

# Advanced Graduate Quantum Mechanics

somer term 2020-21

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Tutor:

dr Ayuki Kamada, Wednesday 11:15-13:00

## Rules

Lectures will be carried out online via Zoom. Every lecture is split into 2 parts with 45 minutes. In the end of each part there will be a quick one minute test or quiz. They will count to the lecture activity points.

Tutorials will be carried out online via Zoom. There will be few problems to be solved and discussed by the tutor. Problems will be given few days ahead before meeting.

Home problems will be given but will not be checked by us. However, some of these problems might occur during a colloquium or an exam.

- Standard way of passing the course
  - Lecture activity, 10pt.
  - Tutorial activity, 10pt.
  - Colloquium, 40 pt.
  - Written exam, 40 pt.
  - Oral exam (in uncertain cases)
- Retaking to pass the course
  - Written exam, 100 pt.
  - Oral exam (in uncertain cases)

Final grade is based on total score points normalized to 100 and determined as follows:

5+ for 99-100 pt.  
5 for 90-98 pt.  
4+ for 81-89 pt.  
4 for 72-80 pt.  
3+ for 62-71 pt.  
3 for 50-61 pt.  
2 for 0-49 pt.

Warning: points from the first and second exams do not sum up.

## Permanent link for lectures:

Krzysztof Byczuk

<https://zoom.us/j/92012043483?pwd=ZkpheDdyQ0pPaDQ3WUN4OW5aL2h0Zz09>

Meeting ID: 920 1204 3483

Passcode: QdZrZ3

## Permanent links for tutorials:

Ayuki Kamada

<https://zoom.us/j/99293586777?pwd=d1ZTeFN4THV6TlAwRnF4anZXMW91dz09>

Meeting ID: 992 9358 6777

Passcode: V87TQh

## Dates of mid-term and final exams:

colloquium , online

written exam I, online

oral exam I, online

written exam II, online

oral exam II, online

## 1 Week I, 01-07/03/2020

### 1.1 Lecture

#### I. Symmetries in Quantum mechanics:

&1. *Symmetry transformation in classical mechanics* - homogeneity of space, conservation of momentum, homogeneity of time, conservation of energy, isotropy of space, conservation of angular momentum.

&2. *Symmetry transformation in quantum mechanics* - unitary symmetry operation  $\hat{U}$  acting on quantum states and wave functions, invariance of norms and average values, conservation law for an observable  $\hat{O}$  expressed by vanishing commutator  $[\hat{O}, \hat{U}] = 0$ , typical symmetry transformations in quantum mechanics: continuous ones - translations in space, translations in time, rotations in space, discrete ones - parity, time reversal; space translations  $\hat{U}_{\vec{a}} = e^{-i\frac{\vec{a}\cdot\hat{p}}{\hbar}}$ , conservation of momentum, time translations  $\hat{U}_{\tau} = e^{i\frac{H\tau}{\hbar}}$ , conservation of energy, to be continued.

### 1.2 Quizes

#### 1.2.1 Quiz 0 - Example

My name is

1. Santa Clause
2. Robin Hood
3. Krzysztof Byczuk
4. Scooby Doo

Your answer is 3.

### 1.3 Tutorial

1. *Conservation laws in homogeneous electromagnetic fields* - Derive the conservation laws corresponding to translational symmetry for a charged particle in a) a homogeneous electrical field  $\vec{E}$ , b) a homogeneous magnetic field  $\vec{B}$ .
2. *Matrix elements of spatially displaced states* - What is a relation between matrix elements of spatially displaced states?
3. *Algebraic relations for translation operators* - Find  $i(\hat{p}/\hbar)^n \hat{B}(x)$  and calculate  $\hat{U}^\dagger \hat{A}(x) \hat{U}$  with  $\hat{U} = e^{-i\frac{a\hat{p}}{\hbar}}$ . Generalize results to three dimensions.
4. Prove the previous results using  $\hat{U}b(\vec{r}) = b(\vec{r} - \vec{a})$  and its conjugated version for any function  $b(\vec{r})$ .

### 1.4 Homework problems

1. *Center of mass* - Show that conservation of momentum allows to define a center of mass in many body nonrelativistic systems.
2. *Angular momenta in different reference frames* - a) What is the connection between the angular momenta in two reference systems which are at rest relative to each other and whose origins are separated by the distance  $a$ ? b) What is the relation between the angular momenta in two inertial reference systems which move with velocity  $V$  relative to each other?

## 2 Literature

- W Greiner, B. Müller *Quantum mechanics - symmetries*.
- More to be added in the course.