

Density

$$\hat{n}(\vec{r}) = \sum_{\vec{r}_i} \delta(\vec{r} - \vec{r}_i)$$

one particle case

$$\{|\psi\rangle = |\vec{k}\rangle\}$$

momentum

$$\hat{n}(\vec{r}) = \int d^3r' \hat{\psi}^\dagger(\vec{r}') \delta(\vec{r} - \vec{r}') \hat{\psi}(\vec{r}') = \hat{\psi}^\dagger(\vec{r}) \hat{\psi}(\vec{r})$$

Fourier transform

$$\hat{n}(\vec{k}) = \int d^3r e^{-i\vec{k}\cdot\vec{r}} \sum_{\vec{k}', \vec{k}''} \langle \vec{k}' | \hat{\psi}^\dagger(\vec{r}) \hat{\psi}(\vec{r}) | \vec{k}'' \rangle a_{\vec{k}'}^\dagger a_{\vec{k}''} =$$

$$= \sum_{\vec{k}', \vec{k}''} \underbrace{\frac{1}{V} \int d^3r e^{i(\vec{k}'' - \vec{k} - \vec{k}')\cdot\vec{r}}}_{\delta(\vec{k}'' - \vec{k} - \vec{k}')} a_{\vec{k}'}^\dagger a_{\vec{k}''} = \sum_{\vec{k}'} a_{\vec{k} + \vec{k}'}^\dagger a_{\vec{k}'}$$