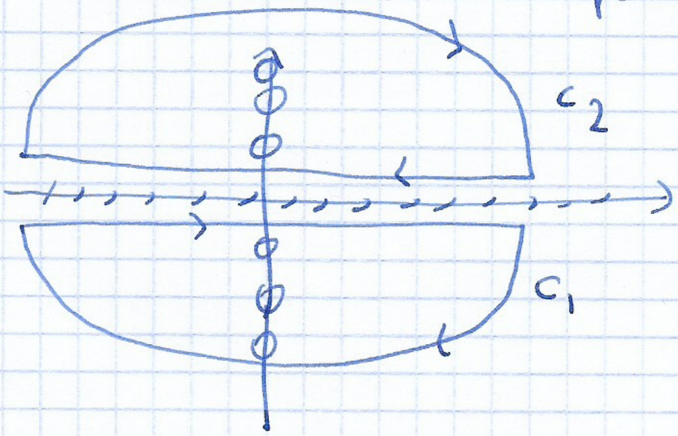
$$\boxed{\text{Matsubara sum over functions with a branch cut}}$$

let $g(z)$ has a branch cut on the real axis and is analytic otherwise



the contributions at C_∞ vanish as before

$$\boxed{S^F(\tau)} = \frac{1}{\beta}\sum_{ik_n} g(ik_n)\, e^{ik_n\tau} =$$

$$= -\int_{C_1+C_2} \frac{dz}{2\pi i}\, n_F(z)\, g(z) e^{z\tau} =$$

$$\boxed{= -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\varepsilon\, n_F(\varepsilon)\left[g(\varepsilon+i\eta) - g(\varepsilon-i\eta)\right] e^{\varepsilon\tau}}$$

with $\eta \to 0^+$

Compute $\quad S_k(\tau) = \frac{1}{\beta}\sum_{ik_n} G_k(ik_n) e^{ik_n\tau} = \quad , \tau > 0$

Green function

$$= S_k(\tau) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\varepsilon\, n_F(\varepsilon)\left[G_k(\varepsilon+i\eta) - G_k(\varepsilon-i\eta)\right] e^{\varepsilon\tau} =$$

$$= -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\varepsilon\, n_F(\varepsilon)\, 2i\left[\mathrm{Im}\, G_k(\varepsilon)\right] e^{\varepsilon\tau} =$$

$$= \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi}\, n_F(\varepsilon)\, A_k(\varepsilon)\, e^{\varepsilon\tau}$$

spectral function

$$\langle c_k^+ c_k\rangle = \lim_{\tau\to 0^-} G_k(\tau) = S_k(0^+) =$$

$A_k^0 = 2\pi\delta(\varepsilon - \tilde{\varepsilon}_k)$ non interacting

$$= \underbrace{\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\, n_F(\varepsilon) A_k(\varepsilon)}_{\text{general}} = n_F(\tilde{\varepsilon}_k)$$

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