Questions for the oral exam for ToFI

1. Adopting the principle of stationary action derive Euler-Lagrange equations for a classical field theory. Assume that there exists a Lagrangian density function $\mathcal{L}$ such that
   - $\mathcal{S} = \int d^4x \mathcal{L}(\phi(x), \partial_\mu \phi(x))$
   - $\mathcal{L}$ depends at most quadratically on $\partial_\mu \phi(x)$.
2. Formulate and derive the Noether theorem (show existence of conserved currents).
3. Derive the form of conserved Noether current in terms of symmetry transformations.
4. Find conserved quantities (Noether charges) implied by the existence of conserved currents.
5. Define the canonical energy-momentum tensor $\theta_{\mu\nu}$, discuss its uniqueness and show its conservation if the Euler-Lagrange equations are satisfied.
6. Find the energy-momentum tensor for $U(1)$ invariant scalar field and for pure electrodynamics (only vector field).
7. Discuss spontaneous symmetry breaking for $U(1)$ invariant scalar field theory.
8. Formulate and derive the Goldstone theorem.
9. Discuss the Higgs mechanism for Abelian gauge theory coupled to complex scalar field.
10. Formulate the canonical quantization of a real scalar field, derive the energy operator and introduce the normal ordering.
11. Derive the electric charge operator for $U(1)$-invariant complex scalar field theory.
12. Define the Feynman propagator $D(x)$ for a real scalar field and show that
$$D(x) = i \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ipx}}{p^2 - m^2 + i\epsilon}\,.$$

13. Discuss symmetries of the Lagrangian density for a fermion field.

14. Formulate the canonical quantization of a fermion field and derive the energy operator.

15. Formulate the canonical quantization of a $U(1)$ gauge field, discuss the gauge fixing condition and derive the energy operator.

16. For a quantum real scalar filed operator $\hat{\phi}(t, \vec{x})$ and its conjugate momentum field $\hat{\pi}(t, \vec{x})$ derive equations of motion.

17. In the interaction picture derive the Schrödinger and Heisenberg equations for quantum scalar field theory.

18. Derive the equation of motion for the evolution operator $U(t, t_0)$ and find its formal solution.

19. Formulate the perturbative solution of the equation of motion for the evolution operator $U(t, t_0)$.

20. Formulate the Wick's theorem and illustrate it for the 2-point function $T\{\phi_A(x_1)\phi_B(x_2)\}$ within the real scalar field theory.

21. Derive Feynman rules in the coordinate space for the QED.

22. Derive Feynman rules in the momentum space for the QED.

23. Discuss processes described by the second order of perturbation expansion in the QED.

24. Using the Feynman rules in the momentum space derive the amplitude for the process $e^+e^- \to \mu^+\mu^-$.

25. Derive the 2-body phase space:

$$d\phi^{(2)} = \frac{1}{32\pi^2}\left(1 - \frac{4m^2}{M^2}\right)^{1/2} d\Omega$$

for a decay of a particle of mass M into two particles of mass m.

26. Formulate the leptonic sector of the SM and determine interactions of leptons and gauge bosons.

27. Formulate the quark sector of the SM and determine interactions of quarks and gauge bosons.

28. Formulate the Yukawa interactions in the leptonic sector of the SM. Discuss the relevance of right-handed neutrinos.

29. Formulate the Yukawa interactions in the quark sector of the SM.

30. Discuss symmetry breaking in the SM.

31. Discuss the Higgs mechanism in the SM and gauge-boson mass matrix diagonalization.

32. Derive the CKM matrix.

33. Derive the number of physical phases in Yukawa matrices.

34. Discuss flavour-mixing in neutral currents in the SM, both in Yukawa couplings and in fermion-gauge interactions.

35. Describe production mechanisms of Higgs bosons in e^+e^- colliders.