§8. Some other rigorous results on HM

Lieb theorem (1989):

Let $\hat{H}$ the Hubbard Hamiltonian with real t_{ij} on a connected graph. Let $U < 0$ (attractive). N - number of particles even. Then the ground state is unique and has a total spin $S = 0$.

Corollary:

Let $\hat{H}$ the Hubbard Hamiltonian with real t_{ij} on a connected and bipartite graph. Let $U > 0$ (repulsive). Let $N = N_L$. Then the ground state is unique in the subspace $S^z = 0$. The total spin $S = \frac{1}{2}(N_L^A - N_L^B)$.

If $N_L^A = N_L^B$ then the ground state is a singlet.

Lieb theorem support antiferromagnetism (ferrimagnetism) like in the t-J, Heisenberg model at half-filling.

Is there a long range order? Mermin-Wagner-Hohenberg '66-'67

Koma, Tasaki theorem (1992)

For the Hubbard model in $d=1$ and 2 with finite range t_{ij}, in the thermodynamic limit ($N_L \to \infty$)

$$|\langle c_{i\uparrow}^{+} c_{i\downarrow}^{+} c_{j\downarrow} c_{j\uparrow}\rangle| \le \begin{cases} |\bar{r}_i - \bar{r}_j|^{-\alpha f(\beta)} & d=2 \\ e^{-\gamma f(\beta)|\bar{r}_i - \bar{r}_j|} & d=1 \end{cases}$$

$$|\langle \bar{S}_i \cdot \bar{S}_j\rangle| \le \begin{cases} |\bar{r}_i - \bar{r}_j|^{-\alpha f(\beta)} & d=2 \\ e^{-\gamma f(\beta)|\bar{r}_i - \bar{r}_j|} & d=1 \end{cases}$$

$\alpha > 0$, $\gamma > 0$, $f(\beta) > 0$, for large $\beta = \frac{1}{k_B T}$, $f(\beta) \sim \frac{1}{\beta}$

No LRO and ODLRO at finite T.
Kosterlitz-Thouless transition may occur.

Nagaoka theorem (1965)

The Hubbard model with non-negative t_{ij}.
$N = N_\Lambda - 1$ (one hole), $U = \infty$ $\forall i \in \Lambda$.
The ground state has a total spin $S = \frac{1}{2} N$,
is unique with the usual degeneracy $(2S+1)$
provided by a certain connectivity condition
for t_{ij}.

→ A unique FM close to AF at half filling!
→ A Nagaoka state is very unstable by changing U or
N (more holes)!