Homework problems # 1

(deadline: 07.11.21)

1. (3 pts) Using the possibility of "improving" the energy-momentum tensor
$$T^{\mu\nu} \to T^{\mu\nu} + \partial_\rho A^{\rho\mu\nu}$$
for $A^{\rho\mu\nu} = -A^{\mu\rho\nu}$ construct the symmetric energy-momentum tensor (Belinfante-Rosenfeld tensor).

2. (1 pt) Show that the replacement of a Lagrangian density $\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$ by
$$\mathcal{L}' = \mathcal{L} + \partial_\alpha \Lambda^\alpha(x)$$
does not influence the equation of motion if $\Lambda^\alpha(x)$ satisfies appropriate conditions at $x \to \pm\infty$. What are they?

3. (2 pts) Let $\phi(x)$ transforms according to a fundamental representation of $SU(2)$, that is
$$\phi(x) \to \phi'(x) = e^{-i\frac{\sigma^a}{2}\alpha_a}\phi(x) \tag{1}$$
where σ^a denotes the Pauli matrices, α_a are parameters of the transformation.
Show that the field $\tilde{\phi}(x) \equiv i\sigma_2\phi^\star(x)$ transforms also according to (1).

4. (3 pts) Derive equations of motion (EoM) for a $U(1)$ symmetric complex scalar field minimally coupled (i.e. through a covariant derivative) to a vector field. Verify current conservation.

5. (1 pt) Construct Lagrangian density for a massive vector field and derive EoM.

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