Homework problems #2

(deadline: 29.11.21)

6. (1 pt) Derive the symmetric and gauge invariant energy-momentum tensor for electrodynamics

$$T^{\alpha\beta}_{\rm em} = -F^{\alpha}{}_{\gamma}F^{\beta\gamma} + \frac{1}{4}\eta^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}$$

   and show its conservation when the Maxwell equations are satisfied.

7. (2 pt) Construct Lagrangian density for renormalizable $SU(2)$ symmetric gauge theory interacting with scalar fields in the adjoint representation.
   For $SU(N)$ group dimension of the adjoint representation is $N^2 - 1$, so in the case of $SU(2)$ the generators are 3×3 matrices satisfying the commutation relation $[T_i, T_j] = i\epsilon_{ijk}T_k$.

8. (3 pt) Derive equations of motion for the Yang-Mills theory with the gauge group SU(2) interacting with SU(2) doublet of scalar fields. Express the equations through appropriate covariant derivatives and currents.

9. (2 pt) For the same theory as above, find the symmetric energy-momentum tensor and show that it is conserved if equations of motion are satisfied.

10. (1 pt) Derive equation of motion for a massive photon, so use the following Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$$

    and show that for $m \neq 0$ the equation implies $A_{\mu}{}^{,\mu} = 0$.

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