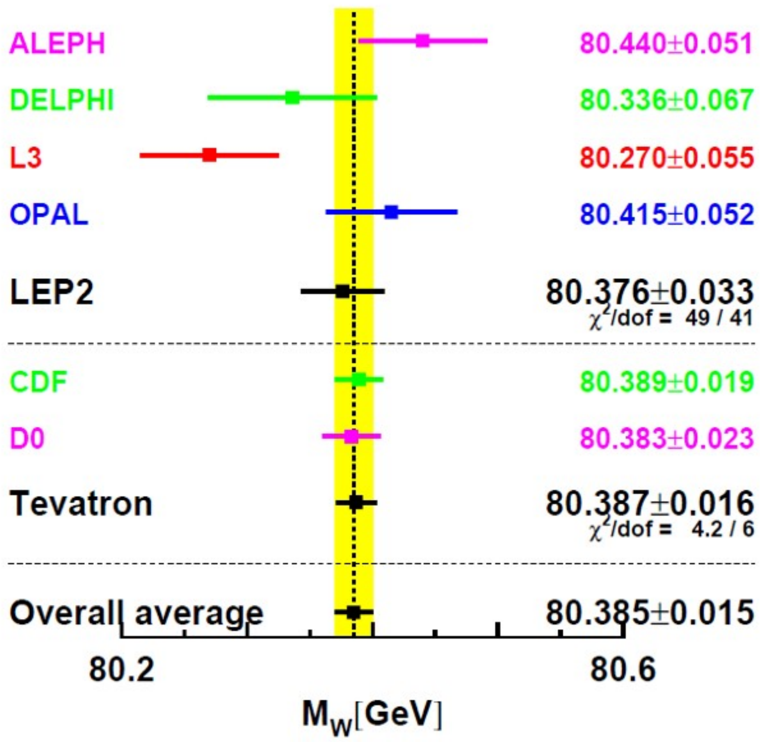
$$\left(\frac{g}{2\sqrt2}\right)^2 \frac{1}{M_W^2} = \frac{G_F}{\sqrt2}$$

$$M_W^2 = \frac{1}{4} g^2 v^2$$

$\Longleftarrow$

$$\frac{\cancel{g^2}}{2\cdot \cancel{8}\,\frac{1}{\cancel{4}}\cancel{g^2}v^2} = \frac{G_F}{\sqrt2}$$

$$\boxed{\begin{array}{c} v = G_F^{-1/2}\, 2^{-1/4} \\ \Downarrow \\ v \simeq 246\ \text{GeV} \end{array}}$$



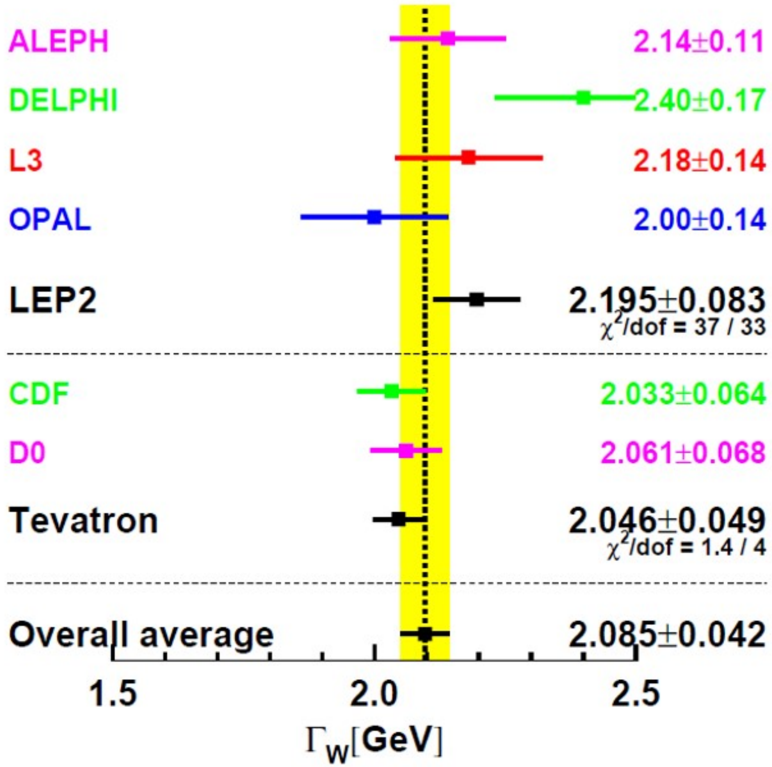


Figure 2: Measurements of the W-boson width by the LEP and Tevatron experiments.

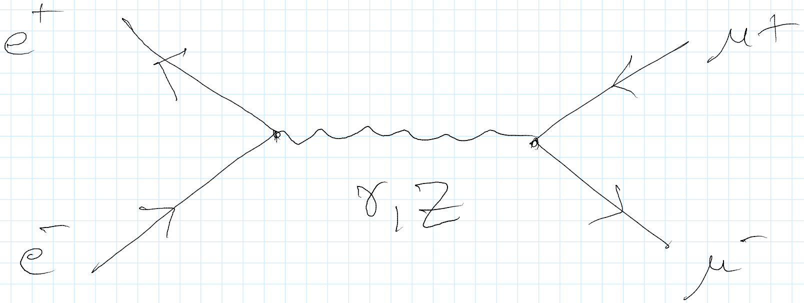
M_W[GeV]

Figure 1: Measurements of the W-boson mass

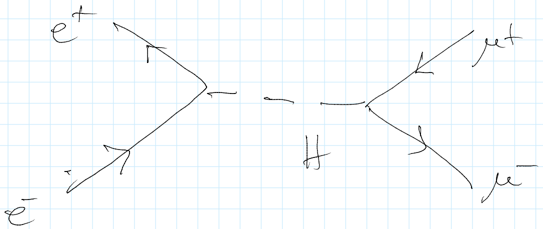
Figure 2: Measurements of the W-boson width by the LEP and Tevatron experiments.

Fermi coupling constant**	$G_F/(\hbar c)^3$	$1.166\,378\,7(6)\times 10^{-5}$ GeV^{-2}
weak-mixing angle	$\sin^2\hat{\theta}(M_Z)$ ($\overline{\text{MS}}$)	0.231 26(5)††
$W^\pm$ boson mass	m_W	80.385(15) GeV/c^2
Z^0 boson mass	m_Z	91.1876(21) GeV/c^2
strong coupling constant	$\alpha_s(m_Z)$	0.1185(6)

- $e^+e^- \to \mu^+\mu^-, \ldots$ (LEP)



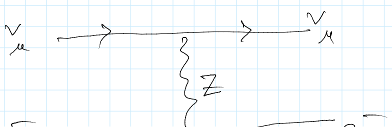
$$\Rightarrow \frac{A}{q^2} + \frac{B}{q^2 - M_Z^2 + i\Gamma_Z M_Z}$$

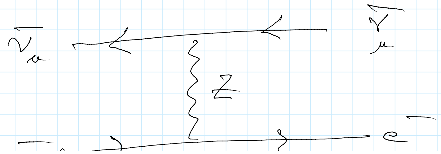


$$\propto \frac{m_e}{v} \cdot \frac{m_\mu}{v}$$

- $\nu_\mu + e^- \to \nu_\mu + e^-$

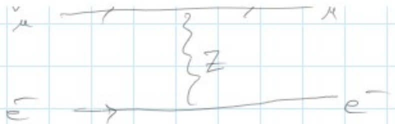
$\bar{\nu} + e^- \to \bar{\nu} + e^-$





$\nu_\mu + e^- \to \nu_\mu + e^-$

$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$



$$\mathcal{L} = -\frac{g}{c_W} Z_\mu J_0^\mu$$

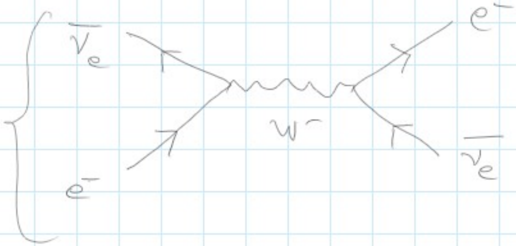
$$J_0^\mu = \sum_{f=u,d} \left[g_L^f \, \bar{f}^L \gamma^\mu f^L + g_R^f \, \bar{f}^R \gamma^\mu f^R \right]$$

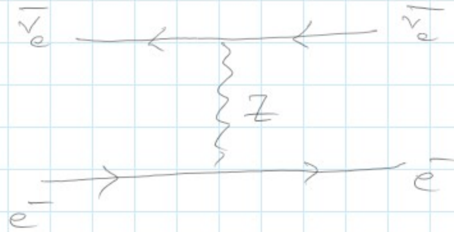
$$\boxed{\begin{array}{ll} g_L^u = \frac{1}{2} - \frac{2}{3} s_W^2 & g_R^u = -\frac{2}{3} s_W^2 \\ g_L^d = -\frac{1}{2} + \frac{1}{3} s_W^2 & g_R^d = \frac{1}{3} s_W^2 \end{array}} \Rightarrow g^f_{L,R} = T_3(f^{L,R}) - s_W^2 \frac{Q(f)}{e}$$

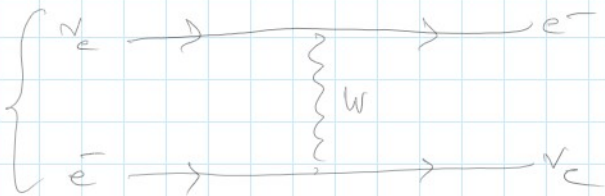
$$\frac{\sigma(\nu_\mu e^-)}{\sigma(\bar{\nu}_\mu e^-)} = \frac{(g_L^e)^2 + \frac{1}{3}(g_R^e)^2}{\frac{1}{3}(g_L^e)^2 + (g_R^e)^2} \Longrightarrow s_W^2 \text{ determination}$$

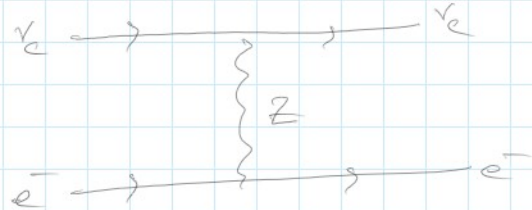
• $\nu_e + e^- \to \nu_e + e^-$

$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$









$e^- \longrightarrow \nu_e \qquad e^- \longrightarrow e^-$

$$\frac{\sigma(\nu_e e^-)}{\sigma(\bar\nu_e e^-)} = \frac{(1+g_L^e)^2 + \frac{1}{3}(g_R^e)^2}{\frac{1}{3}(1+g_L^e)^2 + (g_R^e)^2} \Rightarrow s_W^2 \text{ determination}$$

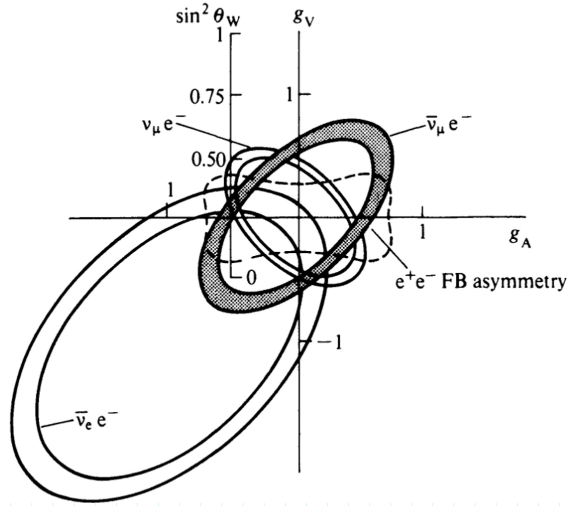
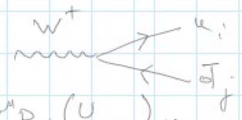


Table 10.7: Values of $\hat{s}_Z^2$, s_W^2, α_s, and M_H [in GeV] for various (combinations of) observables. Unless indicated otherwise, the top quark mass, $m_t = 173.4 \pm 1.0$ GeV, is used as an additional constraint in the fits. The (†) symbol indicates a fixed parameter.

Data	$\hat{s}_Z^2$	s_W^2	$\alpha_s(M_Z)$	M_H
All data	0.23116(12)	0.22295(28)	0.1196(17)	99^{+28}_{-23}
All indirect (no m_t)	0.23118(14)	0.22285(35)	0.1197(17)	134^{+144}_{-65}
Z pole (no m_t)	0.23121(17)	0.22318(60)	0.1197(28)	102^{+133}_{-51}
LEP 1 (no m_t)	0.23152(20)	0.22383(67)	0.1213(30)	191^{+266}_{-105}
SLD + M_Z	0.23067(28)	0.22204(54)	0.1185 (†)	39^{+31}_{-19}
$A_{FB}^{(b,c)} + M_Z$	0.23193(28)	0.22494(76)	0.1185 (†)	444^{+300}_{-178}
$M_W + M_Z$	0.23098(22)	0.22262(47)	0.1185 (†)	75^{+39}_{-30}
M_Z	0.23124(5)	0.22318(13)	0.1185 (†)	124.5 (†)
$Q_W(e)$	0.2332(15)	0.2252(15)	0.1185 (†)	124.5 (†)
Q_W (APV)	0.2311(16)	0.2230(17)	0.1185 (†)	124.5 (†)
ν_μ-N DIS (isoscalar)	0.2332(39)	0.2251(39)	0.1185 (†)	124.5 (†)
Elastic $\nu_\mu(\overline{\nu}_\mu)$-$e$	0.2311(77)	0.2230(77)	0.1185 (†)	124.5 (†)
e-D DIS (SLAC)	0.222(18)	0.214(18)	0.1185 (†)	124.5 (†)
Elastic $\nu_\mu(\overline{\nu}_\mu)$-$p$	0.211(33)	0.203(33)	0.1185 (†)	124.5 (†)

- Discuss possible $W^\pm$ and Z decay at the tree level



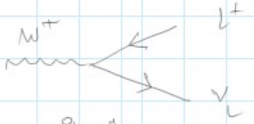
$\frac{g}{\sqrt{2}} \gamma^\mu P_L \left(U_{CKM}\right)_{ij}$

$i = 1,2,3 \quad u_i = (u, c, t)$

$j = 1,2,3 \quad d_j = (d, s, b)$

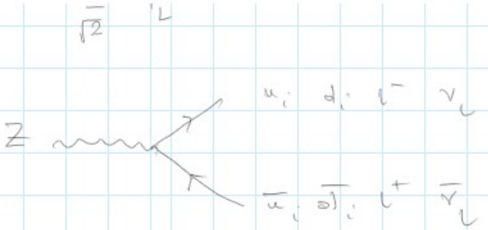
$$U_{CKM} \sim \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & 0 \\ -\lambda & 1-\frac{\lambda^2}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} + \cdots$$

$\lambda \simeq 0{,}22$



$l = e, \mu, \tau$

$\frac{g}{\sqrt{2}} \gamma^\mu P_L$



Fermionic decay of massive vector bosons

$$\hat{A}^\mu(x) = \int \frac{d^3k}{(2\pi)^3(2\omega_k)^{1/2}} \sum_{\lambda=1}^{3} \left[\hat{a}_{\bar{k},\lambda}\, \epsilon^\mu(\bar{k},\lambda)\, e^{-ikx} + \hat{a}^{+}_{\bar{k},\lambda}\, \epsilon^{\mu *}(\bar{k},\lambda)\, e^{ikx} \right]$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

E-L : $\partial_\mu F^{\mu\nu} + m^2 A^\nu = 0$

$$\partial_\mu \left(\partial^\mu A^\nu - \partial^\nu A^\mu \right) + m^2 A^\nu = 0$$

$\partial_\nu \,\big|$ $\quad \Box A^\nu - \partial^\nu (\partial_\mu A^\mu) + m^2 A^\nu = 0$ $\qquad$ Proca equation

$\hookrightarrow \quad m^2 \partial_\nu A^\nu = 0 \;\Rightarrow\; m^2 \neq 0 \;\Rightarrow\; \partial_\mu A^\mu = 0$

$$(\Box + m^2) A^\nu = 0$$

for $A^\nu \propto e^{ikx}$ $\qquad \Rightarrow -k^2 + m^2 = 0$

$$\partial_\mu A^\mu = 0 \;\Rightarrow\; k_\mu \epsilon^\mu(\bar{k},\lambda) = 0$$

$k = (\omega_k, 0, 0, k) \qquad \omega_k = (m^2 + k^2)^{1/2}$

we choose

$$\epsilon^\mu(\bar{k},1) = (0,1,0,0)$$

$$\epsilon^\mu(\bar{k},2) = (0,0,1,0) \qquad \epsilon_\mu(\bar{k},\lambda)\, \epsilon^\mu(\bar{k},\lambda') = \eta_{\lambda\lambda'}$$

$$\epsilon^\mu(\vec{k},3) = \left(\frac{k}{m}, 0, 0, \frac{\omega_k}{m}\right)$$

completeness relation : $\displaystyle\sum_{\lambda=1}^{3} \epsilon_\mu(\vec{k},\lambda)\epsilon_\nu(\vec{k},\lambda) = -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$

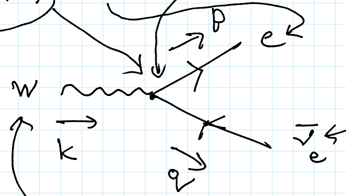
$$\mathcal{L}_{int} = -\frac{g}{2\sqrt{2}}\left(J^{\mu\dagger} W^+_\mu + J^\mu W_\mu\right)$$

$$J^\mu = \bar{\Psi}_\ell \gamma^\mu (1-\gamma_5)\Psi_{\nu_\ell} + \bar{d}\gamma^\mu(1-\gamma_5) U_{CKM}\, u =$$

$$= -\frac{g}{2\sqrt{2}} W^+_\mu \left[(\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\gamma^\mu(1-\gamma_5)\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + (\bar{u}, \bar{c}, \bar{t})\gamma^\mu(1-\gamma_5) U^\dagger_{CKM}\begin{pmatrix} d \\ s \\ b\end{pmatrix} + H.c. \right]$$

Consider $\qquad W(k) \to e(p) + \bar{\nu}_e(q)$

$$i\mathcal{M} = -i\left(\frac{-g}{2\sqrt{2}}\right)\underbrace{\bar{u}^{s_e}_e(p)}\gamma^\mu(1-\gamma_5)\underbrace{v^{s_\nu}_\nu(q)}\,\epsilon_\mu(\vec{k},\lambda)$$



Summing over spins of fermions and averaging over polarizations of W one gets :

$$\frac{1}{3}\sum_{\substack{spin\\pol.}} |\mathcal{M}|^2 = \frac{1}{3}\frac{g^2}{8}\underbrace{\sum_{\lambda=1}^{3}\epsilon_\mu(\vec{k},\lambda)\epsilon^*_\nu(k,\lambda)}_{-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}} \times$$

$$\times \sum_{s_e, s_{\nu_e}} \bar{u}^{s_e}_e(p)\gamma^\mu(1-\gamma_5)\underbrace{v^{s_{\nu_e}}_\nu(q)\cdot\bar{v}^{s_{\nu_e}}_\nu(q)}\gamma^\nu(1-\gamma_5)u^{s_e}_e(p) =$$

$\sum v^s(q)\bar{v}^s(q) = \not{q} - m$

$\sum_{S_e, S_{\nu_e}}$ $\quad \underbrace{\quad}_{= \not{q} - m_\nu}$

$$\sum_s v^s(q)\bar{v}^s(q) = \not{q} - m$$

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m$$

$$= \frac{1}{3}\frac{g^2}{8}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}\right) \mathrm{Tr}\left\{\underbrace{(\not{p}+m_e)\gamma^\mu(1-\gamma_5)(\not{q}-m_\nu)\gamma^\nu(1-\gamma_5)}\right\}$$

$m_e \simeq m_\nu \simeq 0$

$$\to \not{p}\gamma^\mu(1-\gamma_5)\not{q}\gamma^\nu(1-\gamma_5) = \not{p}\gamma^\mu\not{q}\gamma^\nu(1-\gamma_5)^2 = 2\not{p}\gamma^\mu\not{q}\gamma^\nu(1-\gamma_5)$$

$$\mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = 4(\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\sigma})$$

$$\mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5) = -4i\epsilon_{\mu\nu\rho\sigma}$$

$$= \frac{1}{3}\frac{g^2}{8}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}\right) 2\cdot 4\left(p^\mu q^\nu + p^\nu q^\mu - p\cdot q\,\eta^{\mu\nu} + \underbrace{i\epsilon^{\alpha\mu\beta\nu}p_\alpha q_\beta}_{\text{no contribution}}\right) =$$

$$= \frac{1}{3}g^2\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}\right)\left(p^\mu q^\nu + p^\nu q^\mu - p\cdot q\,\eta^{\mu\nu}\right) = \frac{g^2}{3}\Big(-2p\cdot q + 4p\cdot q +$$

$$\frac{1}{m_W^2}\left(k\cdot p\, k\cdot q\cdot 2 - k^2\, p q\right)\Big) =$$

$$k = p + q \Rightarrow m_W^2 = m_e^2 + m_\nu^2 + 2pq \simeq 2p\cdot q$$

$$k - p = q$$

$$k - q = p$$

$$\Rightarrow \quad p\cdot q = \frac{m_W^2}{2}, \quad k\cdot p = \frac{m_W^2}{2}, \quad k\cdot q = \frac{m_W^2}{2}$$

$$= \frac{g^2}{3}\left(2p\cdot q + \frac{1}{m_W^2}\underbrace{\left(\frac{m_W^2}{2}\frac{m_W^2}{\cancel{2}}\cancel{2} - m_W^2\frac{m_W^2}{2}\right)}_{0}\right) = \frac{g^2}{3}m_W^2$$

$$d\Gamma = \frac{1}{2E_p} |\mathcal{M}|^2 \, d\Phi \qquad d\phi^{(2)} = (2\pi)^4 \delta^{(4)}(k-p-q) \frac{d^3p}{(2\pi)^3 2E_p} \frac{d^3q}{(2\pi)^3 2E_q}$$

$$\int d\phi^{(2)} = \frac{1}{8\pi m_W^2} \lambda^{1/2}(m_W^2, \underset{0}{\underset{\shortparallel}{m_e^2}}, \underset{0}{\underset{\shortparallel}{m_\nu^2}})$$

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz$$

$$\Gamma_e = \int d\Gamma = \frac{1}{2\cancel{m_W}} \frac{1}{3} g^2 m_W^{\cancel{2}} \frac{1}{8\pi} \frac{1}{\cancel{m_W^2}} \underbrace{\lambda^{1/2}(m_W^2,0,0)}_{\cancel{m_W^2}} =$$

$$= \frac{g^2 m_W}{48\pi} = \frac{G_F}{\sqrt{2}} \frac{m_W^3}{6\pi} \quad , \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

$\Gamma_e \simeq \Gamma_\mu \simeq \Gamma_\tau$

colour

U_{CKM}

For hadronic decay $\quad \Gamma(W \to u_i \bar{d}_j) \cong 3\, |U_{ij}|^2 \, \Gamma_e$

$m_i = m_j \simeq 0$

$W \not\to t\bar{b}, t\bar{s}, t\bar{d}$

$$\sum_{\substack{i=1,2\\ j=1,2,3}} \Gamma(W \to u_i \bar{d}_j) = 3 \sum_{i=1,2} \underbrace{\sum_{j=1}^{3} |U_{ij}|^2}_{\overset{\shortparallel}{1}} \Gamma_e = 3 \cdot 2\, \Gamma_e = 6\, \Gamma_e$$

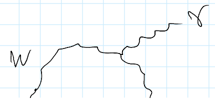
$$\Gamma_{tot} \cong 9\, \Gamma_e \qquad BR(W \to e\nu) \equiv \frac{\Gamma_e}{\Gamma_{tot}} \simeq \frac{1}{9} \simeq 11\%$$

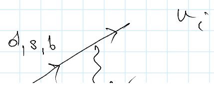
$m_W = 80{,}4$ GeV

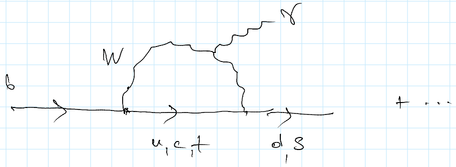
$G_F = 1{,}166 \cdot 10^{-5}$ GeV $\quad \to \quad \Gamma_e \cong 0{,}23$ GeV

$\Gamma_{tot} \cong 2{,}05$ GeV

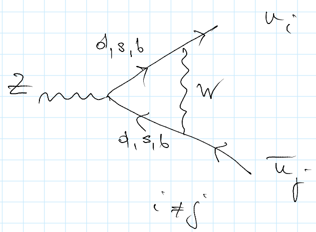
- Are FCNC allowed in the SM? If so, draw diagrams





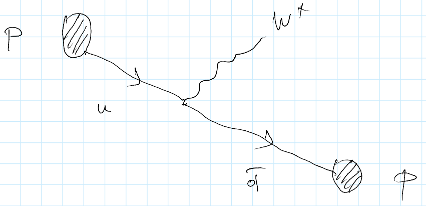


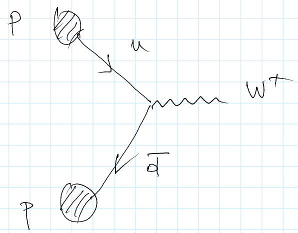
$+ \ldots$,



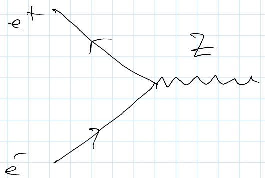
- Vector boson production

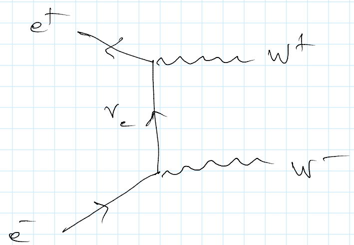
pp:

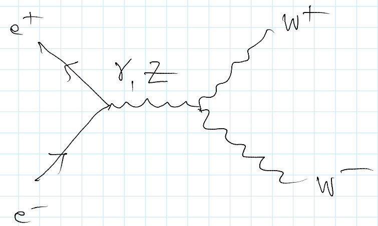


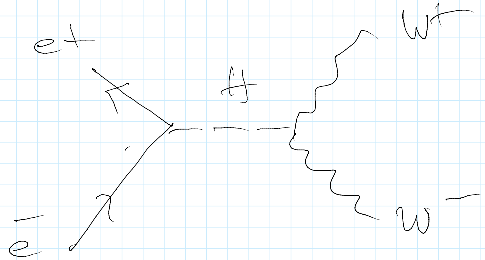


e^+e^-:









- Higgs boson production and decay

- couplings to fermions:

$$\frac{\eta(x)+v}{\sqrt{2}} \cdot \left[\Gamma^{(l)}_{lk}\, \bar\psi^L_l\, \psi^R_k + \Gamma^{(u)}_{lk}\, \bar u^L_l\, u^R_k + \Gamma^{(d)}_{lk}\, \bar d^L_l\, d^R_k + \text{h.c.} \right]$$

$$\Downarrow$$

$g_{hff}\; h \bar f f$ for $g_{hff} = \sqrt{2}\,\frac{m_f}{v}$

$h\bar{f}f$ $\quad g_{hff}$ for $g_{hff} = \sqrt{2}\,\frac{m_f}{v}$

$m_u \simeq 5$ MeV $\quad m_c \simeq 1{,}2$ GeV $\quad m_t = 175$ GeV $\quad \frac{m_{u/d}}{v} \sim \frac{5\cdot 10^{-3}\text{ GeV}}{2\cdot 10^{2}\text{ GeV}} \sim 2{,}5\cdot 10^{-5}$

$m_d \simeq 5$ MeV $\quad m_s \simeq 100$ MeV $\quad m_b \simeq 4{,}3$ GeV

$$\mathcal{L} = \frac{\eta+v}{\sqrt{2}}\left(\overline{u^L}\,\Gamma^{(u)} u^R + \overline{d^L}\,\Gamma^{(d)} d^R\right) \rightarrow -(\eta+v)\left(\bar{u}\,\frac{M_d^{(u)}}{v}\,u + \bar{d}\,\frac{M_d^{(d)}}{v}\,d\right) =$$

mass-eigenstate basis

$$M^{(u)} = -\frac{v}{\sqrt{2}}\,\Gamma^{(u)}, \quad M^{(d)} = -\frac{v}{\sqrt{2}}\,\Gamma^{(d)}$$

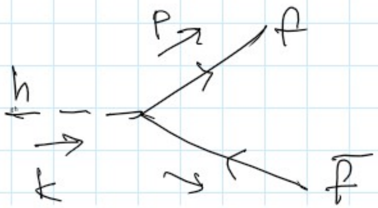
$$= -(\eta+v)\left[(\bar{u},\bar{c},\bar{t})\begin{pmatrix}\frac{m_u}{v} & & \\ & \frac{m_c}{v} & \\ & & \frac{m_t}{v}\end{pmatrix}\begin{pmatrix}u\\c\\t\end{pmatrix} + (\bar{d},\bar{s},\bar{b})\begin{pmatrix}\frac{m_d}{v} & & \\ & \frac{m_s}{v} & \\ & & \frac{m_b}{v}\end{pmatrix}\begin{pmatrix}d\\s\\b\end{pmatrix}\right]$$

$v \simeq 246$ GeV

Fermionic Higgs boson decay

$(\eta \equiv h)$ $\quad h \to f\bar{f}$

$$\frac{1}{v} = \left(G_F\sqrt{2}\right)^{1/2}$$



$$i\mathcal{M} = -i(-)\left(G_F\sqrt{2}\right)^{1/2} m_f \;\overline{u^{s_f}}(p)\, v^{s_{\bar f}}(q) \qquad k = p+q \to m_h^2 = 2m_f^2 + 2p\cdot q$$

$$p\cdot q = \frac{m_h^2}{2} - m_f^2 -$$

$$|\mathcal{M}|^2 = G_F\sqrt{2}\, m_f^2 \underbrace{\mathrm{Tr}\left\{(\not p + m_f)(\not q - m_f)\right\}}_{4(p\cdot q - m_f^2)} = G_F\sqrt{2}\, m_f^2\, 4\left(\frac{m_h^2}{2} - 2m_f^2\right) =$$

$$= \cancel{4}^{\,2}\, G_F\sqrt{2}\, m_f^2\, \frac{m_h^2}{\cancel{2}}\left(1 - \frac{4m_f^2}{m_h^2}\right)$$

$$\Gamma_f = \frac{c_f}{\cancel{2}\cancel{m_h}} \cancel{2}\sqrt{2}\, G_F\, m_f^2\, m_h^{\cancel{2}}\left(1 - \frac{4m_f^2}{m_h^2}\right) \underbrace{\int d\phi^{(2)}\Big|_{m_1 = m_2 = m_f}}_{=\ \frac{1}{8\cancel{32}\pi^{\cancel{2}}}\left(1 - \frac{4m_f^2}{m_h^2}\right)^{1/2}\cancel{4\pi}}$$

$$c_f = \begin{cases} 3 & \text{for quarks} \\ 1 & \text{leptons} \end{cases}$$

$$\Gamma_f = G_F\sqrt{2}\, c_f\, m_f^2\, m_h\, \frac{1}{8\pi}\left(1 - \frac{4m_f^2}{m_h^2}\right)^{3/2}$$

$m_h = 125$ GeV $\qquad m_b = 4{,}2$ GeV

$G_F = 1{,}166 \cdot 10^{-5}$ GeV

$\Gamma_b \simeq 4{,}32 \cdot 10^{-3}$ GeV

$\Gamma^{(exp)}$ ... 10^{-3} GeV ... Γ^{-1} ... -22

$$G_F = 1.166 \cdot 10^{-5}\ \text{GeV}$$

$$\Gamma_{tot}^{(exp)} = 4.21 \cdot 10^{-3}\ \text{GeV} \qquad \tau = \Gamma_{tot}^{-1} = 1{,}56 \cdot 10^{-22}\ \text{sec}$$

$$BR(h \to b\bar{b}) \equiv \frac{\Gamma_b}{\Gamma_{tot}}\Big|_{exp.} \simeq 56{,}1\%$$

– couplings to vector bosons

$$(D_\mu \phi)^+ (D^\mu \phi) \longrightarrow \boxed{g h \left(M_W W^+_\mu W^{-\mu} + \frac{1}{2c_W} M_Z Z_\mu Z^\mu \right)}$$

$$(D_\mu \phi)^+ (D^\mu \phi) = (D_\mu \phi')^+ (D^\mu \phi') = \left\{ \left(\partial_\mu - \frac{i}{2} g \tau_i W_{i\mu} - i g' \frac{1}{2} B_\mu \right) \left[\frac{v + \eta(x)}{\sqrt{2}} \right] \chi \right\}^+$$

$$\cdot \left\{ \left(\partial^\mu - \frac{i}{2} g \tau_i W_i^\mu - i g' \frac{1}{2} B^\mu \right) \left[\frac{v + \eta(x)}{\sqrt{2}} \right] \chi \right\}$$

$$\frac{1}{2} M_Z^2 Z_\mu Z^\mu = \frac{v^2}{8} (g W_{3\mu} - g' B_\mu)(g W^{3\mu} - g' B^\mu) = \frac{v^2}{8} (W_{3\mu}, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} =$$

$$\det(\ ) = g^2 g'^2 - (gg')^2 = 0$$

$$= \frac{1}{2} (Z_\mu, A_\mu) \begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix} \qquad M_Z^2 = \frac{v^2}{4} (g^2 + g'^2)$$

$$\operatorname{tg} \theta_W = \frac{g'}{g}$$

[illegible]

$$\tan\theta_W = \frac{g'}{g}$$

$$(D_\mu\phi)^\dagger D^\mu\phi \longrightarrow \frac{v^2}{2}\left\{ g^2\frac{1}{4} W_\mu W^\mu + g g' W_{3\mu} B^\mu \frac{1}{4}\cdot 2\cdot(-) + g'^2\frac{1}{4} B_\mu B^\mu \right\} =$$

$$= \frac{v^2}{8}\left\{ g^2\left(W_{1\mu} W_1^\mu + W_{2\mu} W^{2\mu}\right) + \left(g W_{3\mu} - g' B_\mu\right)\left(g W_3^\mu - g' B^\mu\right) \right\} =$$

$$= M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

<u>charged bosons:</u>

$$W_\mu^-(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) + i W_{2\mu}(x)\right], \quad W_\mu^+(x) = \frac{1}{\sqrt{2}}\left[W_{1\mu}(x) - i W_{2\mu}(x)\right] \Rightarrow \begin{cases} W_{1\mu} = \frac{1}{\sqrt{2}}\left(W_\mu^- + W_\mu^+\right) \\ W_{2\mu} = \frac{-i}{\sqrt{2}}\left(W_\mu^- - W_\mu^+\right) \end{cases}$$

$$W_\mu^+ W^{-\mu} = \frac{1}{2}\left(W_1\cdot W_1 - i W_1\cdot W_2 + i W_2\cdot W_1 + W_2 W_2\right) =$$

$$= \frac{1}{2}\left(W_{1\mu} W_1^\mu + W_{2\mu} W_2^\mu\right) \qquad \Longrightarrow \qquad M_W^2 = \frac{v^2}{4} g^2$$

$$\boxed{M_W^2 \Rightarrow \frac{g^2}{4}(v+h)^2} = M_W^2 + \frac{g^2}{2\cancel{4}}\cancel{2}vh + \frac{g^2}{4}h = M_W^2 + \underbrace{\frac{gv}{2}\cdot gh}_{M_W\cdot g\cdot h} + \frac{g^2}{4}h$$

$$\boxed{M_Z^2 \Rightarrow \frac{g^2+g'^2}{4}(v+h)^2} = M_Z^2 + \frac{g^2+g'^2}{4}\cdot 2vh + \frac{g^2+g'^2}{4}h^2$$

$p\bar{p}$, pp collider:

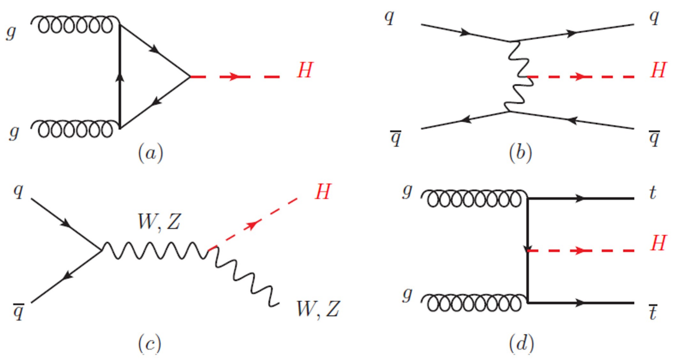
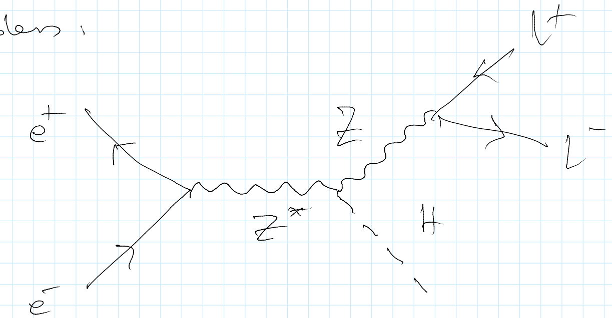
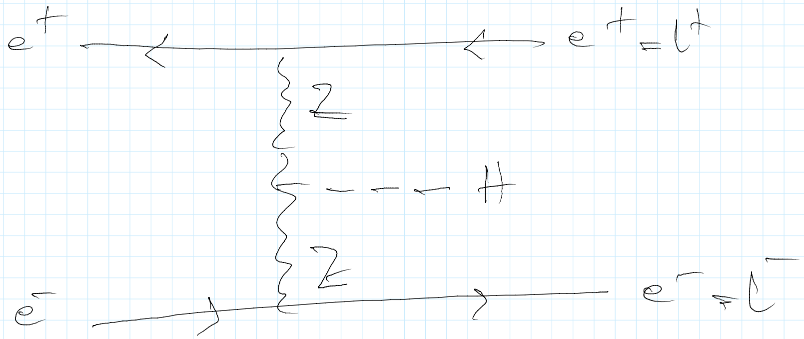
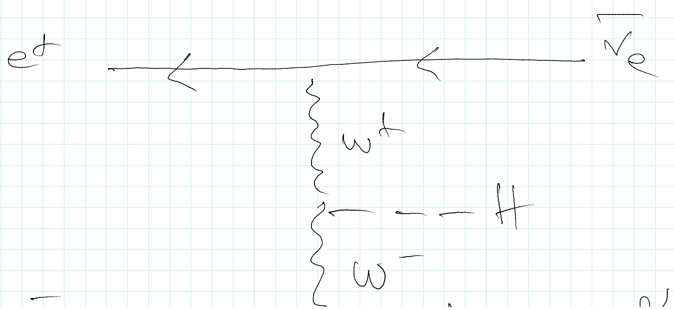


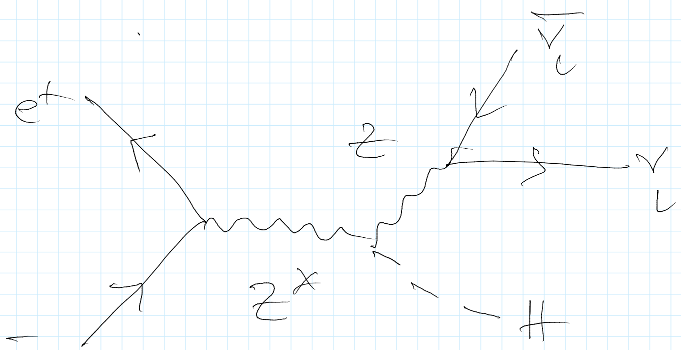
Figure 2: Generic Feynman diagrams contributing to the Higgs production in (a) gluon fusion, (b) weak-boson fusion, (c) Higgs-strahlung (or associated production with a gauge boson) and (d) associated production with top quarks.

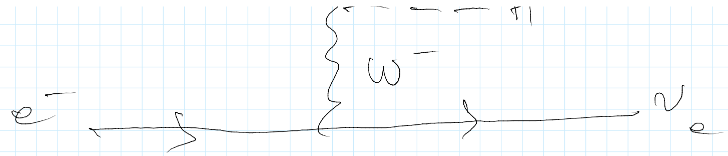
e^+e^- colliders:

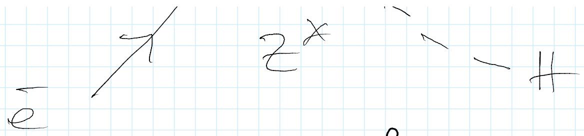




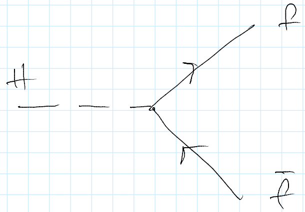




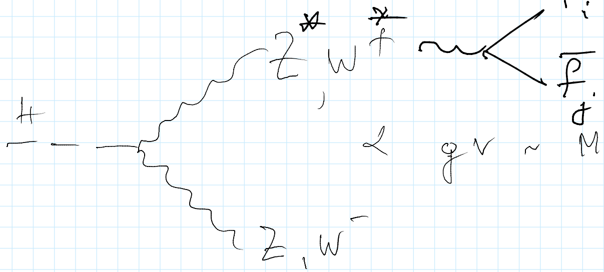




H decays



$\propto \frac{m_f}{v}$



$\propto gv \sim M_{W/Z}$