Connected Feynman diagrams

Again perturbation $V(\vec{x}) = \frac{1}{4!}\sum_{i=1}^{n} x_i^4$

$$\frac{Z(\lambda)}{Z(0)} = 1 - \frac{1}{8}\lambda \sum_i \Delta_{ii}^2 + \frac{1}{128}\lambda^2 \sum_i \Delta_{ii}^2 \sum_j \Delta_{jj}^2 +$$

$$+ \lambda^2 \sum_{ij} \left(\frac{1}{16} \Delta_{ii}\Delta_{jj}\Delta_{ij}^2 + \frac{1}{48}\Delta_{ij}^4 \right) + O(\lambda^3)$$

We observe that up to $O(\lambda^2)$ order this

$$\frac{Z(\lambda)}{Z(0)} = e^{-\frac{1}{8}\lambda\sum_i \Delta_{ii}^2 + \lambda^2 \sum_{ij}\left(\frac{1}{16}\Delta_{ii}\Delta_{jj}\Delta_{ij}^2 + \frac{1}{48}\Delta_{ij}^4\right) + O(\lambda^3)}$$

reproduces earlier result term by term but contains only <u>connected diagrams</u>.

Indeed:

$$e^{(\dots)} = 1 + \left(-\frac{1}{8}\lambda\sum_i \Delta_{ii}^2 + \lambda^2\sum_{ij}\left(\frac{1}{16}\Delta_{ii}\Delta_{jj}\Delta_{ij}^2 + \Delta_{ij}^4\right) + \dots\right) +$$

$$+ \frac{1}{2!}\left(-\frac{1}{8}\lambda\sum_i \Delta_{ii}^2 + \lambda^2\sum_{ij}\left(\frac{1}{16}\Delta_{ii}\Delta_{jj}\Delta_{ij}^2 + \Delta_{ij}^4\right) + \dots\right)^2 + \dots =$$

$$= 1 - \frac{1}{8}\lambda\sum_i \Delta_{ii}^2 + \lambda^2\sum_{ij}\left(\frac{1}{16}\Delta_{ii}\Delta_{jj}\Delta_{ij}^2 + \Delta_{ij}^4\right) +$$

$$+ \frac{1}{\underbrace{2\cdot 64}_{128}}\lambda^2 \sum_i \Delta_{ii}^2 \sum_j \Delta_{jj}^2 + O(\lambda^3)$$

$\square$

This represents <u>linked cluster</u> <u>(cumulant) expansion</u>

$$\boxed{\ln Z(\lambda) - \ln Z(0) = -\frac{1}{8}\lambda\sum_i \Delta_{ii}^2 + \lambda^2\sum_{ij}\left(\frac{1}{16}\Delta_{ii}\Delta_{jj}\Delta_{ij}^2 + \frac{1}{48}\Delta_{ij}^4\right) + O(\lambda^3)}$$

In the logarithm the non-connected contributions are canceled at

1