Homework problems #12

1. Show that the following Lagrangian

$$\mathcal{L} = \sqrt{g} \left(\frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi \right) \tag{1}$$

is invariant under under the local scale (the Weyl transformation)

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x) = \Omega^2(x)g_{\mu\nu}(x) \quad \phi(x) \to \phi'(x) = \Omega^{-1}(x)\phi(x)$$
.

2. Find in N-dimensional space-time analog of (1) that is also invariant under the Weyl transformation.