Correlation functions - averages

$$\langle x_{i_1} x_{i_2} \cdots x_{i_L} \rangle = \frac{1}{Z(\lambda)} Z_{i_1 i_2 \cdots i_L}(\lambda)$$

$$Z_{i_1 i_2 \cdots i_L}(\lambda) = \int d^n x \, x_{i_1} \cdots x_{i_L} e^{-A(x,\lambda)}$$

$$A(x,\lambda) = A(x) + \lambda V(x)$$

<u>Example:</u> two-point function

$$Z_{i_1 i_2}(\lambda) = \int d^n x \, x_{i_1} x_{i_2} e^{-A(x,\lambda)}$$

when $V(x) = \frac{1}{4!} \sum_{i=1}^{n} x_i^4$ we get

$$\frac{Z_{i_1 i_2}(\lambda)}{Z(0)} = \Delta_{i_1 i_2} - \frac{1}{24} \lambda \Delta_{i_1 i_2} \sum_i \langle x_i^4 \rangle_0 - \frac{1}{2} \lambda \sum_i \Delta_{i i_1} \Delta_{ii} \Delta_{i i_2} +$$

$$+ \frac{\lambda^2}{2!(4!)^2} \sum_{ij} \Delta_{i_1 i_2} \langle x_i^4 x_j^4 \rangle_0 + \frac{\lambda^2}{2!\,4!} \sum_{ij} \Delta_{i i_1} \Delta_{ii} \Delta_{i i_2} \langle x_j^4 \rangle_0 +$$

$$+ \lambda^2 \sum_{ij} \Big(\frac{1}{4} \Delta_{i i_1} \Delta_{i i_2} \Delta_{ij}^2 \Delta_{jj} + \frac{1}{6} \Delta_{i_1 i} \Delta_{j i_2} \Delta_{ij}^3 +$$

$$+ \frac{1}{4} \Delta_{i_1 i} \Delta_{j i_2} \Delta_{ij} \Delta_{ii} \Delta_{jj} \Big) + O(\lambda^3)$$

$$\langle x_{i_1} x_{i_2} \rangle_\lambda = \frac{Z_{i_1 i_2}(\lambda)}{Z(\lambda)}$$

The non-connected terms canceled out and we get

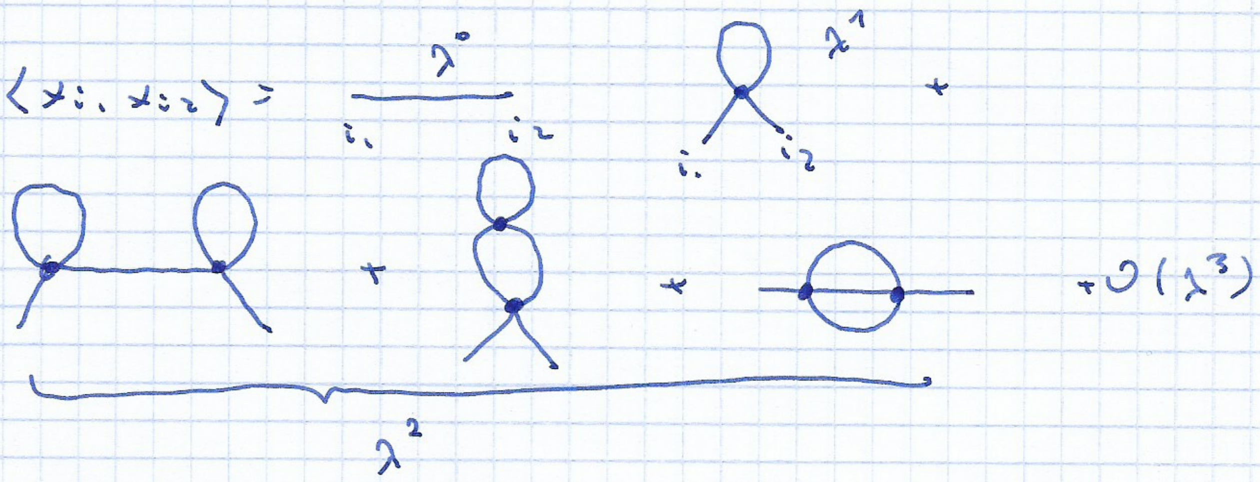
$$\langle x_{i_1} x_{i_2} \rangle = \Delta_{i_1 i_2} - \frac{1}{2} \lambda \sum_i \Delta_{i i_1} \Delta_{ii} \Delta_{i i_2} +$$

$$+ \lambda^2 \sum_{ij} \Big(\frac{1}{4} \Delta_{i_1 i} \Delta_{j i_2} \Delta_{ij} \Delta_{ii} \Delta_{jj} + \frac{1}{4} \Delta_{i i_1} \Delta_{i i_2} \Delta_{ij}^2 \Delta_{jj} +$$

$$+ \frac{1}{6} \Delta_{i_1 i} \Delta_{j i_2} \Delta_{ij}^3 \Big) + O(\lambda^3)$$

Diagramatic representation

$\langle x_{i_1} x_{i_2} \rangle =$



How to justify a factor $\frac{1}{6}$ in  diagram?

$$\frac{1}{2!\,(4!)^2} \sum_{ij} \langle x_{i_1} x_{i_2} x_i^4 x_j^4 \rangle_0$$

x_{i_1} paired on 8 ways with $x_i^4 x_j^4$, one vertex is distinguishe

x_{i_2} paired on 4 ways with remaining vertex

remaining x^3 factor is paired on $3!$ equivalent possibilities

there $\quad \frac{1}{2} \frac{1}{(4!)^2} \times 8 \times 4 \times 3! = \frac{1}{6} = \frac{1}{3!}$

number of permutation of 3 lines that join 2 vertices